Homework 3

1. Write valid proofs of the following arguments. You are permitted to use any of the rules of inference that we have introduced, i.e., any of the rules in Chapter 1 of Lemmon’s book. (Note: you should not use truth tables for these problems.)

   (a) \[ \vdash ((Q \rightarrow P) \rightarrow (Q \rightarrow \neg P)) \rightarrow (Q \rightarrow \neg P) \]
   (b) \[ P \rightarrow (R \lor S) \vdash (P \rightarrow R) \lor (P \rightarrow S) \]

2. Write out full truth tables for the following sentences. Highlight in some way (e.g. draw a circle around) the column under the main connective of the sentence, and say whether the sentence is a contingency, a tautology, or an inconsistency.

   (a) \((P \rightarrow Q) \lor (Q \rightarrow P)\)
   (b) \(\neg (P \lor R) \land (\neg Q \rightarrow (P \land R))\)

3. Determine if the following arguments are valid. You do not need to show all of your work. But if an argument is invalid, then give a counterexample (i.e. a truth-assignment relative to which the premises are true and the conclusion is false).

   (a) \((P \rightarrow Q) \lor (Q \rightarrow R), \neg R \rightarrow \neg (P \land Q) \vdash Q \rightarrow \neg P\)
   (b) \((P \lor Q) \rightarrow (R \lor S), P \leftrightarrow \neg (R \land S), Q \leftrightarrow \neg (P \land R) \vdash (S \land P) \rightarrow \neg (P \lor R)\)
   (c) \(\vdash ((P \leftrightarrow Q) \lor (P \leftrightarrow R)) \lor (Q \leftrightarrow R)\)

4. Which of the following sentences is a substitution instance of the sentence \(P \rightarrow \neg Q\)?

   (a) \(\neg Q \rightarrow \neg P\)
   (b) \(P \rightarrow R\)
   (c) \((P \rightarrow \neg Q) \rightarrow \neg (P \rightarrow \neg Q)\).