Homework 3

1. Which of the following sentences is a substitution instance of the sentence $P \rightarrow \neg Q$.

   (a) $Q \rightarrow \neg P$
   (b) $P \rightarrow R$
   (c) $(P \rightarrow \neg Q) \rightarrow \neg (P \rightarrow \neg Q)$.

2. Write out a full truth table for the following sentence. Highlight in some way (e.g. draw a circle around) the column under the main connective of the sentence.

   $\neg (P \lor R) \land (\neg Q \rightarrow (P \land R))$

3. Use truth tables to determine if the following arguments are valid. You do not need to show all of your work. But if an argument is not valid, give a truth-assignment that witnesses this fact.

   (a) (1) $(P \rightarrow Q) \lor (Q \rightarrow R)$
       (2) $\neg R \rightarrow \neg (P \land Q)$ // $Q \rightarrow \neg P$

   (b) (1) $(P \lor Q) \rightarrow (R \lor S)$
       (2) $P \leftrightarrow \neg (R \land S)$
       (3) $Q \leftrightarrow \neg (P \land R)$ // $(S \land P) \rightarrow \neg (P \lor R)$

4. Determine whether each of the following sentences is consistent. If a sentence is consistent, give a truth-assignment relative to which the sentence is true.

   (a) $(P \lor \neg Q) \rightarrow (P \leftrightarrow (Q \land R))$
   (b) $\neg P \lor (\neg Q \rightarrow R)) \rightarrow ((P \land R) \rightarrow \neg Q)$
5. For each of the following pairs of sentences, determine whether the first sentence implies the second. If the implication fails to hold, give a truth-assignment that witnesses this fact.

(a) \((P \& Q) \iff (Q \& R)\) \ \(P \iff Q\)

(b) \(P \iff (Q \lor R)\) \ \(\neg P \rightarrow (Q \iff R)\)

6. Write out a truth table for the exclusive sense of “or”. That is, fill in the following table:

<table>
<thead>
<tr>
<th></th>
<th></th>
<th>#</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

where \# denotes the exclusive “or” connective. Now write a sentence containing only \(P, Q, \&, \lor, \) and \(\neg\) that has the same truth table.