Firm Experimentation in New Markets

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Abstract

An important feature of the data on new exporters is the evolution over time of their export size and exit rate. As the exporting age of firms increases, their average export volumes grow and the exit rate decreases. In particular, we observe cases of new exporters expanding slowly initially, and switching to high levels of exports after some time. I propose a quantitative model of active learning that can explain these patterns. In the presence of demand uncertainty and high sunk costs of entry, the firm can postpone paying this cost and learn more about its demand by paying for the testing technology. This is the first model in the trade literature to allow for multiple period and varying sample size experimentation. The duration of the learning stage, the intensity of learning, and the total entry costs are determined endogenously. The model permits us to produce rich comparative statics predictions with respect to market characteristics, firm characteristics, and features of uncertainty.

1 Introduction

For a long time, the field of international trade focused on the study of aggregates - country level export and import volumes, determinants of goods traded, factor content of trade and aggregate factor prices. More recently, the trade literature has occupied itself with the firms that trade - the relationships between firm productivity, size, firm-level factor prices and trade activity. This has been mostly an investigation of long-run, static relationships. The newly available firm-level data suggests, however, that looking at the dynamics of export behavior of individual firms is worthwhile as well. Firms behave differently in different parts of their life cycle. New exporters (firms that start exporting for the first time) exhibit patterns that are unlike those revealed by older exporters, and that cannot be explained by standard models. Instead, models that are tailored to the problems that exporters face in new markets are required. The main feature of these models is the presence of demand uncertainty, and the possibility of learning from exporting activity, so that new exporters start out with little information and have to acquire it as they export. Below we will first lay out the evidence that has attracted attention in recent papers, as well as our own evidence (from French firm-level data), and briefly describe a model that we propose to explain these
findings. This model offers unique features, such as a testing technology that allows firms to learn before investing in the sunk cost of entry, and active learning by firms that recognize the information value of their own exports, and optimally choose the duration and intensity of experimentation. It allows for two stages of exporting, where the first stage describes the beginner or new exporter phase, and the second stage captures the old exporter phase of the firm, so it is a complete model of the life cycle of an exporter. We also present the anecdotal evidence we have discovered of test marketing in the domestic and foreign markets.

The first pattern found in the literature is the small initial export volumes (in quantities) of new exporters, and their later expansion over time. This has been documented by Eaton et al. (2007) for Colombia, who find that many new exporters export very small quantities and are likely to drop out of the foreign market in the following few years. This is demonstrated in Figure 1, where we employ French firm-level data (figures using the French firm-level data are borrowed from Akhmetova and Mitaritonna (2010)). This can be explained by the uncertainty that these firms face - they are not sure what the demand for their product is in the foreign market, and start out small. As they learn more about their demand, they either expand (if they find that their demand is higher than expected), or exit the market (if they find that their demand is too low). This also implies that the exit rates will tend to fall over time (although, as we will point out later, not necessarily in a monotonic fashion), as the worst exporters are weeded out gradually, and only the best ones remain. This second pattern was also observed by Eaton et al. (2007), and is reported for the French firm-level data in Figure 2. In Figures 1 and 2, we focus on average export activity as measured by the quantities exported of individual 8-digit products, by destination.

![Figure 1: Ratio of quantity (8-digit product) to average quantity of the firm over the period, averaged over all new exporters, consumer non-durable goods](image)

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Another dimension along which we can examine these dynamics, is the number of products that firms export. Using the French firm-level data, we can examine the number of 8-digit products within a given 4-digit category that firms export. In Figures 3 and 4 we show the expansion in the average (across all new exporters) number of products in a particular sector (Beauty or make-up preparations and preparations for the care of the skin (other than medicaments), including sunscreen or suntan preparations; manicure or pedicure preparations) and destination (Bulgaria). In these figures, the horizontal axis measures the date since first exporting year (1 for the first year when the firm exports), and the vertical axis measures the average number of products exported. In Figure 3, the average is over all new exporters, and in Figure 4, the average runs only over those new exporters that survived for at least 8 years. Note that we define a firm that exports a positive number of products after 8 years (with possible zeros in between) as a firm that survives for at least 8 years. We present Figure 4, to highlight the fact that the expansion in the average number of products is not (purely) due to the selection effect - less productive and smaller firms exiting in the first years, and is also driven by the expansion of surviving firms.

Notice the interesting feature of Figures 3 and 4 - while the average size grows smoothly initially, it jumps to a high level at year 8 (since first exports), and stays there. This tells us something about the learning dynamics of these firms - it looks like they are learning at first, exporting small volumes, and switch to a larger scale later on.

This evidence is consistent with the hypothesis that firms may start small in a foreign market when they are unsure of their profitability there, in order to collect more information about the market and to either expand or quit the market later, depending on their observations. We propose a theory of firm behavior that assumes demand uncertainty in a
Figure 3: Example of average size evolution, Bulgaria, Beauty or make-up preparations

Figure 4: Example of average size evolution, conditional on Survival for at least 8 years, Bulgaria, Beauty or make-up preparations
foreign market, where the firm is uncertain about the mean of the shift demand parameter that affects its profitability. There is a large sunk entry cost one needs to incur to access the entire market. While a firm may be unwilling to export to this market under such conditions, we introduce an additional assumption. We allow for a 'testing technology', so that a firm can access a few consumers in the foreign market if it pays some variable testing costs. The sales to the individual consumers serve as noisy signals about the mean demand parameter, so that the firm uses the information about these sales to update its beliefs about demand. Based on these beliefs, the firm decides whether it should incur the large sunk entry cost to access the entire market or quit, or keep experimenting. The firm can keep experimenting as long and as intensively, as it finds optimal.

Essentially, one is assuming two-tiered costs of accessing the foreign market: the variable testing costs to access a few random consumers, and the sunk costs of entry to access the entire market. One can think of the sunk cost as the cost of establishing a distribution and marketing network in the foreign market, and signing long term shipping contracts, and of the testing costs as the costs of accessing a few consumers by temporarily hiring marketing agencies in the foreign country and locating temporary shipping services. Clearly, we need to assume that the cost of accessing the entire market using the testing technology forever is higher than the sunk cost of entry, and that is quite plausible if we think of these costs the way we described above.

That this kind of experimentation takes place when a firm creates a new product and aims to sell it in the domestic market, has been documented in the marketing literature, as test marketing. As defined in the Handbook of Marketing Research (Ferber, 1974), ”test marketing is a controlled experiment, done in a limited but carefully selected part of the marketplace, whose aim is to predict the sales or profit consequences, either in absolute or in relative terms, of one or more proposed marketing actions. It is essentially the use of the marketplace as a laboratory and of a direct sales measurement which differentiates this test from other types of market research.”

While test marketing is carried out both by small and large firms, examples that get most attention and thus are known to us involve large companies. Procter and Gamble successfully test marketed Tide in October 1946 in six cities: Springfield, Massachusetts; Albany, New York; Evansville, Indiana; Lima, Ohio; Wichita, Kansas; and Sioux Falls, South Dakota. In 1985, Time Inc. announced its test marketing of the Picture Week magazine, in 13 undisclosed markets that represented 10 percent of the national population, ”the most extensive test procedure ever mounted by the corporation and perhaps even the industry” (The NYT, July 3, 1985). More recently (July 2009), Coca Cola has been reported to test market a sweetened fizzy milk beverage called Vio. About 200 retailers in New York city were reported to be carrying the drink. These stories tell us that the notion of a representative consumer can be interpreted in many different ways. A consumer can be viewed as an individual, a household, a retail store (as in the story about the drink Vio), a city (as in the story about Tide), or a state/province. Other examples are available from the author upon request.

Hence, it is clear that using the marketplace as a laboratory is certainly not an unusual
step in the domestic market. There is no doubt that the same kind of demand uncertainty exists when a firm starts exporting to a new foreign market. Even if a product has been produced and sold in the domestic and other foreign markets previously, the firm may be uncertain about the demand for this product in a new destination. The difficulty of expanding into new export markets is acknowledged by government agencies, such as the US Department of Commerce, who offer extensive assistance to businesses wishing to export. Their services include counseling on the rules and regulations in foreign markets, market research (for example, through trade fairs), and identifying potential buyers and distributors in foreign markets. We talked to one of the trade specialists in the New Jersey office of the Department of Commerce, and asked them if they ever helped or observed a firm engage in test marketing in a foreign market. The agent confirmed that it indeed takes place often, for example, US companies may start exporting in the maquiladora area in Mexico, and later expand to the rest of Mexico, if successful.

This model provides a new way of looking at the dynamics of new exporters. It points out that there may be a state of the firm in-between full-market access and non-exporting, where the firm is granted the chance to learn about demand before making a final decision. The duration of this learning stage is moreover determined endogenously - by the firm and market characteristics, and is a random variable, affected by the draws of demand signals that the firm obtains. Similarly, the total entry cost - which here would be the sum of total testing costs and the (one-time) sunk cost of entry, is endogenous and random.

The model can allow to predict the dynamics of exports by new exporters. One specific application would be the study of the response to trade liberalization - once tariffs fall, new firms will be willing to export, and how they do so can be determined within this model. This dynamics will depend on the firm and market variables, as well as features of uncertainty.

**Related literature**

We discuss the papers that are most relevant for us. A model where the decision to export is not a binary decision in the sense described above, is Arkolakis (2006). There the firm may also access a few consumers rather than the entire market, through the marketing technology, so that the fixed cost of entry is no longer ‘fixed’, but endogenously determined. That model is not concerned with learning, however, and the only way that it can produce dynamics in sales is through exogenous shocks to productivity.

Recently, several papers have focused on the role of uncertainty and learning in export markets. In one of the earliest contributions, Rauch and Watson (2003) introduce a model where a firm has to decide whether to contract a supplier in a new destination, under uncertainty about the quality of the producer there. To learn more about this, the firm first places a small order and later either expands (signs a contract with the supplier) or quits the destination. Though this is not a model of export behavior, the idea that a firm might want to start small in a new market is easily applicable to the export setting. Of course, this model lacks such elements as sampling observations over repeated periods of time (rather than over a single period), and choosing an optimal sample size (rather than a predetermined order size). In Eaton et al. (2008), there is uncertainty about the foreign
demand, and firms search for buyers and collect information on their sales. If they collect encouraging information, they expand (search for more buyers), or shrink, until finally they exit. This is a model of passive learning, in the sense that the firm does not take optimally into account the effect its sales have on its learning. Moreover, the sunk cost of entry does not play an important part here. Another paper that deals with the learning behavior of exporters is Albornoz et al. (2009). They assume a sunk cost of entry and uncertain demand in a foreign market. The firm has to incur the sunk cost of entry to export, and once it does so, it learns its demand with complete precision. Thus, learning takes place only over one period, so that this model cannot describe complex patterns of firm’s growth in a given destination over time. However, this model provides a useful way of thinking about the interaction between export behavior in several markets. That is, if the demand parameters are correlated across markets, export experience in one may result in entry into another. Our model can produce the same predictions if we incorporate correlation between the demand parameters across markets. Nguyen (2008) also introduces demand uncertainty, both in the domestic and foreign market, and correlation between the demand parameters in the two markets. The firm can learn about demand with complete precision once it sells there for at least one period. There are no sunk costs of entry into either market, but there are per period fixed costs of selling. Hence, the firm will exit a market, if the demand there is too low to cover the per period fixed cost. Therefore, there is no need for experimentation within a market, but it is possible to improve the knowledge about demand in any given market by first selling in another market. Finally, a paper that provides interesting evidence of learning by exporters is Freund and Pierola(2008). In their model, the firm is uncertain about the per period cost of exporting, which it can learn once it exports. There is a fixed cost of entry that the firm has to pay once it decides to stay in the market, but it can first export a small quantity (a fixed fraction of the potential total sales) for a smaller fixed cost (cost of trial), to learn the per period cost and make the ultimate decision to stay or not. Hence, again, there is a one-period learning with a predetermined trial sales size (which does not depend in any way on the features of uncertainty, such as the distribution of the uncertain parameter, or on the characteristics of the firm or the market).

To summarize, the previous models of learning lack one or more of the main features introduced here: a sunk cost of entry that can be postponed due to the availability of a testing technology, an endogenously determined learning phase duration and endogenously determined experimentation intensity, and active learning - the acknowledgement by the firm of the information value of its own sales, and optimal choice of sales size accordingly.

The rest of the paper proceeds as follows. We study the model and its solution, in Section 2. In Section 3, we discuss the comparative statics predictions. We contrast the model with alternative models of learning in Section 4. Section 5 concludes.

2 The Model

There are two countries, one Home, another Foreign. There are $M$ consumers in the Foreign country, where $M \in Z^+$. For any foreign consumer $i$,
\[ U_{it} = \left[ \int_{\omega \in \Omega} (e^{\mu(\omega)})^{\frac{1}{\epsilon}} c_{it}(\omega) \frac{1}{1-\epsilon} \, d\omega \right]^{1-\epsilon}, \]

where \( \omega \) denotes varieties, and \( \Omega \) the entire set of varieties in the economy. \( \mu(\omega) = \tilde{\mu} \) with probability \( p_0 \), and \( \mu \) with probability \( 1 - p_0 \). Once \( \mu(\omega) \) is drawn for a given variety, it is fixed over time.

Hence,

\[ c_{it}(\omega) = C_{it} e^{\mu(\omega)} \frac{p_t(\omega)}{P_{it}^\epsilon} \]

\[ = e^{\mu(\omega)} y_t(p_t(\omega))^{-\epsilon} P_{it}^{\epsilon - 1}, \]

where \( p_t(\omega) \) is the price of variety \( \omega \), \( P_{it} \) is the ideal price index for the differentiated good for consumer \( i \),

\[ P_{it} = \left[ \int_{\omega \in \Omega} e^{\mu(\omega)} p_t(\omega)^{1-\epsilon} \, d\omega \right]^{\frac{1}{1-\epsilon}}, \]

\( y_t \) is the total income of consumer \( i \), assumed equal across consumers, and \( C_{it} \) is the total consumption of the differentiated good (so that \( C_{it} P_{it} = y_t \)).

Labor is the only factor of production, with the constant marginal cost of producing any variety of the differentiated good given by the ratio of wages, \( w_t \), and productivity, \( \phi_t \). Each firm produces one variety. Hence, the price of any variety is given by

\[ p_t(\omega) = \frac{\epsilon}{\epsilon - 1} \frac{w_t}{\phi_t(\omega)}, \]

where \( \phi(\omega) \) is the productivity of the firm that produces the variety \( \omega \).

We can further calculate the steady state value of the ideal price index for any consumer \( i \):

\[ P_{it} = \left[ \int_{\omega} e^{\mu(\omega)} p_t(\omega)^{1-\epsilon} \, d\omega \right]^{\frac{1}{1-\epsilon}}, \]

\[ = \int_{\phi} \int_{\omega} e^{\mu(\omega)} \left[ \frac{\epsilon}{\epsilon - 1} \frac{w_t}{\phi_t(\omega)} \right]^{1-\epsilon} f(\phi, \omega) d\phi d\omega \]

\[ = \left[ \int_{\phi} \int_{\omega} e^{\mu(\omega)} \frac{\epsilon}{\epsilon - 1} \frac{w_t}{\phi_t(\omega)} \right]^{1-\epsilon} f(\phi, \omega) d\phi d\omega \]

where \( f(\phi, \omega) \) is the steady state joint distribution of \( \phi \) and \( \omega \). We have to specify this because \( \phi \) and \( \omega \) are not independent in the steady state - more low-productivity firms (relative to higher-productivity ones) will not produce the varieties with \( \mu(\omega) = \tilde{\mu} \). In the steady state, the ideal price index is the same for all consumers, even though their variety-specific preferences may be different.

So sales to any consumer \( i \) at time \( t \) are given by:

\[ c_{it}(\omega) = e^{\mu(\omega)} y(p(\omega))^{-\epsilon} P_{it}^{\epsilon - 1}, \]
where all the variables take on their steady state values.

Now drop the $\omega$ notation, and denote the variety produced by firm $j$ by $j$. Operational profits per period of the firm, earned from sales to an individual consumer, can be calculated as

$$\pi_{ijt} = p_{jt} c_{ijt} - w_t \frac{c_{ijt}}{\phi_{jt}}$$

$$= \left(\frac{\epsilon}{\epsilon - 1}\right)c_{ijt} \frac{w_t}{\phi_{jt}}$$

$$= \frac{1}{\epsilon - 1} \frac{w_t}{\phi_{jt}} e^{\mu_j} y_t (p_{jt})^{-\epsilon} P_{it}^{-1}$$

$$= e^{\mu_j} y_t \frac{1}{\epsilon - 1} \left[\frac{\epsilon}{\epsilon - 1}\right]^{-\epsilon} \frac{\phi_{jt}}{w_t} e^{-\epsilon P_{it}}$$

$$= e^{\mu_j} y_t \frac{1}{\epsilon - 1} \left[\frac{\epsilon}{\epsilon - 1}\right]^{-\epsilon} \frac{\phi_{jt}}{w_t} e^{-\epsilon p_{it}}$$

where again all the variables take on their steady state values.

The expected discounted lifetime profits from selling to consumer $i$ are given by:

$$E\pi_{ij} = E\left[\int e^{-rt} e^{\mu_j} y_t \frac{1}{\epsilon - 1} \left[\frac{\epsilon}{\epsilon - 1}\right]^{-\epsilon} \frac{\phi_{jt}}{w_t} e^{-\epsilon P_{it}} dt\right]$$

$$= AG \phi_{jt}^{-1} \int e^{-rt} E[e^{\mu_j}] dt$$

$$= AG \phi_{jt}^{-1} \int e^{-rt} E[e^{\mu_j}] dt$$

$$= AG \phi_{jt}^{-1} \int e^{-rt} [pe^{\bar{\mu}} + (1 - p)e^{\mu}] dt$$

$$= AG \phi_{jt}^{-1} \frac{1}{r} [pe^{\bar{\mu}} + (1 - p)e^{\mu}],$$

where $r$ is the discount (interest) rate, $AG \equiv y \frac{1}{\epsilon - 1} \left[\frac{\epsilon}{\epsilon - 1}\right]^{-\epsilon} \left[\frac{\epsilon}{\epsilon - 1}\right]^{-\epsilon} \frac{\phi_{jt}}{w_t} e^{-\epsilon P_{it}}$, the aggregate demand variables, and $p$ is the belief on the part of the firm that $\mu(\omega) = \bar{\mu}$.

The firm has to choose the optimal number of consumers to target subject to the following cost structure: there are two distribution and marketing technologies available. The first technology has zero or negligible sunk costs, but the cost of selling to $n$ consumers is convex in $n$: $c(n)$ is continuous, strictly increasing and convex. The second technology is linear in $n$, but to use this technology, the firm has to pay the fixed cost $F$, which could reflect expenditures on building own distribution and retail centers, and hiring long-term marketing agencies. Thus, the firm will find it optimal to eventually pay the fixed cost $F$ and utilize the linear technology. To derive the optimal behavior of the firm, consider first the learning of the firm.

The firm does not observe the precise values of quantities sold, i.e. it does not observe the precise value of $\mu$. Instead, it observes the following for an individual consumer indexed by $i$: 
\[ X_{ijt} \equiv \int_0^t \ln\left( \frac{c_{ijt}}{y(p_j)^{-e}P_{e-1}} \right) ds + \sigma_x W_{ijt} = \int_0^t \mu_j ds + \sigma_x W_{ijt}, \]

so that

\[ dX_{ijt} = \mu_j dt + \sigma_x dW_{ijt}, \]

where \( W_{ijt} \) is a Wiener process, and the firm can update its beliefs about \( \mu_j \) from these observations. More precisely, denote by \( p_{jt} \) the (subjective) probability at time \( t \) that firm \( j \)'s variety has high demand, \( \mu_j = \bar{\mu} \). Upon sampling \( n_{jt} \) consumers and observing \( n_{jt} \) values of \( dX_{ijt} \), and the sample average \( \overline{dX}_{jt} \equiv \frac{\sum_{i=1}^{n_{jt}} dX_{ijt}}{n_{jt}} \), the firm updates its beliefs according to:

\[ dp_{jt} = p_{jt}(1 - p_{jt}) \frac{\bar{\mu} - \mu}{\sigma_x} \sqrt{n_{jt}} dW_{jt}, \]

where \( \frac{\bar{\mu} - \mu}{\sigma_x} \equiv \chi \) is the signal-to-noise ratio, and

\[ dW_{jt} \equiv \frac{\sqrt{n_{jt}}}{\sigma_x} \left[ \frac{\sum_{i=1}^{n_{jt}} dX_{ijt}}{n_{jt}} - (p_{jt}\bar{\mu} + (1 - p_{jt})\mu) dt \right] \]

\[ \equiv \frac{\sqrt{n_{jt}}}{\sigma_x} \left[ \overline{dX}_{jt} - (p_{jt}\bar{\mu} + (1 - p_{jt})\mu) dt \right] \]

which is an observation-adapted Wiener innovation process, that is, it follows a standard Wiener process relative to the information at time \( t \). How these assumptions can be intuitively translated into what kind of discrete-time data the firm (and the researcher) observe, is explained in the Appendix.

As can be seen from the equation above, when the firm observes a sample average \( \overline{dX}_{jt} \) higher than the expected value at time \( t \), \( p_{jt}\bar{\mu} + (1 - p_{jt})\mu \), it updates its beliefs upwards \( (dp_{jt} > 0) \), and downwards otherwise. The firm weighs this signal (the difference between observed average and expected average) by the signal-to-noise ratio, \( \chi \), and by the sample size \( n_{jt} \). The higher the signal-to-noise ratio, and the higher the sample size, the more weight the firm puts on the new signal. So one can see that the firm would like to sample as many observations as possible, so as to learn faster, subject to the cost constraints.

Consider now the optimal behavior of the firm once it pays the sunk cost \( F \) and accesses the linear technology, \( \tilde{c}(n) = fn \). Denote this stage as stage 2, and the stage before paying the sunk cost \( F \) as stage 1. In the absence of learning, the firm would sell to the maximum number \( M \) of consumers, as long as the expected profits covered the total costs, \( Mf \), i.e. as long as \( AG_{0}^{\frac{e-1}{e}}[pe^{\tilde{\mu}} + (1 - p)e^{\tilde{\mu}}] \geq f \). This would give us the threshold value of \( p_{jt} \), below which the firm would quit the market (sell to 0 consumers). However, there is also information value to selling to a non-zero number of consumers, so that the value function of the firm \( j \) is given by:
\[ V(p_{jt}) = \max_{n_{jt}>0} E \left[ \int_0^\infty (-fn_{js} + n_{js}AG\phi_j^{-1} [p_{js}e^\mu + (1 - p_{js})e^{\bar{\mu}}])e^{-rs} ds | p_{jt} \right], \]

subject to \( dp_{jt} = p_{jt}(1 - p_{jt}) \frac{\bar{\mu} - \mu}{\sigma_x} \sqrt{n_{jt}} dW_{jt} \), where \( dW_{jt} \) is as above. Note that we do not take into account the possibility that the firm may choose to re-enter the market in the future, since in the model the aggregate variables and firm productivity are constant over time, and once the firm quits it does not get any new signals, and hence \( p_j \) also does not change. So once the firm quits, it stays out of the market. Of course, this is not a very realistic result, but it is easy to add the random shocks to aggregate variables and/or productivity to the model to match the observed re-entry rates in the data. However, our focus right now is on the optimal learning strategy of the firm, and how learning affects the value function, and not the effect of exogenous shocks on the behavior of the firm.

The Hamilton-Jacobi-Bellman equation is as follows (we omit the subscripts below):

\[ rv(p) = \max_{0 \leq n \leq M} \left[ n(-f + AG\phi_j^{-1} [p e^\mu + (1 - p)e^{\bar{\mu}}]) + M \frac{1}{2} (p(1 - p) \frac{\bar{\mu} - \mu}{\sigma_x})^2 v''(p) \right]. \]

It is clear that the stage-2-value function is linear in \( n \), so that the optimal size in stage 2 is \( n^*_1 = M \), as long as the value function is positive. To find the value function \( v(p) \) for this case, plug in \( n = M \) into the HJB equation:

\[ rv(p) = M(-f + AG\phi_j^{-1} [p e^\mu + (1 - p)e^{\bar{\mu}}]) + M \frac{1}{2} (p(1 - p) \frac{\bar{\mu} - \mu}{\sigma_x})^2 v''(p). \]

We need to solve this second-order non-linear ODE, subject to the value matching and smooth pasting conditions:

\[ v(p) = 0, \]
\[ v'(p) = 0, \]

and derive the threshold value \( p_\approx \), below which the firm will quit the market. For any beliefs \( p_{jt} \) above this value the firm will sell to the maximum number of consumers, \( M \). Notice that the value function with \( v''(p) = 0 \) satisfies the ODE above, which gives us:

\[ v(p) = \frac{M}{r} (-f + AG(\phi_j)^{-1} [p e^\mu + (1 - p)e^{\bar{\mu}}]). \]

Intuitively, the only value of learning to the firm comes from the effect of \( p_{jt} \) on the decision to quit the market, where quitting yields a payoff of 0. If we introduced exogenous shocks to the aggregate demand variables or productivity, the firm would possibly want to re-enter the market after quitting, in the case of favorable shocks, so, with the sunk cost of entry \( F \), there would be value to learning resulting from this potential re-entry (and having to pay \( F \) again) in the future. However, for simplicity here we do not allow exogenous shocks, and therefore re-entry. Hence, the value function in the second stage is simply the sum of all
discounted profits net of fixed costs of exporting. Denote this value function by \( \tilde{V}(p) \). The threshold value of \( p_{jt} \), below which the firm would quit the market (sell to 0 consumers) is given by:

\[
p_{j} = \frac{f}{AG\phi^{t+1} T} - \epsilon^{\mu} \frac{1}{e^{\mu} - e^{\mu} - \epsilon^{\mu}}.
\]

Now consider stage 1, when the firm employs the convex costs technology, before paying the sunk cost \( F \). The firm gains profits from selling to \( n \) consumers, and has to pay the costs \( c(n) \), but also gains information value from updating its beliefs and possibly investing in the linear technology in the future as a result. The function \( c(n) \) is twice differentiable on \( (0, M) \), and increasing and strictly convex on \( [0, M] \). We can assume that \( c(0) > 0 \), or that \( c(0) = 0 \), depending on what empirical data we want to match. If there is a fixed cost of maintaining the ability to experiment (think of this for example as the necessity of keeping contracts with marketing agencies), then, given a constant payoff structure over time, whenever it is optimal for a firm to sample zero observations \( (n = 0) \), it is optimal for it to stop experimenting and make a terminal decision (quit the market). This can be proved in a manner similar to Morgan and Manning (1985) (Proposition 2).

**Solution of the problem of Stage 1**

This part of the problem is close to the problem of Moscarini and Smith (2001). One way in which our model is different from that of Moscarini and Smith’s is that the testing phase in our case actually represents productive activity by the firm, which produces the product and ships it to the sample of consumers, even if not to the entire market. Hence, the firm earns profits in the experimentation phase, and these affect the optimal sample size \( n \).

The value function of firm \( j \) takes the form

\[
V(p_{jt}) = \max_{T, <n_{js}>} E[e^{-rT} K(p_T)] + \int_0^T (-c(n_{js}) + n_{js} AG\phi^{t-1} [pe^{\bar{\mu}} + (1 - p)e^{\mu}]) e^{-rs} ds | p_{jt},
\]

where \( K(p) = \max[\tilde{V}(p), 0] \) is the payoff to making the terminal decision, \( \tilde{V}(p) \) is the value function derived for stage 2 above, \( T \) is the stopping time, and \( p_T \) is the value of the belief variable at time \( T \). Then the Hamilton-Jacobi-Bellman equation (HJB) becomes

\[
rv(p) = \max_{0 \leq n \leq M} \left[ -c(n) + n AG\phi^{t-1} [pe^{\bar{\mu}} + (1 - p)e^{\mu}] \right] + \frac{1}{2} (p(1 - p) \frac{\bar{\mu} - \mu}{\sigma_x})^2 n v''(p).
\]

The FOC for the HJB equation give us \( n(p) = z(rv(p)) \), where \( z \equiv g^{-1}, g(n) = nc'(n) - c(n) \), and \( z \) is strictly increasing. The problem can be transformed into a two-point free boundary value problem.
\[ v''(p) = \frac{c'(z(rv(p))) - AG\phi_j^{\epsilon-1}[pe^\mu + (1 - p)e^\mu]}{1/2(p(1 - p)\frac{\mu - \mu^*}{\sigma_x})^2} \]

plus the value matching condition:

\[ v(\bar{p}) = \bar{V}(\bar{p}) - F, v(p) = 0. \]

and smooth pasting condition:

\[ v'(\bar{p}) = \bar{V}'(p), v'(p) = 0. \]

Since the value of experimentation \( v(p) \) should be convex, we require

\[ c'(z(rv(p))) > AG\phi_j^{\epsilon-1}[pe^\mu + (1 - p)e^\mu], \]

for all \( p \).

Intuitively, marginal cost of experimentation is the marginal testing cost net of the marginal profits earned from the sample sales:

\[ MC = c'(n) - AG\phi_j^{\epsilon-1}[pe^\mu + (1 - p)e^\mu] \]

and the marginal benefit of experimentation is the contribution of the sample observations to the updating of beliefs and to the value function as a result:

\[ MB = \frac{1}{2}(p(1 - p)\frac{\mu - \mu^*}{\sigma_x})^2 v'' > 0, \]

so for an optimal \( n \), where MC and MB are equated, MC should be positive. It can be shown that if

\[ c'(z(0)) > AG\phi_j^{\epsilon-1}pe^\mu \geq AG\phi_j^{\epsilon-1}[pe^\mu + (1 - p)e^\mu], \]

then \( v(p) \geq 0 \) and hence

\[ c'(z(rv(p))) > AG\phi_j^{\epsilon-1}[pe^\mu + (1 - p)e^\mu], \]

for all \( p \). Moreover, under this assumption we can extend all the proofs of existence and uniqueness of a solution to the free boundary value problem in Moscarini and Smith (2001) (see Appendix).
The blue line shows the ultimate payoff function, \( \pi \equiv \max[0, \hat{V}(\hat{p}) - F] \), and the green line shows the value function. The value \( P_0 \equiv \bar{p} \), where the value function is tangent to the zero-line (payoff from quitting), determines the cutoff for quitting, and the value \( P_1 \equiv \hat{p} \), where the value function is tangent to the profit line after entry, determines the cutoff for entering the market (paying sunk cost \( F \))
Export participation condition

We consider only a partial equilibrium in this model, taking the domestic side of the economy as given. The firms that start exporting are assumed to have been producing and selling domestically for some time, and therefore they know their productivity. What matters when they make the decision to export, is the prevailing common belief about the distribution of $\mu$ in the foreign market, which is assumed to be the objective probability $p_0 = \text{Prob}(\mu = \bar{\mu})$. To find the cutoff for exporting, we need to know how productivity affects the exporting/experimentation behavior. It can be shown that as $\phi$ rises, both the threshold for switching to stage 2, $\bar{p}$, and the threshold for quitting the market, $\tilde{p}$, fall. The firm will be willing to switch to the linear technology (that requires paying the sunk cost) for lower values of $p$, and will be willing to keep experimenting for lower values of $p$ than before. The optimal experimentation intensity $n(p)$ will also shift up, so that the firm will target more consumers for any given $p$. This is shown in Figure 6.

Thus, there is a monotonic ranking of cutoff thresholds $\bar{p}$ and $\tilde{p}$ over the productivity range. Given the common belief $p_0$, the lowest productivity exporter will have $p = p_0$. Denote this productivity level as $\phi$. All firms with productivity below this cutoff will have $p$ higher than $p_0$, that is it will not be worthwhile for them to start exporting, even as experimenters. Thus, we have

$$\tilde{p}(\phi) = p_0.$$

Notice that this export participation condition is very different from the one we usually work with, where the expected lifetime discounted export profits of the lowest productivity exporter are just high enough to cover the sunk entry cost $F$. Here we cannot apply this rule, since the value of exporting is no longer given by expected lifetime profits for the lowest productivity exporter, but by the total value of exporting - which also includes the information value of export sales. Therefore, we must find the firm whose expected value is exactly zero at the initial belief $p_0$, which would be the firm whose $p$ is $p_0$.

Similarly, it is not possible to tell whether a firm will forego experimenting or not by simply comparing its expected lifetime discounted profits with the sunk entry cost $F$. In fact, the firm that satisfies the equality between its expected lifetime discounted export profits with the sunk entry cost $F$ ($\frac{M}{r} AG\phi_j^{t-1}[p_0 e^{\bar{\mu}} + (1-p_0)e^{\mu}] = F$) will choose to experiment first, since its $\bar{p}$ will be above $p_0$. Even though its expected profits from entry exceed $F$, it chooses to experiment first because of the information value it gains. As productivity increases, one of the two things can happen. Either the profit line for second stage (discounted expected lifetime profits from full-scale exporting) will cross the origin, and all firms with productivity above this level of productivity, denote it as $\phi_1$ (where $\frac{M}{r} AG\phi_1^{t-1}e^{\mu} = F$) will choose to enter right away, or for some $\phi < \phi_1$, the threshold for entering $\bar{p}$ will be just equal to $p_0$, and all firms with productivity at or above this level, call it $\phi_2$, will enter right away. So, we get

$$\frac{M}{r} AG\phi_1^{t-1}e^{\mu} = F,$$

$$\bar{p}(\phi_2) = p_0.$$
Figure 6: Comparing the solutions to the problem for two levels of productivity

The blue line shows the ultimate payoff function, $\pi \equiv \max[0, \bar{V}(\bar{p}) - F]$, and the green line shows the value function. The dotted blue line shows the new ultimate payoff function, for higher productivity, and the dotted green line shows the new value function. The value function shifts up, and the thresholds fall - the thresholds for entering is lower, and the threshold for quitting is lower for the higher productivity firm.
and the productivity of the highest productivity experimenter is given by

\[
\bar{\phi} = \min[\phi_1, \phi_2].
\]

We show the two firms - the lowest productivity exporter, and the highest productivity experimenter in Figure 7.

So, we can conclude that in the presence of a testing technology, the cutoffs for experimenting and entering right away compare as follows with the cutoff for exporting in a standard Melitz-type model: \( \phi < \tilde{\phi} < \bar{\phi} \), where \( \phi \) is the cutoff for exporting (by experimenting first), and \( \bar{\phi} \) is the cutoff for entering right away in the experimentation model, and \( \tilde{\phi} \) is the cutoff for exporting in the standard model.

### 3 Comparative statics predictions

We now consider the comparative statics with respect to the main parameters.

- As \( \phi \) rises, both the threshold for switching to stage 2, \( \bar{p} \), and the threshold for quitting the market, \( p \), fall. The firm will be willing to switch to the linear technology (that requires paying the sunk cost) for lower values of \( p \), and will be willing to keep experimenting for lower values of \( p \) than before. The optimal experimentation intensity \( n(p) \) will also shift up, so that the firm will target more consumers for any given \( p \).

- The same holds for higher \( AG \), i.e. all aggregate variables. As \( y \) and \( P \) increase, or \( w \) falls, the terminal payoff function and value function shift up. Both the threshold for switching to stage 2, \( \bar{p} \), and the threshold for quitting the market, \( p \), fall. The optimal intensity schedule \( n(p) \) shifts up.

- As \( \sigma_x \) falls, so that the signal-to-noise ratio increases, the value function for all \( p \) shifts up, so that the optimal experimentation level \( n \) rises for all \( p \), and the thresholds \( \bar{p} \) and \( p \) shift out. Thus, in a market where the noise in observations for any given individual consumer is less dispersed, a given firm will experiment more intensively, and will require a better observed history of sales growth rates to enter, or a worse history - to quit. That is, experimentation will become more valuable to the firm, since any given amount of experimentation effort now results in more belief updating (remember the expression for the Marginal Benefit of experimentation).

- Similarly, as the testing cost function \( c(n) \) grows less convex and initially (for \( n \) close to 0) weakly lower and less steep, the value function \( v(p) \) rises for all \( p \), so that optimal \( n \) rises for all \( p \), and the thresholds shift out. The intuition is the same as above.

- As \( F \) rises, so that the final payoff to entry falls, the firm will experiment less intensively, for any given value of beliefs, and the threshold for entering the market will increase, that is, the firm will require a more successful history of sales, ceteris paribus, to enter the market. The firm will also quit the market for higher values of \( p_t \), than otherwise,
Figure 7: The cutoffs for exporting (experimenting) and entering right away

The blue curve shows the value function for the lowest productivity exporter, and the green line shows the terminal payoff function for this exporter. Since the threshold for quitting $p$ for this firm is exactly at $p_0$, this firm is just indifferent between exporting (through experimenting first) and not exporting. The red curve shows the value function for the highest productivity experimenter, and the dotted green line - the terminal payoff function for this firm. Since its threshold for entering the market is exactly $p_0$, it is indifferent between experimenting and entering at the initial belief $p_0$. All firms with productivity in-between these two levels of $\phi$ will experiment, and all firms with productivity above the higher level of $\phi$ will enter right away.
since its expected profits from experimentation (which is directly related to the payoff of entry) is now lower.

We consider the implications of these comparative statics results for the observable features of new exporters’ behavior - export size, export volatility (over time), and experimentation phase duration. These results tell us how various characteristics of the firms and the market will affect the size dynamics of new exporters, their export sales volatility, and duration of experimentation phase. The volatility of exports is affected through two channels - the relative length of the two stages (experimentation, where volatility is high, and full-scale, where volatility is zero in the model), and the experimentation intensity. The higher is \( n(p) \), the larger is the variance \( Var(dp) \), that is beliefs change much more with more observations sampled, and hence, the larger is the change in \( n \) in the next period.

Notice that the duration of the experimentation phase becomes endogenous in this model - the experimentation phase is not only multi-period, unlike in other models of learning, but its duration is random, firm-specific, and can be determined in expected value by the productivity of the exporter, the size of the market, the magnitude of the sunk cost, and testing costs, and the signal-to-noise ratio in the market. These effects are also not straightforward and should be determined numerically for each set of values of the parameters, since the duration of the experimentation phase depends on two things: the thresholds for quitting and entering the market, and the intensity of experimentation \( (n(p)) \). The higher the intensity of experimentation \( n(p) \), the faster beliefs \( p \) attain the thresholds for quitting or entering.

We have already discussed the implications of the comparative statics results with respect to productivity for the cutoffs for exporting above. Consider now the implications for volatility. We should see firms with higher productivity that start exporting full-scale right away, and whose export volatility is low. There will be firms with lower productivity that will not start with full-scale exports, and will export small volumes, and exhibit high volatility initially. Some of these, however, will later switch to higher export volumes (full scale), and become more stable. The higher the productivity of these firms, the more likely they are to do so, and the sooner they will do so. We show a result from the empirical paper by Akhmetova and Poncet (2009) in Table 1. In the regression, the dependent variable is the standard deviation of growth rates of quantity exported by 8-digit product and destination for all exporting firms in the manufacturing sector. The sample period is 1995-2005. We define a dummy for a volatile exporter, where a volatile exporter is defined as a new exporter whose standard deviation of growth of quantity exported is higher than the standard deviation of the mean growth rate over the relevant product and destination. The result shows that more productive exporters on average are less volatile (the coefficient on productivity is negative), however, conditional on being a volatile exporter (which in our model means the firm is an experimenter), higher TFP translates into higher volatility (the coefficient on the interaction between TFP and the volatile exporter dummy is positive), since, as predicted in the model, higher productivity implies higher experimentation intensity \( n(p) \) and higher volatility for experimenters.

Second, the market size \( y \), the toughness of the market as measured by \( P \), and any other variables that affect average profitability in the market (for example, tariff rates) will affect
### Table 1: Volatility as depending on productivity

<table>
<thead>
<tr>
<th>Independent Variable</th>
<th>StDev of growth</th>
<th>StDev of growth</th>
<th>StDev of growth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volatile exporter</td>
<td>51.108</td>
<td>0.775</td>
<td>1.917</td>
</tr>
<tr>
<td></td>
<td>(8.52)**</td>
<td>(38.37)**</td>
<td>(56.22)**</td>
</tr>
<tr>
<td>Log-tfp</td>
<td>5.966</td>
<td>-0.179</td>
<td>-0.306</td>
</tr>
<tr>
<td></td>
<td>(1.01)</td>
<td>(7.60)**</td>
<td>(7.64)**</td>
</tr>
<tr>
<td>Volatile exporter* Log-tfp</td>
<td>6.381</td>
<td>0.367</td>
<td>0.675</td>
</tr>
<tr>
<td></td>
<td>(1.15)</td>
<td>(14.81)**</td>
<td>(17.52)**</td>
</tr>
<tr>
<td>Constant</td>
<td>-8.682</td>
<td>1.665</td>
<td>2.052</td>
</tr>
<tr>
<td></td>
<td>(1.01)</td>
<td>(60.27)**</td>
<td>(41.09)**</td>
</tr>
<tr>
<td>Restr. On Std growth</td>
<td>NO</td>
<td>&lt; 11</td>
<td>&lt; 26</td>
</tr>
<tr>
<td>Observations</td>
<td>236680</td>
<td>211390</td>
<td>223922</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.13</td>
<td>0.34</td>
<td>0.33</td>
</tr>
</tbody>
</table>

Robust z-statistics in parentheses.

*significant at 5%; ** significant at 1%.

in the last two columns we bounded the standard deviation of growth rates from above
- by 11 in the second column, and by 26 in the second column

the optimal export/experimentation behavior. The higher the profitability in the market, the more likely are the exporters to forego the experimentation phase, and to transition from the experimentation stage into a full-scale phase, if they do experiment. While they experiment, however, they will do so more intensively, and thus volatility of their (observed) exports will be higher. Thus, we will observe a more prompt switch to a full scale, fewer (relatively) firms with high export volatility, but higher volatility among the volatile firms, in the larger markets, more liberalized markets, and less competitive markets.

Consider the effect of a higher signal-to-noise ratio: as the signal-to-noise ratio increases, so does the optimal \(n(p)\) schedule. Hence, there will be more (proportionally) new exporters that are highly volatile, and these will be more volatile than previously. The effect on the duration of the experimentation phase is ambiguous. The intensity of experimentation \(n(p)\) falls, which speeds up the convergence of beliefs. However, the thresholds for quitting and entering the market shift out as the signal-to-noise ratio increases, and the firm will actually keep experimenting at values of beliefs \(p\) where it previously would have made a terminal decision. Thus, the expected duration of experimentation may go up or down, depending on the relative magnitudes of these effects.

Analogously, the total costs of entry, as measured by the sum of testing costs incurred and the sunk cost of entry (in the case of successful experimentation) may rise or fall with the signal-to-noise ratio. Hence, even though the conditions of market access improve as the signal-to-noise ratio increases, and therefore the firms learn much more from testing, the firm now will pay more in expected value to enter the market. This is an interesting result - we cannot judge how easily accessible the market is just by surveying the exporters.
about the cost of entry into the market, unless we ask them specifically what they mean by the costs of entry, and how they carry out the process of entry. If there is a testing phase, higher total entry costs in a market may not mean that this market is tough to enter - on the contrary, the exporters may optimally pay the higher cost in this market in response to higher information value.

In markets with higher $F$, we expect to see less intensive experimentation, and a higher threshold on beliefs for entering the market ($\bar{p}$). This tells us that fewer firms will start with full-scale exports right away, that firms will take much longer to switch to the full-scale phase from the experimentation phase, and they will be less volatile in the experimentation phase. One proxy of $F$ could be the distance to the market, so that we expect to see lower initial volumes of exports, and slower duration of this phase in more distant markets.

4 Alternative models of learning

Here we solve alternative models of learning in the same framework, and compare their predictions and their implications for the estimates of exporting costs.

Melitz model

While there are two levels of demand, $\bar{\mu}$ and $\mu$, the firms know exactly what their demand is once they start exporting. That is, they only know the probability of a high demand before they enter, but once they pay $F$, they are told what their demand is (before they even export). Hence, there is no learning. There is a convex periodic cost of exporting. Every firm therefore exports a constant optimal number of products, that depends on its productivity, and demand level, $\mu_j$:

$$n^* = (c')^{-1}(AG\phi_j^{\mu_j})$$

where all the variables are as defined above.

Passive learning with linear costs of exporting

If there is demand uncertainty in the market in the same sense as presented in the experimentation model, but the firm has to pay the sunk cost right away, and the exporting costs are linear in $n$, then we have the same problem of the firm as in stage 2 of the experimentation model. That is, the value function of the firm $j$ is given by:

$$V(p_{jt}) = \max_{n_{js} > 0} E\left[ \int_0^\infty (-fn_{js} + n_{js}AG\phi_j^{\mu_j} + (1-p_{js})e^{\mu_j})e^{-rs}ds | p_{jt} \right],$$

subject to $dp_{jt} = p_{jt}(1-p_{jt})\frac{\bar{\mu}-\mu}{\sigma_x} \sqrt{n_{jt}}dW_{jt}$, where $dW_{jt}$ is as above.

The Hamilton-Jacobi-Bellman equation is as follows (we omit the subscripts below):

$$rv(p) = \max_{0 \leq n \leq M}\{n(-f + AG\phi_j^{\mu_j} + (1-p)e^{\mu_j}) + \frac{1}{2}(p(1-p)\frac{\bar{\mu}-\mu}{\sigma_x})^2 v''(p)\}.$$
The value function is linear in $n$, so that the optimal size is $n^*_t = M$, as long as the value function is positive. Plug in $n = M$ into the HJB equation:

$$rv(p) = M(-f + AGf_j^{-1}[pe^{\bar{\mu}} + (1 - p)e^\mu]) + M\frac{1}{2}(p(1-p)\frac{\bar{\mu} - \mu}{\sigma_x})^2 v''(p).$$

The value function with $v''(p) = 0$ satisfies the ODE above, which gives us:

$$v(p) = \frac{M}{r}(-f + AGf_j^{-1}[pe^{\bar{\mu}} + (1 - p)e^\mu]).$$

The threshold value of $p_{jt}$, below which the firm would quit the market (sell to 0 consumers) is given by:

$$p_{jt} = \frac{f AGf_j^{-1}}{e^{\mu} - e^{\bar{\mu}}}.\,$$

Thus, the only response to demand signals (beliefs) takes the form of exit by the firm if beliefs fall below this threshold, and beliefs do not affect the export size.

**Passive learning with convex costs of exporting**

Assume that the firm has to pay the sunk cost right away, to start exporting, and that there are convex, strictly increasing fixed costs of exporting in every period, $c(n)$. The firm faces the same kind of uncertainty as before, and its beliefs about $\mu_j$ evolve in the same fashion as before, but now the firm ignores the effect of its size $n$. That is, it ignores the effect of $n_t$ on $\text{Var}(dp_t)$, shown in the main theoretical part. This means that the firm engages only in passive learning, and maximizes only current profits in every period. Formally, the firm’s value function is

$$V(p_{jt}) = \max_{n_{jt}} E\left[\int_0^\infty (-c(n_{jt}) + n_{jt}AGf_j^{-1}[p_{jt}e^{\bar{\mu}} + (1 - p_{jt})e^\mu])e^{-rs}ds|p_{jt}\right],$$

subject to

$$dp_{jt} = p_{jt}(1 - p_{jt})\frac{\bar{\mu} - \mu}{\sigma_x}\sqrt{n_{jt}}dW_{jt},$$

where $\frac{\bar{\mu} - \mu}{\sigma_x} \equiv \chi$ is the signal-to-noise ratio, $dX_{ijt}$ is the $i$-th observation at time $t$ of firm $j$, $d\bar{X}_{jt} \equiv \frac{\sum_{i=1}^{n_{jt}} dX_{ijt}}{n_{jt}}$ and

$$dW_{jt} \equiv \sqrt{\frac{n_{jt}}{\sigma_x}}\left[\frac{\sum_{i=1}^{n_{jt}} dX_{ijt}}{n_{jt}} - (p_{jt}\bar{\mu} + (1 - p_{jt})\mu)dt\right]$$

$$\equiv \sqrt{\frac{n_{jt}}{\sigma_x}}[d\bar{X}_{jt} - (p_{jt}\bar{\mu} + (1 - p_{jt})\mu)dt]$$
which is an observation-adapted Wiener innovation process, that is, it follows a standard Wiener process relative to the information at time $t$.

The Hamilton-Jacobi-Bellman equation is

$$rv(p) = \max_{0 \leq n \leq M} \left[ -c(n) + nAG\phi_j^{e-1}[pe^\mu + (1 - p)e^\tilde{\mu}] 
+ \frac{1}{2}(p(1 - p)\frac{\tilde{\mu} - \mu}{\sigma_x})^2nv''(p) \right].$$

Now, we impose the assumption that the firm ignores the presence of $n$ in the second term in the HJB equation, and the FOC for the HJB equation gives us

$$AG\phi_j^{e-1}[pe^\mu + (1 - p)e^\tilde{\mu}] = c'(n),$$

$$n^*(p) = [c']^{-1}(AG\phi_j^{e-1}[pe^\mu + (1 - p)e^\tilde{\mu}]) \equiv q(p).$$

The SOC is satisfied due to the convexity of $c(n)$. The problem then becomes

$$v''(p) = \frac{rv(p) + c(q(p)) - q(p)AG\phi_j^{e-1}[pe^\mu + (1 - p)e^\tilde{\mu}]}{\frac{1}{2}(p(1 - p)\frac{\tilde{\mu} - \mu}{\sigma_x})^2q(p)},$$

with the free boundary conditions

$$v(p) = 0,$$

$$v'(p) = 0,$$

for some $p$.

Moreover, there is an upper bound on the value function: the best the firm can hope for is that at some point in the future, say, $s > t$, its beliefs $p$ will converge to 1 and stay there. In that case, its value function at time $s$ will be simply

$$AG\phi_j^{e-1}e^{\tilde{\mu}}n - c(\tilde{n})$$

where $\tilde{n} \equiv (c')^{-1}(AG\phi_j^{e-1}e^\mu)$.

The best scenario would be the one where $s = t + 1$, that is, the firm’s beliefs converge to 1 already in the next period. Therefore, the value function is bounded above by

$$AG\phi_j^{e-1}e^{\tilde{\mu}}n^*(p) - c(n^*) + (1 - r)AG\phi_j^{e-1}e^{\tilde{\mu}}n - c(\tilde{n})$$

since

$$\int_0^\infty e^{-rs}[AG\phi_j^{e-1}e^{\tilde{\mu}}n - c(\tilde{n})]ds = \frac{AG\phi_j^{e-1}e^{\tilde{\mu}}n - c(\tilde{n})}{r}.$$
Notice that $v(p)$ is undefined at $p = 1$ (from the ODE for $v(p)$). Instead, the value function goes in the limit to this maximum value as $p \to 1$:

$$\lim_{p \to 1} v(p) = \frac{AG\phi_j^{t-1} e^\hat{\mu} \hat{n} - c(\hat{n})}{r}.$$ 

This gives us the following observations:

$$\lim_{p \to 1} v''(p) = \infty,$$

$$\lim_{p \to 1} v''(p)p^2(1 - p)^2 = 0.$$

which provide additional conditions on the threshold $p$ and the value function $v(p)$. We need to solve the BVP (plus the two limits) above to find the optimal $v(p)$ and the threshold for exiting the market $p$. This can be done easily numerically.

The gist of the solution derived is that the firm learns every period, decides whether to exit or not, and if it stays, sets $n$ to maximize current profits. However, now, unlike in the model of Passive Learning with linear costs, the firm sets export size optimally in response to beliefs - if its beliefs improve, it expands, and contracts, if its beliefs deteriorate. This happens because the firm now expects higher sales per consumer, which can cover the higher marginal cost of exporting, but not because the firm wants to learn more (as in the active learning models).

**Active learning with convex costs throughout, and no postponing sunk entry cost $F$**

Assume the following environment for the firm. The firm has to pay the sunk cost of entry right away to access the market - i.e. in order to sell a non-zero quantity in the market, the firm pays sunk cost $F$. The firm then gains access to the marketing and distribution technology that is convex in the number of consumers it sells to, say $c(n)$, where $n$ is again the number of consumers. The demand for the product and production technology are just as above, that is, only the foreign market distribution technology is different this time. The firm, therefore, again is uncertain about its demand - whether it has drawn a high or low $\mu$ - $\tilde{\mu}$ or $\mu$. The firm has to decide on the optimal $n$, and whether to export or not (quit). This time the firm recognizes the information value of its sale. The value function of the firm is:

$$V(p_{jt}) = \max_{n_{js}} E[ \int_0^\infty (\phi_j^{t-1}[pe^{\mu} + (1 - p)e^{\mu}]) e^{-rs} ds \mid p_{jt}],$$

subject to

$$dp_{jt} = p_{jt}(1 - p_{jt}) \left( \frac{\hat{\mu} - \mu}{\sigma_x} \right) \sqrt{n_{jt}} dW_{jt},$$
where \( \frac{\bar{\mu} - \mu}{\sigma_x} \equiv \chi \) is the signal-to-noise ratio, 
\( dX_{jt} \equiv \frac{\sum_{i=1}^{n_{jt}} dX_{ijt}}{n_{jt}} \) and 
\[
dW_{jt} \equiv \frac{\sqrt{n_{jt}}}{\sigma_x} \left[ \sum_{i=1}^{n_{jt}} dX_{ijt} - (p_{jt} \bar{\mu} + (1 - p_{jt}) \mu) dt \right]
\]
which is an observation-adapted Wiener innovation process, that is, it follows a standard Wiener process relative to the information at time \( t \).

The Hamilton-Jacobi-Bellman equation is

\[
rv(p) = \max_{0 \leq n \leq M} \left[ -c(n) + nAG\phi_j^{-1}\left[p\bar{\mu} + (1 - p)e\mu\right] + \frac{1}{2} (p(1 - p)\frac{\bar{\mu} - \mu}{\sigma_x})^2 n^\prime\prime(p) \right].
\]

The FOC for the HJB equation give us \( n^{**}(p) = z(rv(p)) \), where \( z \equiv g^{-1} \), \( g(n) = nc'(n) - c(n) \), and \( z \) is strictly increasing. The problem then becomes

\[
v''(p) = \frac{c'(z(rv(p))) - AG\phi_j^{-1}\left[p\bar{\mu} + (1 - p)e\mu\right]}{\frac{1}{2} (p(1 - p)\frac{\bar{\mu} - \mu}{\sigma_x})^2}
\]

plus the value matching condition:

\( v(p) = 0 \),

and smooth pasting condition:

\( v'(p) = 0 \).

Just as in the Passive Learning with convex costs model above, the value function is bounded above by

\[
\frac{AG\phi_j^{-1}e\bar{\mu} - c(\bar{n})}{r},
\]

where \( \bar{n} \equiv (c')^{-1}(AG\phi_j^{-1}e\mu) \).

The value function goes in the limit to this maximum value as \( p \to 1 \):

\[
\lim_{p \to 1} v(p) = \frac{AG\phi_j^{-1}e\bar{\mu} - c(\bar{n})}{r}.
\]

This gives us the following conditions:
\[
\lim_{p \to 1} v''(p) = \infty, \\
\lim_{p \to 1} v''(p)p^2(1-p)^2 = 0.
\]

These two equations impose additional conditions on the optimal \(n_{js}\), and \(p\), apart from the BVP above.

This is intuitive: the information value comes from two sources - first, the value of not exiting the market unnecessarily, while future sales may reveal that staying is actually profitable (this value is especially high when \(p\) is close to 1), and second, the value of correctly setting the optimal sales size \(n\). As \(p\) goes to 1, the first source of information value becomes less important, since \(p\) is very high, and the second source becomes less important, since it is very likely that \(p\) will converge to 1 and stay there, in which case the optimal size is simply \(\tilde{n}\). Hence, the information value \(\frac{1}{2}(p(1-p)\frac{\mu - \bar{\mu}}{\sigma_x})^2 n v''(p)\) goes to 0. This is depicted in Figure 19.

**Contrasting the four models of learning**

We would like to compare the predictions of the four models of learning, given the same structural parameters (such as \(F, c(n), f, \bar{\mu}, \mu, \sigma_x\)).

First, compare the predictions for export size \(n\). Clearly, the interesting comparison here is that between the export size under Passive Learning with convex costs, and Active Learning. Export size for the same (convex) cost schedule \(c(n)\) will be lower under Passive Learning with convex costs than, for any given beliefs value \(p\), under Active Learning. From the FOC for PL,

\[
c'(n_{pl}) = \pi [pe^\mu + (1-p)e^\mu],
\]

and from FOC and the HJB equation for AL,

\[
n^a c'(n^a) - c(n^a) = rv^a(p),
= -c(n^a) + n^a \pi [pe^\mu + (1-p)e^\mu]
+ \frac{1}{2}(p(1-p)\chi)^2 n^a [v^{a\prime\prime}(p)].
\]

Simplifying the last equation, we get

\[
c'(n^a) = \pi [pe^\mu + (1-p)e^\mu]
+ \frac{1}{2}(p(1-p)\chi)^2 [v^{a\prime\prime}(p)] \geq \pi [pe^\mu + (1-p)e^\mu]
= c'(n_{pl}),
\]
The information value, given by the expression $\frac{1}{2} (p(1-p)\frac{\mu - \bar{\mu}}{\sigma_x})^2 v''(p)$, attains a maximum for some $p \in (0, 1)$, and goes to 0 as $p$ goes to 1.
since \((p(1 - p)\chi)^2[v^{al}]''(p) \geq 0\) for all \(p \in [\underline{p}^{al}, 1)\) for the solution \(v^{al}(p)\). Since \(c(n)\) is convex, we obtain

\[ n^{al}(p) \geq n^{pl}(p) \]

for all \(p\) where both solutions are defined.

Also, in both models the solution \(v(p)\) satisfies the condition

\[ \lim_{p \to 1} v(p) = \frac{\tilde{\pi} c^{\mu} \tilde{n} - c(\tilde{n})}{r}. \]

where \(\tilde{n} \equiv (c')^{-1}(AG\phi_j^{-1}e\tilde{\mu})\).

Therefore, both the value functions are bounded above by and go to this value as \(p\) goes to 1. Knowing that in the Passive Learning model the firm (by assumption) ignores the effect of its export size \(n\) on learning, and sets it only to maximize current profits in every period, we can conclude that \(v^{al}(p) > v^{pl}(p)\) for all \(p\) where both value functions are defined.

Combining this with the boundary conditions for \(v^{pl}(p)\), we get

\[ v^{al}(\underline{p}^{pl}) > v^{pl}(\underline{p}^{pl}) = 0, \]

that is, to satisfy the boundary condition on \(v^{al}(p)\), we need

\[ \underline{p}^{al} < \underline{p}^{pl}. \]

Thus, we can compare the solutions to the PL and AL models: in the AL model, the threshold for quitting is lower, and the export size and the value function are higher for all values \(p\), where the PL solution is also defined. By the same token, the export size is higher in the experimentation stage in the Experimentation model than the export size in the Passive Learning model (with convex costs), given the same exporting costs \(c(n)\) in the experimentation stage.

This also implies that volatility of exports will be higher under the active learning models, than under the passive learning ones. Starting from the same initial beliefs, the firm will set lower size \(n\) under Passive Learning. Since \(Var(dp_t)\) is proportional to \(n\), that is, the variance in beliefs is higher the higher the sample size, we will get smaller changes in future beliefs and future export size, under the Passive Learning assumption.

Another way to compare the models is to consider the implications for the estimates of export costs they would provide. Given the same data, one would require lower exporting cost schedule \(c(n)\) to match the same export sizes of firms in the Passive Learning model with convex costs of exports, than in the Active Learning models (single-stage or two-stage), because of the observation we just made that the export size is lower in the former under the same cost schedule. So to generate the same export sizes in both models, one needs a lower cost schedule in the Passive Learning model.

Second, the estimate of the sunk cost of entry \(F\) will in general be higher in the Experimentation model than in the Passive Learning or single-stage Active Learning model,
or a Melitz type with no learning model. This can be seen intuitively by thinking of the way we measure sunk entry costs. When we measure sunk entry costs in standard models, we take note of the lowest productivity exporter in the market, and evaluate this firm’s expected lifetime discounted profits from exporting. We use the free entry condition (or export participation condition) to estimate $F$ as equal to this value. Suppose the lowest productivity exporter has not yet incurred the sunk entry cost $F$, and is only testing the market. Then this firm’s lifetime discounted profits may be actually much lower than the sunk entry cost $F$, at the initial belief $p_0$. So we will underestimate $F$ based on observing these lowest productivity exporters. We need to fit the experimentation model to correctly evaluate exporting costs.

5 Conclusion

We present a model with demand uncertainty and sunk costs of entry. The firm is able to learn more about the demand before incurring the sunk cost of entry, by using a costly testing technology that allows to sample individual sales observations with some noise, and update its beliefs about the parameter of interest. Thus, the decision for new exporters is no longer a binary choice between entry and non-entry, but an optimal control and optimal stopping problem where the scale and duration of the initial testing phase is optimally chosen, depending on the characteristics of the firm, the destination and the product. In particular, we present interesting comparative statics results with respect to the features of uncertainty, such as the signal-to-noise ratio, where a higher signal-to-noise ratio in the market may either prolong or shorten the duration of the experimentation phase. This also gives us the observation that the expected total costs of entry (the sum of testing costs and sunk entry cost) may be higher in markets with higher signal-to-noise ratio. Therefore, measuring total costs of entry is not sufficient to judge how easy it is to enter the market. We show that the zero-cutoff profit condition for exporters is very different in this model, than in standard Melitz type models, or in other models of learning.

Overall, the model proposed is very intuitive, very flexible, in that it allows two stages, that can accommodate different phases of the life cycle of an exporter, and allows for endogenously determined duration and intensity of the experimentation phase. We believe that it can provide a framework for numerous further applications.
References


Akhmetova Z. and S.Poncet (2009), New Exporter Dynamics, Working Paper, CEPII.


Akhmetova Z. and S. Poncet (2009), New Exporter Dynamics, Working Paper, CEPII.


Appendix

Verifying some comparative statics results of Moscarini and Smith (2001)

To study the effect of including profits in the testing phase in the value function on the behavior of the testing phase profits, which is assumed to be continuous, and range from 0 to 1. If $\Delta$ is 0, testing profits do not enter the value function, if it is 1, they do so fully.

\[
V(p_0) = \max_{T, <n_t>} E[e^{-\delta T} \pi(p_T) + \int_0^T (-c(n_t) + \Delta n_t A \frac{p_t \bar{\mu} + (1 - p_t) \mu}{\bar{\mu} + \mu}) e^{-\delta t} dt | p_0].
\]

Take the derivative of the value function with respect to $\Delta$, using the envelope theorem:

\[
V_\Delta(p_0) = E[\int_0^{T^*} n_t^* A \frac{p_t \bar{\mu} + (1 - p_t) \mu}{\bar{\mu} + \mu}) e^{-\delta t} dt | p_0] > 0,
\]

where $T^*$ and $n_t^*$ are the optimal values of the stopping time and sample size, respectively. Hence, the value function increases for all $p_t$ when $\Delta$ goes from 0 to 1. This implies, by the Theorem in Moscarini and Smith (2001), that in the case with testing profits in the value function, compared with the case without, $n(p_t) = f(rv(p_t))$ goes up for all $p_t$, and $\bar{p}$ goes up and $p$ goes down. Hence, the firm experiments more intensively and the thresholds for it to take a terminal decision expand, when we allow the firm to gather the profits in the experimentation phase.

To make sure that all the comparative statics of the solution with respect to key parameters derived in Moscarini and Smith still hold, we check if all the proofs of these results can be replicated.

Consider the comparative statics with respect to $A \equiv y \frac{1}{\epsilon - 1} \left[\frac{\epsilon}{\epsilon - 1}\right]^{-\epsilon} \left[\frac{\phi v}{w}\right] e^{-1} P^{e-1}$, which will effectively give us the comparative statics with respect to any of $\phi$, $P$, and $w$. The derivative of the value function (with $\Delta$ set to 1) with respect to $A$, using the envelope theorem, is

\[
V_A(p_0|\Delta = 1) = E[\int_0^{T^*} n_t^* A \frac{p_t \bar{\mu} + (1 - p_t) \mu}{\bar{\mu} + \mu}) e^{-\delta t} dt | p_0] + \text{Prob}(p_T = \bar{p}|p_0, n^*, T^*) E[e^{-r T^*} | p_T = \bar{p}, p_0] *
\]

\[
* (\bar{p} \frac{\bar{\mu}}{\bar{\mu} + \mu} + (1 - \bar{p}) \frac{\mu}{\bar{\mu} + \mu} > 0).
\]

Hence, we already know that the value function increases for all $p_0$ as $A$ rises, and since $n(p_0)$ is increasing in $v(p_0)$, so does the experimentation intensity $n$ for each $p$. 

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Let $A_1 > A_0$, and denote the corresponding thresholds for quitting as $\overline{p}_1$ and $\overline{p}_0$, and for entering as $\overline{p}_1$ and $\overline{p}_0$, respectively. Since the value function of the problem for $A = A_1$, evaluated at $\overline{p}_0$ is, by linear approximation,

$$V(\overline{p}_0|A_1) \simeq V(\overline{p}_0|A_0) + V_A(\overline{p}_0|A_0)(A_1 - A_0) > 0,$$

by value matching ($V(\overline{p}_0|A_0) = 0$), and $V(p)$ being increasing, we get $\overline{p}_1 < \overline{p}_0$. That is, the threshold for quitting goes down as $A$ increases. This result is the same as in Moscarini and Smith (2001).

Next, we want to see what happens to the threshold for entering, $\overline{p}$. Consider

$$V_p(\overline{p}_0|A_1) \simeq V_p(\overline{p}_0|A_0) + V_{pA}(\overline{p}_0|A_0)(A_1 - A_0)$$

$$= \frac{A(\overline{\mu} - \mu)}{(\overline{\mu} + \mu)\delta} + V_{pA}(\overline{p}_0|A_0)(A_1 - A_0).$$

If $V_{pA}(\overline{p}_0|A_0) > 0$, then $V_p(\overline{p}_0|A_1) > \frac{A(\overline{\mu} - \mu)}{(\overline{\mu} + \mu)\delta}$, and since the smooth pasting condition has to be satisfied by the new solution, and by convexity of the value function $V(p)$, we will get $\overline{p}_1 < \overline{p}_0$. To check if this is true:

$$V_{pA}(p) = V_{Ap}(p)$$

$$= E\left[\int_0^{T^*} \left(\frac{n_t \overline{\mu} - \mu}{\overline{\mu} + \mu} \frac{dn_t p_t \overline{\mu} + (1 - p_t)\mu}{\overline{\mu} + \mu} + e^{-\delta t} dt\right) + E\left[n_{T^*}^* \frac{p_{T^*} \overline{\mu} + (1 - p_{T^*})\mu}{\overline{\mu} + \mu} e^{-\delta T^*} \mid p_0 \right] \frac{dE(T^*)}{dp_0} + E\left[p_{T^*} \mid p = \overline{p}, p_0 \right] dP(p_{T^*} = \overline{p} \mid p_0) \right] + \text{Prob}(p_{T^*} = \overline{p} \mid p_0) \frac{dE[e^{-r T^*} \mid p = \overline{p}, p_0]}{dp_0}$$

$$* (\overline{p} \frac{\overline{\mu}}{(\overline{\mu} + \mu)\delta} + (1 - \overline{p}) \frac{\mu}{(\overline{\mu} + \mu)\delta}).$$

For $p_0$ small enough, $\frac{dE(T^*)}{dp_0} > 0$, and for $p_0$ close to 1, $\frac{dE(T^*)}{dp_0} < 0$. However, even if $p_0$ is close to 1,

$$\frac{dP(p_{T^*} = \overline{p} \mid p_0)}{dp_0} > 0,$$

and

$$\frac{dE[e^{-r T^*} \mid p = \overline{p}, p_0]}{dp_0} > 0.$$
Hence, if the last term is large enough relative to the preceding term, the cross-derivative should be positive. That is, if the optimal experimentation intensity is not too large compared to the entire market size (normalized by us to 1), so that the profits in the second to last term are small compared to the profits in the last term, then the threshold for entering the market \( \bar{p} \) falls as \( A \) increases. This is intuitive: since \( A \) is a measure of profits from an average consumer in the market, if the optimal testing sample size is large and approaches the entire market size, then the firm will have little incentive to rush towards a terminal decision to enter the market, and even less so as \( A \) rises, so that the profits in the testing phase dominate the pure testing costs \((c(n))\).

**Interpreting the assumptions about the demand signals of the firms**

The firm \( j \) does not observe the precise values of quantities sold, i.e. it does not observe the precise value of \( \mu \). Instead, it receives an imperfect signal from an individual consumer indexed by \( i \):

\[
X_{ijt} = \int_0^t \ln \left( \frac{c_{ij}}{y(p_j)^{-\epsilon} P^{\epsilon - 1}} \right) ds + \sigma_x W_{ijt} = \int_0^t \mu_j ds + \sigma_x W_{ijt},
\]

so that

\[
dX_{ijt} = \mu_j dt + \sigma_x dW_{ijt},
\]

where \( W_{ijt} \) is a Wiener process. We now assume that the firm (and the researcher) observe the signals only at discrete time intervals, \( t \in 0, 1, 2, 3, 4, \ldots \). We can approximate the signal process as

\[
\Delta X_{ijt} = \ln \frac{c_{ij}}{y(p_j)^{-\epsilon} P^{\epsilon - 1}} \Delta t + \sigma_x \Delta W_{ijt},
\]

and with \( \Delta t = 1 \), and denoting \( \Delta X_{ijt} = \alpha_{ijt} \), and \( \Delta W_{ijt} = \eta_{ijt} \sim N(0, 1) \), we get

\[
\alpha_{ijt} = \ln c_{ij} - \ln y + \epsilon p_j - (\epsilon - 1) \ln P + \sigma_x \eta_{ijt}.
\]

Now, our assumption will be that the way this happens is that the firm observes not the true quantities \((\ln c_{ijt})\), but noise-contaminated ones:

\[
\ln \tilde{c}_{ijt} = \ln c_{ijt} + \sigma_x \eta_{ijt},
\]

so that

\[
\ln \tilde{c}_{ijt} - \ln y + \epsilon p_j - (\epsilon - 1) \ln P = \mu_j + \sigma_x \eta_{ijt} \equiv \alpha_{ijt},
\]

\[
\ln \tilde{c}_{ijt} = \ln y - \epsilon p_j + (\epsilon - 1) \ln P + \alpha_{ijt} = \ln c_{ijt} + \alpha_{ijt},
\]

where \( \alpha_{ijt} \sim N(0, 1) \). Therefore, we can interpret the assumptions made about the demand signals as saying that in discrete time the firm observes log quantities sold with some normally distributed error.