Practice Midterm Exam

1. Complete the following sentences.
   (a) An argument with premises \(A_1, \ldots, A_n\) and conclusion \(B\) is valid if . . .
   (b) A sentence \(A\) is an inconsistency if . . .

2. Short answer: Explain, using words and/or pictures, the \(\lor\)-Elimination rule of inference, including how dependency numbers are tabulated.

3. Translate the following English sentences into sentence logic. Use the suggested letters for elementary sentences.
   (a) Fanny loves Edmund, but Edmund loves Miss Crawford. \((F, E)\)
   (b) Fanny loves Mr. Crawford only if he helps her brother. \((F, H)\)
   (c) If Fanny loves Mr. Crawford then Edmund is happy, unless Miss Crawford doesn’t love him. \((F, E, M)\)

4. Prove the validity of the following arguments. You may use any of the rules of inference that we have learned.
   (a) \(\neg P, \neg Q \vdash \neg (P \lor Q)\)
   (b) \((P \to Q) \lor (P \to R) \vdash P \to (Q \lor R)\)
   (c) \(\vdash P \leftrightarrow (P \land (Q \lor \neg Q))\)

5. Is the following sentence a tautology, an inconsistency, or a contingency? Justify your answer.
   \((\neg P \to P) \to (Q \to (R \to (S \to (T \to P))))\)

6. True or false (justify your answer): There is an inconsistent sentence of the form \(A \to B\), where \(A\) is a contingency.

7. Does sentence (a) imply sentence (b)? Justify your answer.
   (a) \((P \lor Q) \to (R \lor S)\)
   (b) \((P \to R) \lor (P \to S)\)

8. Is the English sentence connective “It is possible that . . .” truth-functional? (e.g., “It is possible that Harvard will go bankrupt in the near future.”) Justify your answer.

9. Find a sentence \(A\) containing only \(\neg, \to, P, Q\) that has the truth table below:

\[
\begin{array}{ccc}
  P & Q & A \\
  T & T & F \\
  T & F & F \\
  F & T & F \\
  F & F & T \\
\end{array}
\]
10. True or False (explain and justify your answer): There could be a correctly written proof with the following line fragments (where $n$ is some number greater than 1):

\[
\begin{align*}
1 & \quad (1) \quad (P \rightarrow Q) \rightarrow Q \quad \text{A} \\
\vdots & \\
1 & \quad (n) \quad \neg P \rightarrow Q
\end{align*}
\]