Homework 5

A. Show that the following arguments are valid by constructing formal proofs. You may use any of the inference rules from the first half of the semester, as well as Universal Elimination (UE) and Universal Introduction (UI).

1. \((x)(Fx \rightarrow Gx), (x)(Hx \rightarrow \neg Gx) \vdash (x)(Fx \rightarrow \neg Hx)\)
2. \((x)((Fx \lor Gx) \rightarrow Hx), (x) \neg Hx \vdash (x) \neg Fx\)
3. \((x)(Fx \rightarrow Gx) \vdash (x)Fx \rightarrow (x)Gx\)
4. \((x)(Fx \& Gx) \vdash Fm \& Gn\)
5. \(Gm \rightarrow (x)Fx \vdash (x)(Gm \rightarrow Fx)\)
6. \((x) \neg Fx \vdash \neg (x)Fx\)

B. Consider the following argument form:

\((x)(Fx \rightarrow Gx) \vdash (x)(y)(Fx \rightarrow Gy)\)

1. Write out the two sentences in English without using variables \(x\) and \(y\). For example, we would write \((\exists x)(Fx \& Gx)\) as “Something is both \(F\) and \(G\).”
2. Provide a counterexample to this argument by replacing \(F\) and \(G\) with predicates. For example, a counterexample to \((\exists x)Fx \vdash (x)Fx\) is given by setting “\(Fx \equiv x\) is a Princeton student.”