Midterm Exam Key (Version W)

A.1. ... there is a truth assignment relative to which both $A_1, \ldots, A_n$ are true and $B$ is false.

A.2. ... there is a truth assignment relative to which $A$ is true, and another truth assignment relative to which $A$ is false.

B.1. $L \rightarrow -D$

B.2. $L \& (M \rightarrow I)$

B.3. $-L \& -R$

C.1.

1. (1) $R \rightarrow -(P \rightarrow Q)$  
2. (2) $-(-Q \lor -R)$  
3. (3) $-R$  
3. (4) $-Q \lor -R$  
2,3  
2. (5) $-Q \lor -R) \& -(-Q \lor -R)$  
2. (6) $- - R$  
2. (7) $R$  
1,2  
 (8) $-(P \rightarrow Q)$  
9. (9) $Q$  
10. (10) $P$  
9. (11) $P \rightarrow Q$  
1,2,9  
 (12) $(P \rightarrow Q) \& -(P \rightarrow Q)$  
1,2  
 (13) $-Q$  
1,4  
 (14) $-Q \lor -R$  
1,2  
 (15) $-Q \lor -R) \& -(-Q \lor -R)$  
1 (16) $- - (Q \lor -R)$  
1 (17) $-Q \lor -R$  

C.2.

1. (1) $(P \rightarrow Q) \& (-P \rightarrow Q)$  
1. (2) $P \rightarrow Q$  
1. (3) $-P \rightarrow Q$  
4. (4) $-Q$  
1,4  
 (5) $-P$  
1,4  
 (6) $Q$  
1,4  
 (7) $Q \& -Q$  
1. (8) $- - Q$  
1. (9) $Q$  
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 (10) $((P \rightarrow Q) \& (-P \rightarrow Q)) \rightarrow Q$
D.1. It is invalid. Consider the truth assignment:

\[ v(P) = T, v(Q) = F, v(R) = F, v(S) = T \]

This truth assignment makes the premise true and the conclusion false.

D.2. False. For example, let \( A \) be the sentence “\( P \)” and let \( B \) be the sentence “\( Q \& \neg Q \)”. Then “\( P \rightarrow (Q \& \neg Q) \)” is contingent although “\( Q \& \neg Q \)” is not contingent.

D.3. The sentence “\( \neg(P \& Q) \& \neg(\neg P \& Q) \)” is equivalent to “\( \neg(P \leftrightarrow Q) \)”.

E.1. True. The argument with Line 1 as premise and Line n as conclusion is valid because Line 1 is an inconsistency. (There is no case where Line 1 is true, hence whenever Line 1 is true, so is Line n.) By the completeness of the propositional calculus, it follows that there is a correctly written proof of this form.