Home-Market Effects on Innovation

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Abstract

We propose a two-country model of international trade with non-homothetic preferences to study how local tastes influence innovation. We consider separately three mechanisms: First, firms in rich countries invent disproportionately more varieties to produce income-elastic (luxury) goods. Second, technological improvements in rich countries are biased toward luxury goods. As a result, Ricardian differences in technologies across countries and goods arise endogenously. Third, new technologies are influenced by the availability of factors. Since the production of luxury goods is concentrated in the rich country, the technologies to produce them complement skilled labor and substitute unskilled labor. The data are consistent with predictions of the model: Rich countries produce and consume relatively more income-elastic goods, and they have a comparative advantage in them. Comparative advantage is partly explained by observed differences in variety and factor intensities.

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1 Introduction

Economists and historians alike have long argued that local economic conditions influence innovation in various ways. Firms in all countries tailor their goods to local tastes. In developed countries, firms specialize in goods with a high income elasticity of demand, such as high-quality and luxury items. This specialization provides incentives for the development of new expertise, techniques and equipment suited to the production of luxury goods. Cost-reducing efforts are also influenced by the availability of factors. Since developed countries have abundant skilled labor and scarce unskilled labor, the technologies to produce luxury goods tend to complement skills and substitute unskilled labor. Analogously, firms in developing countries generally produce more basic consumer goods. Their technological improvements are geared to the production of these goods and to the utilization of unskilled labor.

These home-market effects on innovation shape patterns of demand, production and trade. With increasing returns to scale at the firm level, firms from developed countries export their luxury goods, and developing countries export basic goods. If technologies diffuse better within than across countries, then innovation that is biased toward domestic tastes renders developed countries more efficient at making luxury goods. And even if technologies diffuse perfectly across firms and countries, developed countries may exhibit a comparative advantage in luxury goods because factor bias in innovation makes the production of these goods more skill intensive.

We develop a model to study these mechanisms. The model has two countries, North and South. Individuals in North are endowed with more efficiency units of labor than in South. There are two types of goods, basic and luxury, and a continuum of varieties of each type. Preferences are non-homothetic and the relative demand for luxury goods increases with consumer income. We propose three technology set ups to isolate three different aspects of the argument above. First, \textit{ex ante} symmetric firms develop varieties of basic or luxury goods, and compete monopolistically. Second, there are constant returns to scale technologies to produce final goods, and an innovation sector develops machines that are specific to a type of good (basic or luxury) and a country. As a result, Ricardian differences in technologies across countries and goods arise endogenously. Third, final-goods production uses skilled and unskilled labor, and an innovation sector develops machines that are specific to a factor (skilled or unskilled labor) and to a type of good (basic or luxury), but not specific to a country. Exogenous differences in factor endowments across countries and endogenous differences in skill intensity across goods give rise to comparative advantages à la Heckscher-Ohlin.
The three set ups have similar and particular positive predictions. In all three cases, advanced North is richer, produces and consumes relatively more luxury goods. North is a net exporter of luxury goods in the monopolistic-competition set up, and in the other two set ups if economies of scale in innovation are sufficiently large. There is a greater variety of luxury goods in North under monopolistic competition. Northern firms are relatively more productive in making luxury goods in the Ricardian set up. If North is skill abundant in the Heckscher-Ohlin set up, then the production of luxury goods is more skill intensive than basic goods in all countries, and North uses skilled labor more intensively than South to produce both luxury and basic goods.

We find support for these predictions in the data. Compared to poor countries, rich countries allocate more resources to the production of income-elastic goods (luxury goods). Bilateral trade data suggest that rich countries have a comparative advantage in producing these goods and that all three proposed mechanisms contribute to this comparative advantage. Within sectors, the number of exporters (variety) in rich countries is relatively larger in income-elastic sectors. Income-elastic goods are more skill intensive. And even after controlling for variety and factor usage, rich countries still reveal a comparative advantage in producing income-elastic goods, suggesting a role for productivity differences.

Home-market effects in innovation also have welfare implications. As in other models with economies of scale, population growth increases welfare. In the Heckscher-Ohlin set up, whether trade increases or decreases relative wages of skilled workers depends critically on whether technology diffuses with trade. In the open economy, technologies are tailored to the average of Northern and Southern endowments, weighted by the volume of production. If there is no diffusion in autarky, then a country’s technologies to produce all goods is tailored to its own factor endowment. Then as trade costs decrease, technologies in North become less skill bias, and relative wages of skilled workers decrease. In South, trade makes technologies more skill bias and increases relative wages of skilled workers. In contrast, if technologies diffuse prior to trade openness, then the effect of international trade on wages flips and it is the same as in a standard Heckscher-Ohlin model: Trade increases the relative wages of skilled workers in North and decreases it in South.

The paper is linked to several strands of literature. Basic elements of the model hark back to Linder (1961) and Vernon (1966). While these authors propose biases in innovation to describe patterns of production and trade after World War II, historians apply these biases to earlier periods. During the First Industrial Revolution, the dramatic growth of the textile industry was facilitated by factor-biased technological progress and demand.

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1Machines can be interpreted as patentable intangible technologies (Acemoglu (2002)). The exercise consists of letting machines be traded or not before the decrease in the cost of trading final goods.
patterns according to Landes (2003). Machines substituted scarce weavers and gifted spinsters. Local firms succeeded in expanding production with fashionable items, colorful muslin for summer and thicker fabrics for winter not least because the British populace was much richer than their counterparts in continental Europe. During the Second Industrial revolution, between 1870 and World War I, newly-found conglomerates—Procter & Gamble, Lever Brothers, Nestlé, General Electric and Ford—supplied an increasingly affluent middle class with pre-processed food, hygiene items, electricity, small home appliances, and the automobile. The massive scale and scope under which these firms operated was enabled by technological innovations in metallurgy, transportation and telecommunications as well as factor-biased innovations. Synthetic dyes, rubber and plastics substituted raw materials that were erratically supplied by developing countries. The spread of professional managers and clerks, marketing and advertisement is associated with a well-educated (white) American population.\(^2\)

Our work builds on Fajgelbaum, Grossman and Helpman (2011) who develop a model where firms choose between creating new varieties of high- or low-quality goods. In equilibrium, North has a greater variety, produces, consumes, and exports relatively more luxury goods. This set up and results are akin to our monopolistic-competition set up, which like their model, does not speak to systematic differences in firm productivity and in factor intensity across countries and goods.\(^3\) Earlier models of international trade with non-homothetic preferences assume that rich countries have a comparative advantage in income-elastic goods. They are relatively more efficient at making these goods in Flam and Helpman (1987), Stokey (1991), Murphy and Shleifer (1997) and Matsuyama (2000), and income-elastic goods use intensively factors that are abundant in rich countries in Markusen (1986), Bergstrand (1990) and Verhoogen (2008).\(^4\)

Also related are Matsuyama (2002) and Murphy, Shleifer and Vishny (1989) who study the effect of domestic demand on growth in closed economies. Bernard, Redding and Schott (2007) show that firm heterogeneity augment differences in comparative advantages due to factor endowments. Their mechanism may magnify the endogenous differences in comparative advantage studied here. Dingel (2015) empirically estimates the effect of

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\(^2\)See Chandler, Hikino and Chandler (2009) and Goldin and Katz (2009). Mokyr (1998) and Landes (2003) argue that inventions since the Second Industrial Revolution have been more complex and tightly linked to scientific knowledge, which is itself linked to higher education.

\(^3\)The two papers are complementary due to other differences. On the one hand, Fajgelbaum, Grossman and Helpman (2011) study the effect of inequality and trade policy on welfare. On the other hand, our monopolistic model is in general equilibrium, while their model is in partial equilibrium—relative wages of North to South are exogenous.

\(^4\)Fieler (2011) and Caron, Fally and Markusen (2014) do not assume but show through estimation exercises that a model where rich countries have a comparative advantage in producing income-elastic goods explains well data on bilateral trade and national accounts.
demand on specialization and find a dominant role for local demand.

The remainder of the paper is organized as follows. Section 2 documents stylized facts in the data to motivate the model. Section 3 presents the model and its results. Extensions are in section 4 and the conclusion is in section 5.

2 Stylized facts

We document stylized facts linking the characteristics of demand of a good—income-elasticity of demand—to the characteristics of supply—productivity, variety and factor intensity—to trade patterns. Most trade models do not offer an explanation for these links because assumptions on demand and supply are made independently. Our theory is based on non-homothetic preferences and trade costs: The technologically advanced country is endogenously richer and consumes more income-elastic goods. With trade costs, a greater consumption translates into greater production, which in turn biases innovation and determines the characteristics of production. While the theory is consistent with the systematic relation between demand and supply below, we do not claim that it is the only explanation.\(^5\) The findings are intended to further motivate the theory, and they do not have a structural interpretation. Some findings are novel and others are found in previous work—they are compiled here and confirmed with alternative sources of data. We describe the data in section 2.1 and the results in section 2.2.

2.1 The data

We combine several data sets. Since some of them have overlapping information, we use more than one data source for robustness whenever a specification allows. We focus on year 2006 which maximizes country coverage in the combined data. The first data set is the Global Trade Analysis Project, GTAP version 7 (Narayanan and Walmsley, 2008). It contains data on production, consumption, expenditure, factor usage, input-output linkages, and bilateral trade by country and broad industry categories. The data contain 57 sectors and 109 countries. The World Input-Output Database, WIOD (Timmer et al (2015)) contains the same information as GTAP for a narrower set of countries and industries: 40, mostly wealthy, countries and 40 industries. The advantage of the WIOD

\(^5\)While preferences shape technology here, in Atkin (2013) it is the reverse: Habit formation imply that exogenous comparative advantages shape preferences. An additional explanation was proposed to us by Gene Grossman. Goods of higher quality and higher income elasticity are more complex to produce. As a result, they require more skilled workers and a better institutional environment to produce—both of which are found in rich countries. In this explanation, the link between demand and supply is plausible but exogenous. A natural experiment would be ideal to disentangle these explanations.
is that it reports the usage of labor and capital in physical and value terms. GTAP reports only the total wage bill for skilled and unskilled workers.\textsuperscript{6}

Our main set of trade data is BACI (Gaulier and Zignago, 2010), a database collected by the United Nations (UN-COMTRADE), compiled and cleaned by CEPII. The data cover more than 150 countries and contain all trade flows by country pair at a finely disaggregated product category, four-digit Harmonized System (HS4).\textsuperscript{7} We estimate productivity and income elasticity of demand from the WIOD and GTAP. But because sector classifications are very coarse in these data, we also estimate income elasticity of demand and revealed comparative advantage at the HS4 product category using BACI. The drawback of these BACI estimates is the missing information on domestic consumption and production. We henceforth refer to product categories simply as products.

We also use the Exporter Dynamics Database, EDD, provided by the World Bank (Cebeci et al. 2012). EDD reports the number of exporters by HS4, which we take as a measure of variety. There are only 42 countries in EDD. We use data on population and real GDP per capita from the Penn World Tables, schooling years from Barro and Lee (2010), and distance, common language and colony from CEPII (Mayer and Zignago 2011). Before turning to the results, we estimate revealed comparative advantage in section 2.1.1 and the income elasticity of demand in section 2.1.2.

\subsection*{2.1.1 Estimating comparative advantage from trade data}
Separately for each product, we estimate the following gravity equation using a Poisson Pseudo-Maximum-Likelihood estimator (PPML):

$$X_{nik} = A_{ik} FM_{nk} (d_{ni})^{-\theta_k}$$

where $X_{nik}$ is the value of trade flows from exporter $i$, importer $n$ in product $k$, $A_{ik}$ is an exporter fixed effect, $FM_{nk}$ is an importer fixed effect, $d_{ni}$ is a vector containing the distance between countries $n$ and $i$, and dummies for whether they share a common language, border or colonial link, and $\theta_k$ is a vector with the corresponding coefficients to be estimated. These geopolitical characteristics capture trade frictions between countries $n$ and $i$. We estimate equation (1) using, separately, the BACI and WIOD databases. BACI has 1191 products and WIOD has 40. Since the WIOD report domestic spending, $d_{ni}$

\textsuperscript{6}WIOD classifies workers into high-, middle- and low-skilled. We define “skilled” as the aggregation of middle- and high-skilled labor. All our results are robust to using high-skilled labor only. We calculate capital intensity as nominal gross fixed capital formation divided by labour compensation (both in millions of national currency). In GTAP, we define skill intensity as the ratio the skilled-labor compensation to the total labor compensation.

\textsuperscript{7}BACI is available at HS6, but we only use HS4 to be consistent with EDD (below).
contains an additional element indicating whether \( n = i \). PPML is the most recommended estimator for gravity equations using data sets with a lot of zeros.\(^8\)

Estimated separately by product, equation (1) follows the structure of a wide range of trade models, including the models in Arkolakis et al. (2014), Costinot, Donaldson, Komunjer (2011), Fiele, Caliendo, Parro (2014), and Caron, Fally, Markusen (2014). The latter model is relevant because it shares the general structure of our theory (section 4). Following these models, we use exporter fixed effects \( A_{ik} \) as country \( i \)'s comparative advantage in product \( k \). These fixed effects combine information on product variety, productivity and factor prices.\(^9\) Their advantage over measuring productivity from domestic data is that equation (1) controls for demand through importer fixed effects.\(^10\)

### 2.1.2 Estimating income elasticity of demand from trade data

We follow Caron et al. (2014) to estimate the income elasticity of demand of product \( k \), as the coefficient \( \delta_k \) in regression

\[
\log \left( \frac{X_{nk}}{L_n} \right) = \alpha_k + \delta_k \log y_i + (\sigma_k - 1) \log P_{nk}. \tag{2}
\]

where \( \left( \frac{X_{nk}}{L_n} \right) \) is per capita final consumption of product \( k \) in country \( n \), \( \alpha_k \) is a product fixed effect, \( y_i \) is country \( i \)'s income per capita, \( \sigma_k \) is the price elasticity of demand, and \( P_{nk} \) is the price index of product \( k \) in country \( n \). In appendix A, we detail the strategy for estimating \( P_{nk} \), which uses results from the gravity equation to construct proxies and instruments for prices. Equation (2) arises from assuming preferences with constant relative income elasticity (CRIE), to which we extend our model in section 4.\(^11\)

Estimating income elasticities \( \delta_k \) from equation (2) requires data on expenditures by country and product, \( X_{nk} \). These data are available at broad sector categories from WIOD and GTAP. In addition, appendix A develops a method to estimate \( \delta_k \) at a more

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\(^8\)See Santos Silva and Tenreyro (2006) and Fally (2015). In BACI, zeros account for 90% of importer-exporter-product observations. Aggregating across exporters (for importer-fixed effects), only 10% of potential importer-product observations are zeros.

\(^9\)With Krugman (1980), \( A_{ik} = M_{ik} T_{ik} \sigma_k^{-1} w_{ik} \) where \( M_{ik} \) is the mass of varieties, \( T_{ik} \) is an exporter-product productivity parameter, \( w_{ik} \) is the cost of inputs, \( \sigma_k \) is the elasticity of substitution between varieties. This formula holds in Armington (Anderson and Van Wincoop 2003) with \( N_{ik} = 1 \), and in Melitz (2003) and Chaney (2008) with \( \sigma_k \) reflecting the shape of the distribution of firm productivity. In Eaton and Kortum (2002), \( A_{ik} = T_{ik} w_{ik} \theta_k \) where \( \theta_k \) reflects the dispersion in productivity.

\(^10\)In the models above, these importer fixed effects are \( FM_{nk} = \frac{X_{nk}}{\Phi_{nk}} \), where \( X_{nk} \) is country \( i \)'s spending on product \( k \) and \( \Phi_{nk} = \sum_i A_{ik} d_i \theta_k \) is the inward multilateral resistance term.

\(^11\)These preferences are used in Bils, Klenow (1998) and Fieler (2011) among others. An expression similar to equation (2) holds in Comin, Lashkari and Mestieri (2015).
disaggregated HS4 level from trade data. We include all products in all regressions.\textsuperscript{12}

\subsection*{2.2 Empirical regularities}

\textbf{Fact 1: Rich countries allocate more resources to the production of income-elastic goods}

Given that rich countries, by definition, demand relatively more income-elastic goods, this finding is not surprising. We report it because it is the \textit{sine qua non} for innovation in rich countries to be biased toward income-elastic goods. Specifically, using WIOD, we regress

\begin{equation}
\log RE_{ik} = \beta \cdot \delta_k \cdot \log y_i + \alpha_i + \alpha_k + \epsilon_{ik}
\end{equation}

where $\delta_k$ is the income elasticity of demand from section 2.1.2 above, and $y_i$ is income per capita, $\alpha_i$ and $\alpha_k$ are, respectively, country and product fixed effects, and $\epsilon_{ik}$ is the residual. The dependent variable $RE_{ik}$ is the resources allocated to product $k$ in country $i$, which depending on the specification is value added, labor or capital.

\begin{table}[h]
\centering
\begin{tabular}{lcccc}
\hline
 & (1) & (2) & (3) & (4) \\
Dependent var.: & log $VA_{ik}$ & log $L_{ik}$ & log $K_{ik}$ \\
$\delta_k \cdot \log y_i$ & 0.799\textsuperscript{**} & 0.858\textsuperscript{**} & 0.858\textsuperscript{**} & 0.627\textsuperscript{**} \\
 & [0.081] & [0.109] & [0.109] & [0.120] \\
skill interaction$\dagger$ & -0.002 & & & \\
 & [0.001] & & & \\
Product FE & Yes & Yes & Yes & Yes \\
Country FE & Yes & Yes & Yes & Yes \\
R2 & 0.962 & 0.962 & 0.920 & 0.939 \\
N Obs. & 1353 & 1353 & 1353 & 1353 \\
\hline
\end{tabular}
\caption{Allocation of resources}
\end{table}

Notes: \textsuperscript{*} significant at 5\%; \textsuperscript{**} at 1\%. \textsuperscript{$\dagger$} skill interaction is skill intensity of product $k$ times average years of schooling in country $i$. The table uses WIOD.

The results are on table 1. In all specifications, the coefficient on the interaction term $\delta_k \cdot \log y_i$ is positive and statistically significant, indicating that rich countries produce relatively more income-elastic goods. In specifications (1) and (2), $RE_{ik}$ is value added.

\textsuperscript{12}Results are robust to dropping any one of the Broad Economic Categories (BEC): Capital goods, processed goods, parts and components, etc.
Column (2) adds an interaction between skill intensity of product $k$ and the skill endowment of country $i$ to ensure that the result is not driven by differences in education across countries and skill intensity across products. In columns (3) and (4), $RE_{ik}$ is labor $L_{ik}$ (hours of work) and fixed capital stock $K_{ik}$, respectively: Rich countries allocate relatively more labor and capital to produce income-elastic goods.

**Fact 2: Exports are positively correlated with imports**

In standard trade models focusing on the supply side, countries tend to export their comparative advantage goods and import their comparative disadvantage goods. In contrast, the data suggest that countries have a comparative advantage in producing the very same goods that they import. We use two measures of a country’s imports by product—the value of imports, and the importer fixed effect estimated in equation (1)—and three measures of exports—the value of exports, an export dummy, and the exporter fixed effect estimated in equation (1).

<table>
<thead>
<tr>
<th>Dependent var.:</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>log Exports $A_{ik}$ (exporter FE)</td>
<td>0.210**</td>
<td>0.025**</td>
<td>0.140**</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[0.006]</td>
<td>[0.001]</td>
<td>[0.007]</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>log $FM_{ik}$</td>
<td>0.090**</td>
<td>0.019**</td>
<td>0.013*</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[0.006]</td>
<td>[0.001]</td>
<td>[0.006]</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Product FE</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Country FE</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>R2</td>
<td>0.662</td>
<td>0.657</td>
<td>0.447</td>
<td>0.444</td>
<td>0.644</td>
<td>0.641</td>
</tr>
<tr>
<td>N Obs.</td>
<td>93745</td>
<td>93745</td>
<td>121676</td>
<td>121676</td>
<td>93745</td>
<td>93745</td>
</tr>
</tbody>
</table>

*Notes: Data from BACI; * significant at 5%; ** significant at 1%.

Table 2 shows a positive correlation between imports and exports using all combinations of these measures and controlling for country and product fixed effects. The advantage of the fixed effects from the gravity equation is that the results are not driven by geography—i.e., by countries being physically close to other countries with similar geographic proximity.

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13Skill intensity is defined as average share of skilled workers in the wage bill and skill endowment as the average years of schooling in country $i$.

14Similarly in the model, countries have a comparative advantage in the goods they value the most. But fact 2 cannot strictly hold in a two-country model. For example, if South exports relatively more basic goods, than North imports relatively more basic goods and exports more luxuries.
comparative advantages or similar demand patterns.

Fact 3: Rich countries have a comparative advantage in income-elastic goods

In the model, there are three mechanisms through which differences in comparative advantage arise: Variety, productivity, and factor-bias innovation. The data suggest that rich countries have a comparative advantage in income-elastic goods, and that all three mechanisms contribute to this finding. Using BACI trade data, we regress

\[ EX_{ik} = \beta \cdot \delta_k \cdot \log y_i + \alpha_i + \alpha_k + \varepsilon_{ik} \]  

(4)

where \( \delta_k \) is the income elasticity of demand from section 2.1.2 above, \( y_i \) is income per capita; \( \alpha_i \) and \( \alpha_k \) are, respectively, country and product fixed effects, and \( \varepsilon_{ik} \) is the residual. The dependent variable \( EX_{ik} \) is either \( \log(X_{ik}) \) the log of total exports of country \( i \) in product \( k \), or it is \( \log(A_{ik}) \) the exporter fixed effect from gravity equation (1). The latter is a better measure of comparative advantage because it controls for geography and demand effects.

Table 3 presents the results. The positive coefficient on \( \delta_k \cdot \log y_i \) indicates that rich countries export more income-elastic goods. To understand the mechanism, columns (2) and (4) control for the interaction between skill intensity of product \( k \) and skill endowment of country \( i \). The value and statistical significance of the original coefficient on \( \delta_k \cdot \log y_i \) do not change, suggesting that differences in factor intensities alone cannot explain the specialization of rich countries in income-elastic goods.

Table 4 examines the role of product varieties. We proxy variety with \( NX_{ik} \), the number of exporters by country and product from EDD database. Since we control for product and country fixed effects, this proxy does not bias the results if either the number of varieties per firm is larger in some products or in some countries. It biases the results if, say, firms in rich countries have a particularly large variety in income-elastic products.\(^{16}\) Column (1) regresses variety \( NX_{ik} \) on the interaction between income elasticity of the product \( \delta_k \) and income per capita of country \( i \). The positive coefficient indicates that rich countries have relatively more firms exporting income-elastic goods—in support of the variety mechanism of section 3.2. Since the EDD has a much smaller set of countries and products than BACI, column (2) repeats and confirms results from regression (4) with the smaller sample. Columns (3) and (4) introduce controls for variety \( NX_{ik} \) and

\(^{15}\)The results also hold for WIOD data.

\(^{16}\)If this were the case, we would underestimate the effect of variety in governing comparative advantage: In columns (3) and (4), the coefficient on \( NX_{ik} \) would be biased downward, and the coefficient on \( \delta_k \cdot \log y_i \) would be biased upward.
Table 3: Comparative advantage in income-elastic goods

<table>
<thead>
<tr>
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<th>(1)</th>
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<tbody>
<tr>
<td>Dependent var.:</td>
<td>log Exports</td>
<td>log $A_{ik}$ (Gravity FE)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\delta_k \cdot \log y_i$</td>
<td>0.282**</td>
<td>0.296**</td>
<td>0.175**</td>
<td>0.175**</td>
</tr>
<tr>
<td></td>
<td>[0.016]</td>
<td>[0.016]</td>
<td>[0.017]</td>
<td>[0.016]</td>
</tr>
<tr>
<td>skill interaction</td>
<td>0.184**</td>
<td>0.207**</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>[0.005]</td>
<td>[0.006]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Product FE</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Country FE</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>R2</td>
<td>0.647</td>
<td>0.654</td>
<td>0.630</td>
<td>0.638</td>
</tr>
<tr>
<td>N Obs.</td>
<td>95493</td>
<td>95493</td>
<td>95493</td>
<td>95493</td>
</tr>
</tbody>
</table>

Notes: * significant at 5%; ** significant at 1%. The table uses BACI data.

Table 4: Comparative advantage in income-elastic goods

<table>
<thead>
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<th>(1)</th>
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<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dependent var.:</td>
<td>log $NX_{ik}$</td>
<td>$A_{ik}$ (Gravity Exporter FE)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\delta_k \cdot \log y_i$</td>
<td>0.132**</td>
<td>0.271**</td>
<td>0.133**</td>
<td>0.144**</td>
</tr>
<tr>
<td></td>
<td>[0.016]</td>
<td>[0.034]</td>
<td>[0.030]</td>
<td>[0.029]</td>
</tr>
<tr>
<td>log $NX_{ik}$</td>
<td>1.078**</td>
<td>1.058**</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>[0.014]</td>
<td>[0.014]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Skill interaction</td>
<td>0.969**</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>[0.097]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Product FE</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Country FE</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>R2</td>
<td>0.720</td>
<td>0.605</td>
<td>0.705</td>
<td>0.706</td>
</tr>
<tr>
<td>N Obs.</td>
<td>23894</td>
<td>26931</td>
<td>22320</td>
<td>22320</td>
</tr>
</tbody>
</table>

Notes: * significant at 5%; ** significant at 1%. The table uses EDD data.
skill intensity interacted with skill endowments. The coefficient on $\delta_k \log y_i$, decreases but remains positive and statistically significant, suggesting again that skill intensity and product variety only partly explain rich countries’ comparative advantage in income-elastic goods. In the model, the remainder is explained by productivity differences across countries. Unfortunately, it is difficult to test this hypothesis directly without a measure of productivity that is comparable across countries and sectors.\footnote{The most common and simple measure of productivity, value added per worker, is misleading once country and sector fixed effects are included. In models with perfect labor markets, such as our own, value added per worker simply equals wages divided by the labor share in production, which is entirely absorbed by the fixed effects. In the model and in the WIOD data, value added per worker is uncorrelated with the interaction $\delta_k \log y_i$ once we control for country and sector fixed effects. A similar issue applies to the Solow residual, and similar findings are in Yeaple and Golub (2007).}

While results in tables 3 and 4 focus on the patterns of trade across products, the literature on quality has highlighted similar patterns of specialization. Even within finely defined product categories, researchers find that rich countries import and export goods of higher quality, based on unit values or price-equivalent demand shifters.\footnote{See Schott (2004), Hummels, Klenow (2005), Hallak (2006) for evidence based on unit values. These results are robust to estimating quality as a price-equivalent demand shifter in Khandelwal (2010), Hallak and Schott (2011), Feenstra and Romalis (2014).} Taken together, these findings suggest that rich countries have a comparative advantage in income-elastic goods, within and across sectors, and that these patterns are not entirely driven by differences in factor intensities or product variety.

**Fact 4: Income-elastic goods are skill intensive**

<table>
<thead>
<tr>
<th>Dependent Variable:</th>
<th>Income elasticity $\delta_k$</th>
<th>WIOD</th>
<th>GTAP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Skill intensity</td>
<td></td>
<td>0.456$^*$</td>
<td>0.441$^*$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.273)</td>
<td>(0.286)</td>
</tr>
<tr>
<td>Capital intensity</td>
<td></td>
<td>0.055</td>
<td>0.068</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.017)</td>
<td>(0.268)</td>
</tr>
<tr>
<td>Natural resources int.</td>
<td></td>
<td></td>
<td>-0.005</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.180)</td>
</tr>
<tr>
<td>N Obs. (sectors)</td>
<td></td>
<td>34</td>
<td>34</td>
</tr>
</tbody>
</table>

*Notes: Beta coefficients; $^*$ significant at 5%; $^{**}$ significant at 1%. GTAP-based coefficients are taken from Caron et. al 2014.*

In the model of section 3.4, innovation in a certain sector is biased towards the fac-
tor that is abundant in countries that specialize in the sector. Because richer countries produce more income-elastic goods and are abundant in skilled labor, the production of income-elastic goods is more intensive in skilled labor than the production of basic goods in all countries. This prediction is well supported by the data. Using GTAP data, Caron et al (2014) find a striking correlation between income elasticity and skill intensity across sectors. We report these results in table 5 and confirm them using the WIOD data.\footnote{Capital intensity is almost uncorrelated with income elasticity once we control for skill intensity. A lower elasticity of substitution between capital and labor than between skilled and unskilled labor can explain this finding.}

**Fact 5:** Products use more skilled workers in skill abundant countries

### 3 The model

#### 3.1 Environment & Preferences

There are two countries, North (N) and South (S). Each individual in North supplies inelastically one unit of labor, and each Southerner supplies $e < 1$ units. The population in country $i$ is $L_i$. With increasing returns to scale, we also assume that effective labor supply is larger in North, $L_N \geq eL_S$, to ensure that North is richer than South in equilibrium. Northern wages are the numeraire.

There are two types of goods $\tau \in \{1, 2\}$, each with a continuum of varieties $\omega \in \tau$. Consumers observe prices and choose quantities $q(\omega)$ to maximize utility

$$U = \begin{cases} Q_1 & \text{if } Q_1 < 1 \\ 1 + Q_2 & \text{otherwise} \end{cases}$$

where $Q_\tau = \left[ \int_{\omega \in \tau} q(\omega) \frac{\sigma - 1}{\sigma} d\omega \right]^{\frac{\sigma}{\sigma - 1}}$ for $\tau = \{1, 2\}$.

The consumer needs one unit of the aggregate basic good 1 in order to enjoy luxury good 2. The elasticity of substitution across varieties $\sigma > 1$ is the same for both types for simplicity. We assume throughout that consumers in North and South consume type 2. Next sections present three production set ups. Section 3.2 has monopolistic competition. Technologies differ across types and countries in section 3.3, and there are two factors of production in section 3.4. Sections 3.3 and 3.4 have endogenous innovation, and we make slight changes in preferences and the environment as needed. Section 4 shows that the results hold with other, less stylized preferences.
3.2 Production I: Monopolistic competition

The production setup follows Krugman (1980). There is a large pool of potential entrepreneurs who can pay a fixed cost of $f_\tau$ units of labor to invent and have monopoly rights over a new variety of type $\tau$. After fixed costs, producing each unit requires $c_\tau$ units of labor. There are symmetric iceberg trade costs: To deliver each unit of a good from one country to another requires the shipment $d > 1$ units. An equilibrium is a mass of firms $M_{i\tau}$ for each country $i = S, N$ and type $\tau = 1, 2$, and a Southern wage $w$ such that firms make zero profits and labor markets clear.

We study here the case where both countries produce both types, and show in appendix B.2 that all results hold when at least one country specializes. We use small case letters to denote the ratio of North to South—i.e., $v_{\tau} = \frac{V_{N\tau}}{V_{S\tau}}$ for any variable $V$. Firms charge a constant markup $\mu = \frac{\sigma}{\sigma - 1}$ over marginal cost. Price indices for $\tau \in \{1, 2\}$ are

$$P_{N\tau} = \mu c_\tau \left[ M_{N\tau} + M_{S\tau} (dw)^{1-\sigma} \right] \frac{1}{1-\sigma}$$

$$P_{S\tau} = \mu c_\tau \left[ M_{N\tau} + M_{S\tau} w^{1-\sigma} \right] \frac{1}{1-\sigma}.$$  

Zero-profit conditions are

$$\frac{1}{\sigma} (\mu c_\tau)^{1-\sigma} \left[ (P_{N\tau})^{\sigma-1} X_{N\tau} + d^{1-\sigma} (P_{S\tau})^{\sigma-1} X_{S\tau} \right] = f_\tau$$

$$\frac{1}{\sigma} (\mu c_\tau w)^{1-\sigma} \left[ (P_{S\tau})^{\sigma-1} X_{S\tau} + d^{1-\sigma} (P_{N\tau})^{\sigma-1} X_{N\tau} \right] = w f_\tau,$$

where $X_{i\tau}$ is spending of country $i$ on type $\tau$. Taking ratios of prices and zero-profit conditions, we get respectively

$$p^{1-\sigma}_\tau = \frac{m_\tau + (dw)^{1-\sigma}}{d^{1-\sigma} m_\tau + w^{1-\sigma}}$$

$$p^{1-\sigma}_\tau = \left( \frac{1 - d^{1-\sigma} w^{-\sigma}}{w^{-\sigma} - d^{1-\sigma}} \right) x_\tau$$

$$\Rightarrow m_\tau = w^{1-\sigma} \left[ x_\tau (w^{\sigma} - d^{1-\sigma}) \right]$$

$$\frac{1 - d^{1-\sigma} w^{-\sigma}}{w^{-\sigma} - d^{1-\sigma} x_\tau (w^{\sigma} - d^{1-\sigma})} (1 - d^{1-\sigma} w^{\sigma})$$

Lemma 1 states that the economy exhibits home-market effects: The country that consumes relatively more of a good, produces relatively more of it and exports it on net. The type of good that each country produces is specified in proposition 2.

**Lemma 1** If the South spends relatively more on type $\tau$, $x_\tau < x_{\tau'}$, then it has relatively more firms of type $\tau$, $m_\tau < m_{\tau'}$, and it is a net exporter of type $\tau$.  

14
proof. From the zero-profit conditions, the ratio of revenue in goods of type $\tau$ in the North relative to the South is $\frac{M_{N\tau}}{wM_{S\tau}} = \frac{m_\tau}{w}$. Then, the North is a net exporter of type 2 if and only if $\frac{M_{N1}/X_{N1}}{wM_{S1}/X_{S1}} < \frac{M_{N2}/X_{N2}}{wM_{S2}/X_{S2}} \equiv \frac{m_1}{x_1} < \frac{m_2}{x_2}$. Dividing equation (8) by $x_\tau$ yields

$$\frac{m_\tau}{x_\tau} = w^{1-\sigma} \left[ \frac{(w^\sigma - d_1^\sigma) - (1/x_\tau)d_1^\sigma(1 - d_1^\sigma w^\sigma)}{(1 - d_1^\sigma w^\sigma) - x_\tau d_1^\sigma(w^\sigma - d_1^\sigma)} \right],$$

which is increasing in $x_\tau$. Hence, $\frac{m_1}{x_1} < \frac{m_2}{x_2} \iff x_1 < x_2$ and $m_1 > m_2 \iff x_1 > x_2$. ■

Proposition 2 South consumes, produces and exports relatively more type 1, and North, type 2. Per capita utility is lower in South than North.

The proof is in appendix B. The challenge is that multiple equilibria may exist, as in typical models of trade with non-homothetic preferences. In particular, we prove that there are no equilibria where South specializes in luxury goods and is richer than North: If $w \geq 1$, then type 1 becomes very expensive in South because production is small, $eL_S \leq L_N$, and labor is expensive. Then, to fulfill its demand for type 1, South disproportionately spends its income on type 1. If on the other hand, $w < 1$, the relative price of type 1 in South is sufficiently low for North to be a net importer. Lemma 1 has the empirical implication that rich countries consume and produce relatively more income-elastic goods. They also export relatively more of these goods because they have a greater variety. These predictions are supported by the data—facts 1 and 3. Proposition 2 links these patterns to unobserved, exogenous absolute advantage.

3.3 Production II: Ricardo & Innovation

Firms that produce the same type of good in a country often use similar technologies, produce varieties with similar characteristics and pool from a labor market where workers have similar expertise. To the extent that this local knowledge does not diffuse perfectly across country borders, it gives rise to Ricardian comparative advantages—technologies to produce final goods differ across countries and goods.

The second production set up captures the link between demand patterns and the development of local knowledge. It features exogenous comparative advantages across

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20See Stokey (1991) for a discussion. Fajgelbaum, Grossman and Helpman (2011) have a unique equilibrium but their wages are pinned down by a sector with homogeneous goods and no trade frictions. Similarly here, equilibrium production and consumption allocations are unique given $w$. Appendix B derives these allocations as a function of wages $w$ in closed form. The new model may be put in partial equilibrium if we assume that type 2 is a Cobb-Douglas between a differentiated sector and a sector with a homogeneous good and no trade frictions.
varieties within industries à la Eaton and Kortum (2002) and endogenous comparative advantages across industries due to an innovation sector à la Acemoglu (2002). The innovation sector produces machines that are only suitable to production in a specific industry and country. Inventing a machine is costly, but once invented, its reproduction is costless. As a result, the larger is an industry in a country, the more machines are invented, and the more efficient is its production.

We make changes to the environment relative to section 3.1. First, like other models of external economies of scale, there are multiple equilibria, some of which are inefficient and counterintuitive. To avoid these equilibria, we follow Grossman and Rossi-Hansberg (2010) and allow firms to be sufficiently sophisticated to realize that they can become large and profit whenever the equilibrium displays inefficiencies. That is, without decreasing returns to scale, firms may become large and internalize inefficiencies. Second, we augment the model and assume a continuum of infinitesimally-small symmetric industries of each type. The purpose is to keep the focus on demand effects and eliminate specialization on the basis of size. If industries were large, then the large country could specialize in a large industry even when it does not consume much of its goods.

Last, we impose no restrictions on the magnitude of Northern labor endowment $L_N$ relative to South $L_S$ but make assumptions on countries’ exogenous productivity below. Individuals in both countries are endowed with one unit of labor.

Preferences. Goods are divided into two types $\tau = 1, 2$ as before. Type 1 contains basic goods and type 2 contains luxuries. The difference is that now, each type comprises a continuum of industries $\iota \in \tau$, and each industry $\iota$, a continuum of varieties. All industries have a measure one of varieties and all types have a measure one of industries. We use lower case for industry variables and upper case for type variables. Preferences are:

$$U = \begin{cases} Q_1 & \text{if } Q_1 < 1 \\ 1 + Q_2 & \text{otherwise} \end{cases}$$

where

$$Q_{\tau} = \left[ \int_{\iota \in \tau} q_{\iota}^{\frac{\sigma - 1}{\sigma}} d\iota \right]^{\frac{\sigma}{\sigma - 1}}$$

for $\tau = \{1, 2\}$,

$$q_{\iota} = \left[ \int_{\omega \in \iota} \hat{q}(\omega)^{\frac{\sigma - 1}{\sigma}} d\omega \right]^{\frac{\sigma}{\sigma - 1}}$$


\(^{22}\)Hanson and Xiang (2004) study and find evidence that size matters for production location: Large countries are more likely to specialize in producing goods in large industries. Grossman and Rossi-Hansberg (2010) also assume a continuum of small industries.
\( Q_\tau \) is the aggregate quantity of type \( \tau \), \( q_\iota \) is the aggregate quantity of industry \( \iota \), \( \hat{q}(\omega) \) is the quantity of variety \( \omega \) in industry \( \iota \), and \( \eta > 1 \) is the elasticity of substitution between industries.

**Production of final goods.** Production of final goods follows Eaton and Kortum (2002). The unit cost of producing variety \( \omega \) of industry \( \iota \) in country \( i \) is \( \frac{c_{\iota \omega}}{z_{i}(\omega)} \) where \( z_{i}(\omega) \) is an exogenous country- and variety-specific productivity parameter, and \( c_{\iota \omega} \) is the endogenous cost of inputs used to produce goods of industry \( \iota \) in country \( i \).\(^{23}\) Final-goods markets are perfectly competitive and trade costs are variable—to deliver one unit of a good from country \( i \) to country \( j \neq i \), \( d > 1 \) units must be shipped. Industries are symmetric within a type. The measure of goods of industry \( \iota \in \tau \) with productivity less than or equal to \( z \) in country \( i \) follows a Fréchet cumulative distribution:

\[
F_\tau(z) = \exp(-T_\tau T_\tau z^{-\theta})
\]

where \( T_\tau \) governs efficiency in industries of type \( \tau \), and \( T_i \) governs the efficiency of country \( i \). Since these parameters enter multiplicatively, there is no exogenous comparative advantage across types and countries. Heterogeneity parameter \( \theta > 1 \) does not depend on type to avoid comparative advantages between South and North arising from differences in \( \theta \), as in Fieler (2011). Aggregate price index of industry \( \iota \in \tau \) in country \( i \) is

\[
p_{\iota \iota} = \gamma(\Phi_{\iota \iota})^{-1/\theta}
\]

where

\[
\Phi_{\iota \iota} = T_\tau \left[ T_i(c_{\iota \iota})^{-\theta} + T_j(d_{\iota \iota})^{-\theta} \right],
\]

\[
\gamma = \left[ \Gamma \left( \frac{1 + \theta - \sigma}{\theta} \right) \right]^{1/(1-\sigma)}
\]

where \( \Gamma \) is the gamma function and \( j \neq i \). Denote with \( x_{j\iota} \) country \( j \)'s spending on industry \( \iota \). Country \( j \)'s spending on industry \( \iota \) from country \( i \) is

\[
x_{j\iota} = \frac{T_i(d_{\iota \iota})^{-\theta}}{T_j(c_{\iota \iota})^{-\theta} + T_i(d_{\iota \iota})^{-\theta}} x_{j\iota}. \tag{11}
\]

**Innovation.** Innovators in country \( i \) can invent a machine for industry \( \iota \in \tau \) at a fixed cost \( F_\tau \) units of local labor and reproduce the machine at zero cost. Innovators have monopoly rights over their machines and can only be used in the country where they are located. Alternatively, innovation could shape the distribution of productivity \( z_i(\omega) \) as in Eaton and Kortum (2001) and Somale (2014).
invented. Final goods producers combine labor and machines into an input bundle used to make final goods $\omega$. Machines are bought separately for each bundle so that there is constant returns to scale in final goods production. Buying more than one identical machine per bundle does not increase output. Given prices of machines $p_{ii}^M$ and labor $w_i$, producers of final goods in industry $\iota$ choose the measure of different machines $a$ and labor $l$ to minimize input costs:

$$c_{ii} = \min_{a,l}\{p_{ii}^M a + w_i l \text{ subject to } a^\alpha l \geq 1\}$$

In equilibrium, the number of machines chosen equals the stock of invented machines $a_{ii}$.

From the first order condition, machine innovators charge price $p_{ii}^M = \alpha w_i (a_{ii})^{-(1+\alpha)}$ and the cost of one input bundle is

$$c_{ii} = (1 + \alpha)w_i a_{ii}^{-\alpha}$$

where machine producers get a share $\frac{\alpha}{1+\alpha}$ of revenue and labor in final production gets a share $\frac{1}{1+\alpha}$. Free entry into innovation implies:

$$\frac{\alpha}{1+\alpha}r_{ii} - a_{ii}F_\tau w_i = 0 \quad \text{for } \iota \in \tau, \tau = 1, 2 \text{ and } i = S, N. \tag{12}$$

where $r_{ii} = x_{ii} + x_{ji}$ is the total revenue of firms in industry $\iota$ in country $i$. Since the market of final goods is perfectly competitive, total labor allocated to (production and innovation of) industry $\iota$ in country $i$ is $w_i l_{ii} = r_{ii}$ and the stock of machines is $a_{ii} = \frac{\alpha}{1+\alpha} \frac{w_i}{F_\tau}$. The cost of inputs decreases in the size of the industry, $l_{ii}$, according to:

$$c_{ii} = (1 + \alpha)^{1+\alpha} \alpha^{-\alpha} w_i \left( \frac{l_{ii}}{F_\tau} \right)^{-\alpha}. \tag{13}$$

**Definition 1** An *equilibrium* is a wage in South $w$, a set of labor allocations $l_{ii}$ and consumption allocations $q_{ii}$ for all countries $i = N, S$, industries $\iota \in \tau$ and types $\tau = 1, 2$ such that:

(i) Firms make zero profit and industry price indices $p_{ii}$ are a function of labor allocations, given by equations (10) and (13).

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24Equivalently, the cost of transferring the technology from one country to the other is sufficiently high that it takes at least as much labor to imitate as to innovate. A more realistic model would have the North having a comparative advantage in innovating and the South using labor to imitate, but our purpose is to highlight the heterogeneity in outcomes arising from demand, not from differences in labor efficiencies to produce and to innovate in North vis-à-vis South.
(ii) Consumers maximize utility and split their income between countries according to equation (11).
(iii) Firms in industry $\iota$ are sufficiently sophisticated to take over the whole industry if they find it profitable.
(iv) Labor markets clear, $w_i l_i = r_i$ in all countries $i$ and industries $\iota$.

**Proposition 3** An equilibrium exists if $\alpha(\eta - 1) \in (0,1)$. If $\alpha \theta < 1$, then all countries produce varieties in all industries (diversification). If $\alpha \theta > 1$ then all production in an industry occurs in one country (specialization).

The formal proof is in appendix C.1. Ignoring trade costs, the profit of a representative firm in industry $\iota$ is:

$$\max_i \left[ w - \theta l_{\iota S_i} + \log S_i \right]^{\frac{\theta - 1}{\theta}} - w l_{\iota S_i} - l_{N_i}$$

where $D_\tau$ captures demand for type $\tau$ and is exogenous to the firm. If $\alpha(\eta - 1) \in (0,1)$ then the problem is well behaved and the demand for countries’ labor satisfies standard existence conditions.\footnote{See Mas-Colell et al. (1995), chapter 17.C.} If $\alpha(\eta - 1) > 1$, then the first term is convex and the firm chooses infinite labor (profit goes to infinity if the firm sets $l_i = 0$ and $l_j = \infty$). So, no equilibrium exists—there is a tendency for all production to concentrate on a measure 0 of industries $\iota$ with infinite labor demand each. If $\alpha \theta < 1$, then term in brackets is the sum of two concave functions where the marginal product of labor is infinite as $l_i$ tends to zero. So, the representative firm diversifies between South and North. If $\alpha \theta > 1$, the term in brackets is convex and the firm concentrates all labor in only one country. Economically, specialization occurs if either economies of scale in innovation $\alpha$ is large or exogenous heterogeneity in production technologies is small ($\theta$ is large). Because the procedure and some of the results are different for the diversification and specialization cases, we cover them separately in sections 3.3.1 and 3.3.2, respectively.

### 3.3.1 Production II: Diversification

**Assumption 1** $\alpha \theta < 1$ and $\eta < \theta + 1$.

Appendix C.2 shows that if $\eta < \theta + 1$ then all (potentially multiple) equilibria are symmetric across industries of the same type $\tau$. For simplicity, we cover only this symmetric case. Price indices $P_{i\tau} = p_{i\iota}$, labor allocation $L_{i\tau} = l_{i\iota}$, consumption allocation $Q_{i\tau} = q_{i\iota}$,
spending $X_{iτ} = x_{iτ}$ and revenue $R_{iτ} = r_{iτ}$ for all industries $i \in τ$, types $τ = 1, 2$, and countries $i = S, N$. Using equations (11) and (13), labor market clearing of type $τ$ is

$$L_{Nτ} = T_N L_{Nτ}^αθ \left[ d^{-θ} X_{Sτ} \left( \frac{T_N L_{Nτ}^αθ + w^{-θ} T_N L_{Sτ}^αθ}{T_N L_{Nτ}^αθ + (dw)^{-θ} T_S L_{Sτ}^αθ} \right) + \frac{X_{Nτ}}{T_N L_{Nτ}^αθ + (dw)^{-θ} T_S L_{Sτ}^αθ} \right]$$

$$w L_{Sτ} = w^{-θ} T_S L_{Sτ}^αθ \left[ d^{-θ} T_N L_{Nτ}^αθ + w^{-θ} T_N L_{Sτ}^αθ + \frac{w^{-θ} X_{Sτ}}{T_N L_{Nτ}^αθ + (dw)^{-θ} T_S L_{Sτ}^αθ} \right]$$

Let $x_τ = \frac{X_{Nτ}}{X_{Sτ}}$, $l_τ = \frac{L_{Nτ}}{L_{Sτ}}$ and $t = \frac{T_N}{T_S}$. Taking ratios and rearranging, we have

$$x_τ = \left( 1 - d^{-θ} tw^{θ+1} l_τ^{αθ-1} \right) \left( tl_τ^{αθ} + (dw)^{-θ} d^{-θ} tw^{θ+1} l_τ^{αθ} + w^{-θ} \right) \quad (14)$$

Lemma 4 In any equilibrium, if country $i$ spends relatively more type $τ$, $\frac{X_{iτ}}{X_{jτ}} > \frac{X_{iτ}'}{X_{jτ}'}$ then it also produces relatively more type $τ$, $\frac{L_{iτ}}{L_{jτ}} > \frac{L_{iτ}'}{L_{jτ}'}$ and it is relatively more productive in all industries $i \in τ$.

proof. In equation (14), $x_τ$ is an increasing function of $l_τ$ whenever $αθ < 1$. The relative productivity statement follows because input cost $c_{iτ}$ decreases with labor $L_{iτ} = L_{iτ}$ in equation (13).\[26\]

A larger labor allocation $\frac{L_{iτ}}{L_{jτ}} > \frac{L_{iτ}'}{L_{jτ}'}$ implies that prices of type $τ$ are lower in country $i$. Hence, its real consumption of type $τ$ is also larger, $\frac{Q_{iτ}}{Q_{jτ}} > \frac{Q_{iτ}'}{Q_{jτ}'}$. From preferences, the country that consumes relatively more type 2 enjoys higher utility. So, lemma 4 has the empirical implication that richer countries produce, consume and are relatively more efficient in making income-elastic goods—as per facts 1 and 3 of the data. But the proposition does not link these patterns to exogenous productivity differences. In general, if external economies of scale are sufficiently large, they could overturn exogenous productivity—there exist equilibria where the country with lower $T_i$ consumes and produces relatively more type 2. These equilibria have the flavor of Hausman, Hwang and Rodrik (2007), where a country’s specialization determines its income.

This type of equilibrium is eliminated in proposition 5, where the ratio of technology parameters $\frac{T_N}{T_S}$ is sufficiently large for North always to be richer than South. This ratio increases with $\frac{L_N}{L_S}$ because, as in Eaton and Kortum, a larger population forces a country to expand production to varieties $ω$ where it is relatively unproductive (low $z_i(ω)$). The ratio also increases in $d$ because larger trade costs enable a naturally comparative disadvantage industry to develop under external economies of scale.\[26\]

\[26\]This is consistent with the infant-industry tariff protection. See Baldwin (1969), Melitz (2005).
Assumption 2 \( \left( \frac{L_N}{L_S} \right)^{1-\alpha \theta} \frac{T_S}{T_N} < d^{-2\theta-1} \)

Proposition 5 Under assumption 2, North consumes relatively more type 2, \( \frac{X_{N2}}{X_{N1}} > \frac{X_{S2}}{X_{S1}} \), produces relatively more type 2, \( \frac{L_{N2}}{L_{N1}} > \frac{L_{S2}}{L_{S1}} \), and it is relatively more productive in type 2, \( c_{N1} < c_{S1} \) for all \( i \in 2 \) and \( i' \in 1 \).

proof. In order for markets to clear in equation (14),

\[
1 - d^{-\theta} tw^{\theta + 1} \tau^{\theta - 1} > 0 \quad \text{for } \tau = 1, 2
\]

\[
\Rightarrow w < \left[ d^{\theta} \frac{T_S}{T_N} \min \left\{ \frac{L_{N1}}{L_{S1}}, \frac{L_{N2}}{L_{S2}} \right\}^{1-\theta \alpha} \right]^{\frac{1}{1+\theta}}
\]

\[
w \leq \left[ d^{\theta} \frac{T_S}{T_N} \left( \frac{L_N}{L_S} \right)^{1-\theta \alpha} \right]^{\frac{1}{1+\theta}} \quad (15)
\]

Suppose by contradiction that South spends relatively more on type 2:

\[
\frac{X_{S1}}{X_{N1}} = \frac{L_SP_{S1}}{L_NP_{N1}} < \frac{L_S(w - P_{S1})}{L_N(1 - P_{N1})} = \frac{X_{S2}}{X_{N2}} \quad \Leftrightarrow \quad w > \frac{P_{S1}}{P_{N1}}
\]

Since \( \frac{P_{S1}}{P_{N1}} > d^{-1} \), this equation contradicts equation (15) under assumption 2. ■

Technologically-advanced North is richer, produces, consumes and is more efficient at making luxury goods. To see why this is not sufficient for North to be a net exporter of luxury goods, let \( \frac{X_{SN}}{X_{S2}} \) be the market share of North in Southern absorption of type \( \tau \). Lemma 4 and proposition 5 together imply \( \frac{X_{S2}^2}{X_{S1}} > \frac{X_{S1}^1}{X_{S1}} \), but North is a net exporter of type 2 if

\[
\left( \frac{X_{S2}^2}{X_{S2}} \right) X_{S2} > \left( \frac{X_{S1}^1}{X_{S1}} \right) X_{S1}
\]

which requires Southern consumption of luxuries to be sufficiently large or endogenous innovation to be sufficiently large for \( \frac{X_{S2}^2}{X_{S2}} \) to be much greater than \( \frac{X_{S1}^1}{X_{S1}} \).

Proposition 6 If \( X_{S1} \geq X_{S2} \) or if \( \alpha \theta \) is sufficiently close to one, then North is a net exporter of type 2.

proof. From proposition 5, \( \frac{X_{S2}^2}{X_{S2}} > \frac{X_{S1}^1}{X_{S1}} \) and so equation (16) holds whenever South is sufficiently rich to spend most of its income on type 2.\(^{27}\) Otherwise, from equation (12),

\(^{27}\)It is easy to find sufficient conditions for type 1 to be a small share of total income in both countries, using the exogenous parameters. Since the autarky case has closed form solution, one can assume, for example, that in autarky both countries consume relatively more type 2.
revenue in type $\tau$ is proportional to labor allocation. Then, North is a net exporter of type 2 if $\frac{x_{2\tau}}{l_{2\tau}}$ is decreasing in $l_{2\tau}$. This relationship holds if we divide equation (14) by $l_{2\tau}$ and set $\alpha \theta = 1$. \[\square\]

### 3.3.2 Production II: Specialization

**Assumption 3** $\alpha \theta > 1$

By proposition 3 above, countries are specialized if $\alpha \theta > 1$. The case of specialization is simpler than diversification. The production of one of the types $\tau \in \{1, 2\}$ takes place only in the country that demands it the most, and $R_{i\tau} = 0$ for the other country. For the remaining type $\tau'$, firms are indifferent between allocating production in one country versus the other, but within an industry $i \in \tau'$ production is concentrated in only one country to take advantage of local innovation spillovers. This indifference condition and labor market clearing pin down relative wages of North to South and the share of industries $i \in \tau'$ that allocate in each country. Given this structure of production, it is straightforward to prove results analogous to the diversification case: Production is concentrated where relative demand is highest; North consumes relatively more type 2 if its exogenous productivity is sufficiently high, and North is a net exporter of the good it produces the most. We just present the results here and prove them in appendix C.3.

**Lemma 7** In any equilibrium, if country $i$ spends relatively more type $\tau$, $\frac{X_{i\tau}}{X_{j\tau}} > \frac{X_{i\tau'}}{X_{j\tau'}}$, then it also produces relatively more type $\tau$, $\frac{L_{i\tau}}{L_{j\tau}} > \frac{L_{i\tau'}}{L_{j\tau'}}$ and it is typically more productive in industries of type $\tau$.

**Assumption 4** $\frac{T_S}{T_N} < d^{2-\alpha}$

**Proposition 8** Under assumption 4, North consumes relatively more type 2, $\frac{X_{N2}}{X_{N1}} > \frac{X_{S2}}{X_{S1}}$. It produces relatively more type 2, $\frac{L_{N2}}{L_{N1}} > \frac{L_{S2}}{L_{S1}}$, and it is a net exporter of type 2.

Lemma 7 is almost identical to lemma 4 of the diversification case.28 Proposition 8 is analogous to propositions 5 and 6 put together. The conditions to ensure that North is in equilibrium richer than South do not depend on population because when a country produces in an industry, it produces all varieties of that industry. As before, the assumed ratio $\frac{T_S}{T_N}$ increases in trade costs $d$.

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28The only difference is that under proposition 4, country $i$ was relatively more productive in making goods in all industries $i \in \tau$ relative to $i' \in \tau'$. Here, country $i$ may only produce a subset of industries of type $\tau$ and be infinitely unproductive at making goods in other industries.
The sharper differences between specialization and diversification lie in their empirical implications. It is difficult to map the specialization case to data because one would never simultaneously observe production of North and South in an industry. Under diversification, all countries produce all goods; richer countries consume, produce and are relatively more efficient at making luxury goods. In the data, most product categories are produced and consumed by rich and poor countries, and these predictions linking demand to production are consistent with data (facts 1 and 3 of section 2). The disadvantage of the diversification case is that the richer country only exports on net luxury goods under restrictive assumptions (proposition 6).

Given these differences, we turn to the empirical literature for values of our parameters. In the model, $\eta$ is the elasticity of substitution between industries within types. Take $\eta \in [2, 4]$ to be the estimates of the elasticity of substitution within broad product categories SITC-3 in Broda and Weinstein (2006). Estimates of $\theta$ typically range between 4 and 9.\(^{29}\) Griliches (1998, table 3.6) estimates the elasticity of output with respect to cumulated R&D spending, $\alpha \in [0.07, 0.14]$. Given these estimates, the conditions for existence $\alpha(\eta - 1) < 1$ and symmetry $\eta < \theta + 1$ are easily met. Taking $\theta = 4.4$ from Simonovska and Waugh (2013) and $\alpha \in [0.07, 0.14]$, we get $\alpha \theta \in [0.3, 0.6]$. But other reasonable estimates push $\alpha \theta$ closer to one.\(^{30}\) In sum, diversification ($\alpha \theta < 1$) is more likely to hold than specialization: Richer countries consume, produce and have a comparative advantage in luxury goods (facts 1 and 3). But it is unclear whether $\alpha \theta$ is sufficiently close to one for North to be a net exporter of luxuries, unless the mechanism here is combined with sections 3.2 and 3.4.

### 3.4 Production III: Factor proportions & Innovation

To the model in section 3.3 we add two factors of production: Skilled and unskilled labor. There are two countries, North and South. Individuals in each country are endowed with one unit of skilled or unskilled labor. The endowment of skilled and unskilled labor in country $i$ is $H_i$ and $L_i$, respectively. North is skill abundant $H_N > H_S$, $L_N > L_S$. Preferences and the production of final goods are given in section 3.3 above. Utility is represented by equation (9), price index of industry $\iota$ is given by equation (10) and trade shares are given by equation (11). The difference is in the innovation sector.

Creating a new machine that complements skilled labor in industry $\iota$ costs $F_h$ units

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\(^{30}\)The value $\theta = 4.4$ is in the lower range of the values of $\theta$ in the literature. As for $\alpha$, Griliches notes that his estimate may be low because he only observes returns on the firm that has incurred R&D spending, not potential spillovers to other firms.
of the industry’s input bundle, and creating a machine that complements unskilled labor costs $F_l$ units. The assumption that these costs are not specific to the type $\tau$ is made for simplicity.\textsuperscript{31} Skilled labor is more efficient than unskilled labor if $F_h < F_l$.\textsuperscript{32} Once invented, a machine can be reproduced at zero cost and used in any country with no efficiency loss. So, as in the classic Heckscher-Ohlin model, countries have identical factor intensities across industries, which in turn generate comparative advantages across countries with different factor endowments.

An innovator has monopoly rights over his machine and may price it to market. Denote with $w_{ih}$ and $w_{il}$ the wages of skilled and unskilled labor in country $i$, and with $p^M_{ih}$ and $p^M_{il}$ the price of machines that complement skilled and unskilled labor in industry $i$ and country $i$. Final-goods producers choose quantities of skilled and unskilled labor, $h$, $l$, and quantities of machines $a_h$, $a_l$ to minimize the cost of the input bundle:

$$c_{ii} = \min_{h, l, a_h, a_l} \left[ w_{ih} h + w_{il} l + p^M_{ih} a_h + p^M_{il} a_l \right]$$

subject to \[
\left[ (a_h^\alpha h) \frac{\rho-1}{\rho} + (a_l^\alpha l) \frac{\rho-1}{\rho} \right]^{\frac{\rho}{\rho-1}} \geq 1
\]

where $\alpha \in (0, 1)$ and $\rho$ is the elasticity of substitution between skilled and unskilled labor. Assuming $\rho > 0$ and $\alpha(\rho - 1) < 1$, the solution is interior and satisfies\textsuperscript{33}

$$c_{ii} = (1 + \alpha)\frac{\alpha}{1+\alpha} \left[ (w_{ih}(p^M_{ih})^\alpha)^{-\frac{1+\rho}{1-\alpha(\rho-1)}} + (w_{il}(p^M_{il})^\alpha)^{-\frac{1+\rho}{1-\alpha(\rho-1)}} \right]^{\frac{1-\alpha(\rho-1)}{(1-\rho)(1+\alpha)}}$$

$$h = (1 + \alpha)^{\rho} \left( \frac{w_{ih}}{c_{ii}} \right)^{\frac{\rho}{\rho-1}} a_h^{\alpha(\rho-1)}$$

$$l = (1 + \alpha)^{\rho} \left( \frac{w_{il}}{c_{ii}} \right)^{\frac{\rho}{\rho-1}} a_l^{\alpha(\rho-1)}$$

$$p^M_{ih} = \frac{\alpha}{1+\alpha} c_{ii} h^{\frac{\rho-1}{\rho}} a_h^{\alpha \left( \frac{\rho-1}{\rho} \right)^{-1}}$$

$$p^M_{il} = \frac{\alpha}{1+\alpha} c_{ii} l^{\frac{\rho-1}{\rho}} a_l^{\alpha \left( \frac{\rho-1}{\rho} \right)^{-1}}.$$ 

Cost $c_{ii}$ takes the form of a CES price index. It combines the prices of skilled and unskilled inputs, $[w_{ih}(p^M_{ih})^\alpha]$ and $[w_{il}(p^M_{il})^\alpha]$, with a constant elasticity. The exponent $\frac{1}{1+\alpha}$ outside the brackets implies homogeneity of degree one in prices: If $w$ and $p$ double, input cost $c$

\textsuperscript{31}Nothing changes if we assume more generally that $F_{ih} = F_{il}$ so that there is no exogenous difference in factor intensity across types.

\textsuperscript{32}Wages of skilled workers is higher than wages of unskilled workers in both countries if $H_i < L_i$ in $i \in \{S, N\}$ and $F_h < F_l$.

\textsuperscript{33}Values of $\alpha$ and $\rho$ in the literature satisfies these assumptions. See section 3.3 and footnote 34.
doubles. Rearranging, labor and machinery get constant shares of input costs $c_i$:

$$w_{ih}h + w_{il}l = \left(\frac{1}{1 + \alpha}\right) c_i$$

$$a_h p_{ih}^M + a_l p_{il}^M = \left(\frac{\alpha}{1 + \alpha}\right) c_i$$

### 3.4.1 Production III: Results

**Proposition 9** An equilibrium exists if $\alpha \eta < 1$. All countries produce goods in all industries in equilibrium.

The proof is in appendix D.1. In section 3.3, the innovation sector uses only labor as inputs. Here, it uses labor and machinery. As a result, economies of scale are larger here, and the existence condition $\alpha \eta < 1$ is stronger than $\alpha(\eta - 1) < 1$ in section 3.3. Because machines can be used in all countries, input cost $c_i < \infty$ even if the country does not innovate. Then, heterogeneity in production technologies across varieties à la Eaton and Kortum (2002) implies that all countries produce goods in all industries. We henceforth consider only the symmetric equilibrium, where production and consumption in all industries $\iota$ of the same type $\tau$ is the same. Denote with $R_{i\tau}$ the revenue of firms of type $\tau$ from country $i$, with $C_{i\tau} = c_i$ the cost of input bundles in industries $\iota \in \tau$. Using equation (20), free entry into innovation implies:

$$\alpha(1 + \alpha)^{-\rho} A_{i\tau}^{\alpha(\rho-1)-1} \left[ \left(\frac{w_{Si}}{C_{S\tau}}\right)^{1-\rho} R_{S\tau} + \left(\frac{w_{NI}}{C_{N\tau}}\right)^{1-\rho} R_{N\tau} \right] = F_l \min_{N,S} \{C_{S\tau}, C_{N\tau}\},$$

$$\alpha(1 + \alpha)^{-\rho} A_{h\tau}^{\alpha(\rho-1)-1} \left[ \left(\frac{w_{Sh}}{C_{S\tau}}\right)^{1-\rho} R_{S\tau} + \left(\frac{w_{Nh}}{C_{N\tau}}\right)^{1-\rho} R_{N\tau} \right] = F_h \min_{N,S} \{C_{S\tau}, C_{N\tau}\}.$$  

Taking ratios,

$$\left(\frac{A_{i\tau}}{A_{h\tau}}\right)^{1-\alpha(\rho-1)} \frac{F_l}{F_h} = \left(\frac{w_{Si}}{C_{S\tau}}\right)^{1-\rho} + \left(\frac{w_{NI}}{C_{N\tau}}\right)^{1-\rho} r_{\tau} \frac{r_{\tau}}{r_{\tau}} \left(\frac{w_{Sh}}{C_{S\tau}}\right)^{1-\rho} + \left(\frac{w_{Nh}}{C_{N\tau}}\right)^{1-\rho}$$

where $r_{\tau} = \frac{R_{N\tau}}{R_{S\tau}}$ is the ratio of revenue of North to South. It governs the weights of equilibrium wages in North relative in South in determining the equilibrium stock of machines $\frac{A_{i\tau}}{A_{h\tau}}$. If all production takes place in country $i$, $r_{\tau} \in \{0, \infty\}$ and $\frac{A_{i\tau}}{A_{h\tau}} = \left[ \frac{F_h}{F_l} \left(\frac{w_{ih}}{w_{il}}\right)^{\rho-1} \right]^{1-\alpha(\rho-1)}$ depends on the relative cost of inventing skill-complementary machines, $F_h/F_l$, and on the skill premium, $w_{ih}/w_{il}$. The effect of wages on technologies $\frac{A_{i\tau}}{A_{h\tau}}$
depends on $\rho$. To avoid a taxonomy of cases, we consider only the empirically-relevant case $\rho \in (1, 2)$.34

**Lemma 10**  
(i) The skill premium in North is smaller than South, $\frac{w_{Nh}}{w_{Nl}} < \frac{w_{Sh}}{w_{Sl}}$.  
(ii) Production in North is more skill intensive than in South for both types of goods, $\frac{H_{N\tau}}{L_{N\tau}} < \frac{H_{S\tau}}{L_{S\tau}}$ for $\tau = 1, 2$.  
Let $\tau$ be the type of good that is disproportionately produced in North, $r_\tau > r_\tau'$.  
(iii) The production of type $\tau$ goods is more skill intensive than good $\tau'$: $\frac{H_{i\tau}}{L_{i\tau}} > \frac{H_{i\tau'}}{L_{i\tau'}}$ in all countries $i \in \{S, N\}$.  
(iv) North has a comparative advantage in producing type $\tau$: $\frac{C_{N\tau}}{C_{N\tau'}} < \frac{C_{S\tau}}{C_{S\tau'}}$.  

**proof.** The derivative of equation (21) with respect to $r_\tau$ is positive if and only if $\frac{w_{Nh}}{w_{Nl}} < \frac{w_{Sh}}{w_{Sl}}$. Then, $\frac{A_{h\tau}}{A_{l\tau}}$ increases in $r_\tau$ if and only if the skill premium in North is smaller than South, $\frac{w_{Nh}}{w_{Nl}} < \frac{w_{Sh}}{w_{Sl}}$. Contradicting claim (i), suppose $\frac{w_{Nh}}{w_{Nl}} \geq \frac{w_{Sh}}{w_{Sl}}$. Then, (a) goods disproportionately produced in North have lower $\frac{A_{h\tau}}{A_{l\tau}}$ and, from equation (19), these goods also have lower skill intensity $\frac{H_{N\tau}}{L_{N\tau}} \leq \frac{H_{S\tau}}{L_{S\tau}}$. Again from equation (19), (b) the skill intensity in North is lower than in South for both types $\tau$ when $\frac{w_{Nh}}{w_{Nl}} > \frac{w_{Sh}}{w_{Sl}}$. But since North is skill abundant, labor markets would not clear under (a) and (b).

Part (ii) again follows from equation (19): Skill intensity is decreasing in the skill premium $w_h/w_l$ for any stock of machines $A_{h\tau}/A_{l\tau}$. Part (iii) follows because $\frac{A_{h\tau}}{A_{l\tau}}$ is increasing in $r_\tau$. Appendix D.2 proves part (iv). It shows that production costs in North decrease in $r_\tau$ and are minimized when $r_\tau = \infty$.□

In words, lemma 10 states that the skill premium is lower in the skill abundant country, North (i). As a result, the production of all goods is more skill intensive in North than in South (ii). Because of economies of scale, goods disproportionately produced in North are more efficiently made in North (iv), and they are more skill intensive in all countries (iii). Next, lemma 11 and proposition 12 link production to demand patterns and to exogenous technology levels $\frac{T_N}{T_S}$. Together with lemma 10, these propositions imply that the skill-intensive goods disproportionately made in North are luxury goods—they rationalize facts 1, 3 and 4 of section 2.

**Lemma 11** In any equilibrium, if country $i$ consumes relatively more type $\tau$, $\frac{Q_{ix}}{Q_{ix}} > \frac{Q_{ix'}}{Q_{ix'}}$, then it also produces relatively more type $\tau$, $\frac{R_{ix}}{R_{ix}} > \frac{R_{ix'}}{R_{ix'}}$.

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34Estimates of the elasticity of substitution between skilled and unskilled labor $\rho$ range from 1.1 to 1.8. See Katz and Murphy (1992), Lee and Wolpin (2006), Acemoglu and Autor (2011).
The proof is in appendix D.3. Suppose by contradiction $\frac{R_{i\tau}}{R_{j\tau}} \leq \frac{R_{i\tau}'}{R_{j\tau}'}$. The appendix shows switching the ratio of labor allocations between an industry of type 1 and one of type 2, while maintaining the size of each industry, yields a profitable deviation in one of the two industries if the other industry is at its optimal. The small industry size is important because we can change industry-level labor allocations without affecting the rest of the labor market.

**Proposition 12** If $\frac{T_N}{T_S}$ is sufficiently large, North consumes relatively more type 2 goods.

Since the stock of machines is the same in both countries, it is straightforward to impose sufficient conditions for North to be richer than South in all equilibria. Specifically, real wages in North relative to South is lowest in the hypothetical scenario where all machinery is tailored to Southern endowments—i.e., $\frac{A_{h\tau}}{A_{l\tau}}$ equals the ratio of when South is in autarky—and where South holds a disproportionate amount of world spending so that all Northern goods are subject to iceberg costs. We leave the derivation of this lower bound on $\frac{T_N}{T_S}$ as an exercise to the reader. The final two propositions study the implications of the model for inequality and contrast them to the standard Heckscher-Ohlin model.

**Proposition 13** Relative to the autarky ($d = \infty$ and machines are not traded), the skill premium $\frac{w_h}{w_l}$ decreases in North and increases in South.

**Proof.** By setting $r_\tau = 0$ (or $r_\tau = \infty$) in equation (21), we get the autarky level of technologies:

$$\frac{A_{l\tau}}{A_{h\tau}} = \left[ \frac{E_h}{E_l} \left( \frac{w_{h\tau}}{w_{l\tau}} \right)^{\rho-1} \right]^{\frac{1}{\rho-1}}$$

which does not depend on type $\tau$. From equation (19), skill intensity is also independent of type $\tau$ and must then equal endowments $\frac{H_{N\tau}}{L_{N\tau}} = H_L$ for $\tau = 1, 2$. Now consider the trade equilibrium. If by contradiction $\frac{w_{N\tau}h}{w_{N\tau}l} > \frac{w_{N\tau}h_{autarky}}{w_{N\tau}l_{autarky}}$, then $\frac{w_{S\tau}h}{w_{S\tau}l} > \frac{w_{S\tau}h_{autarky}}{w_{S\tau}l_{autarky}}$ by lemma 10(i).

Then, by equation (21), $\frac{A_{l\tau}}{A_{h\tau}} > \frac{A_{l\tau}}{A_{h\tau}}_{\frac{w_{N\tau}h}{w_{N\tau}l}}$ for $\tau = 1, 2$. Together with $\frac{w_{N\tau}h}{w_{N\tau}l} > \frac{w_{N\tau}h_{autarky}}{w_{N\tau}l_{autarky}}$, this inequality implies that skill intensity of both types is lower under trade than autarky, $\frac{H_{N\tau}}{L_{N\tau}} < \frac{H_{N\tau}}{L_{N\tau}}_{\frac{w_{N\tau}h}{w_{N\tau}l}} = H_L$ for $\tau = 1, 2$. Labor markets then cannot clear in North. Using the symmetric argument, we conclude that the skill premium increases in South.  

Assumption $\frac{Q_{S\tau}}{Q_{N\tau}} < \frac{Q_{S\tau}'}{Q_{N\tau}'}$ is stronger than the equivalent statement on spending used in lemma 1 and lemma 4. Northern relative prices in type $\tau$ could be sufficiently low so that spending $\frac{X_{N\tau}}{X_{N\tau}'} > \frac{X_{S\tau}}{X_{S\tau}'}$ even though quantities $\frac{Q_{S\tau}}{Q_{N\tau}} < \frac{Q_{S\tau}'}{Q_{N\tau}'}$.  

27
This prediction is the exact opposite of a standard Heckscher-Ohlin model. Openness here makes technologies in North less skill biased and technologies in South more skill biased. As a result, it decreases the skill premium in North and increases it in South. But the term openness here is much broader than the usual exercise of decreasing trade costs—it presumes that no technology diffuses in autarky. The usual result is recovered, even exacerbated, if we allow technologies to diffuse perfectly (machines are frictionlessly traded) when costs of trading final goods are prohibitively high. Then, decreasing trade costs in final goods has two effects. First is the standard effect: Trade shifts production in North toward more skill-intensive goods. Second, this shift in production increases skill-bias innovation in originally skill-intensive goods. Both of these effects increase the skill premium in North and decrease it in South.

Proposition 13 is not intended to be realistic. Evidence suggests that international trade increases the skill premium in both developed and developing countries, and economists have proposed several theoretical mechanisms to explain this pattern. The objective is to understand the effect of factor-biased innovation on the skill premium. Proposition 14 further contrasts the predictions of our model to the standard model:

**Proposition 14** Factor prices never equalize. If \( d = 1 \), the skill premium in North is smaller than South, \( \frac{w_{Nh}}{w_{Nl}} < \frac{w_{Sh}}{w_{Sl}} \).

**Proof.** Suppose by contradiction that factor prices equalized, \( \frac{w_{Nh}}{w_{Nl}} = \frac{w_{Sh}}{w_{Sl}} \). Then from equation (21), the ratio of machines would be equal in the two types, \( \frac{A_{h1}}{A_{l1}} = \frac{A_{h2}}{A_{l2}} \), and from equation (19), relative demand for skilled labor would be equal for all types and countries, \( \frac{H_{N1}}{L_{N1}} = \frac{H_{N2}}{L_{N2}} = \frac{H_{S1}}{L_{S1}} = \frac{H_{S2}}{L_{S2}} \). Then, labor markets in North and South could not simultaneously clear. Hence, in equilibrium when \( d = 1 \), \( \frac{w_{Nh}}{w_{Nl}} < \frac{w_{Sh}}{w_{Sl}} \) and some industries must be more skill intensive than others, \( \frac{a_{h1}}{a_{l1}} > \frac{a_{h1}}{a_{l1}}' \). North is relatively more productive and produces more in the skill intensive industries. But with \( d = 1 \), there is no home-market effect, and patterns of production are independent of local patterns of demand.

To summarize, we study three sources of home-market effects on innovation. In all cases, non-homothetic preferences and trade costs together imply that technologically-advanced North disproportionately produces and consumes income-elastic goods. With endogenous innovation, North develops a comparative advantage in producing these goods.

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36See Stolper and Sammelson (1941).
37See, for example, Feenstra, Hanson (1996), Antrás, Garicano, Rossi-Hansberg (2006), Burstein, Vogel (2012). Because the development of new technologies is slow, it would be difficult to observe in the data, the predictions of proposition 13.
but the source of comparative advantage differs across specifications. In section 3.2, North produces a greater variety of income-elastic goods. In section 3.3, firm productivity in North is relatively higher in income-elastic goods. In section 3.4, income-elastic goods use intensively skilled labor, North’s abundant factor. Our empirical results in section 2 suggest that all three sources of comparative advantage are present in the data (fact 3). Directly testing the prediction from section 3.4 that skill premium is lower in rich countries is difficult because it requires the comparison of educational levels across countries. Still, the prediction is suggested by the finding that the production of all goods—luxury and basic goods—disproportionately use skilled labor in rich countries.

4 Extensions

We discuss extensions of the model that relax preferences and number of countries. In all preference specifications, $Q_\tau$ is the aggregate consumption of type $\tau$ defined in equation (5) for the monopolistic set up and in equation (9) for the other set ups.

**Multiple types.** Let the set of types be $\{1, ..., \Gamma\}$ ordered from most to least essential. Utility is

$$U = \sum_{t=1}^{\tau-1} B_t + Q_\tau$$

where $\tau = \min\{t \in \{1, ..., \Gamma\} : Q_t < B_t\}$

Consumers need $B_\tau$ units of type $\tau$ before enjoying utility from type $\tau+1$, where $B_\tau \in \mathbb{R}_+$ for $\tau \leq \Gamma - 1$ and $B_\Gamma = \infty$. Throughout the text, we assumed that consumers in both countries consumed both types 1 and 2. All results are readily extended to multiple types if North and South consume the same set of types $\{1, ..., \bar{\Gamma}\}$, with $\bar{\tau} \leq \Gamma$. All types $\tau < \bar{\tau}$ may be treated as type 1 in the previous section: South consumes, produces and has a comparative advantage in all $\tau < \bar{\tau}$ relative to $\bar{\tau}$.

The case where the set of types consumed is different between North and South is also analogous to the case where only North consumes the income-elastic type $\tau = 2$ in the set ups of section 3. Since we did not consider this possibility above, we discuss it here. In all three set ups, North would produce relatively more type $\tau = 2$. It would have a greater variety of type 2, $\frac{M_{N2}}{M_{N1}} > \frac{M_{S2}}{M_{S1}}$, in the monopolistic-competition set up, and a lower input cost, $\frac{C_{N2}}{C_{N1}} < \frac{C_{S2}}{C_{S1}}$, in the other two set ups. Despite this difference in comparative advantage, North would still be a net importer of type 2 since Southern consumption is
assumed to be zero.\textsuperscript{38}

**Stone-Geary preferences.** Appendix E.1 shows that all results hold if preferences take the Stone-Geary form:

\[
U = (Q_1 - B)\gamma Q_2^{1-\gamma}
\]

where \( B \) is a minimum consumption requirement for basic goods \( \tau = 1 \) and \( \gamma \in (0, 1) \). The original preference specification is the limiting case where \( B = 1 \) and \( \gamma \to 0 \).

**Constant Relative Income Elasticity (CRIE) preferences.** Let utility be:

\[
U = \left( \frac{\delta_1}{\delta_1 - 1} \right) Q_1^{\delta_1 - 1} + \left( \frac{\delta_2}{\delta_2 - 1} \right) Q_2^{\delta_2 - 1}
\]

where \( 1 < \delta_1 < \delta_2 \). At all levels of income and prices, the income elasticity of demand for goods of type 1 relative to type 2 is \( \frac{\delta_1}{\delta_2} < 1 \). Per capita spending on type \( \tau \) in country \( i \) is

\[
X_{i\tau} = \lambda_i^{-\delta_r} P_{i\tau}^{1-\delta_r}
\]

where \( \lambda_i \) is the Lagrange multiplier, implicitly defined through the budget constraint. The original preference specification was used only in propositions 2, 5 and 8, which establish sufficient conditions for North to consume relatively more of the income-elastic goods \( \tau = 2 \)—i.e., \( \frac{X_N}{X_S} > \frac{X_S}{X_S} \). With the original preference structure, demand for type 1 is inelastic, and the location of production only has an income effect. Here, it has an additional substitution effect: If the production of type 2 moves to South, it decreases the relative price of type 2 in South and increases relative demand, \( \frac{X_S}{X_S} \). So, the sufficient condition for North produce more type 2 and to be richer than South is more restrictive under CRIE preferences (see appendix E.2).\textsuperscript{39}

\textsuperscript{38}The exception is if North is the only producer of type 2, which may occur in the monopolistic competition and occurs in the Ricardian set up if \( \alpha \theta > 1 \).

\textsuperscript{39}The conditions in the appendix are

\[
e < d^{\delta - 1}
\]

\[
\left[ d^\theta \frac{T_S}{T_N} \left( \frac{L_N}{L_S} \right)^{1-\delta \alpha} \right] ^{1/\theta} < d^{\tilde{\delta}}
\]

\[
\left[ d \frac{T_S}{T_N} \right] ^{1/\alpha} < d^{\tilde{\delta}}
\]

where \( \tilde{\delta} = \frac{\delta_1 + \delta_2 - 2\delta_1 \delta_2}{\delta_2 - \delta_1} < 0 \). These conditions converge to the original ones in section 3 as \( d \to 1 \).
**Multiple countries**  Highly stylized assumptions allow us to make stark predictions about inequality, consumption and production patterns. These predictions are lost if we extend the model to multiple countries and arbitrary trade costs. As in usual quantitative trade models, the demand for a country’s goods depends not only on its own income level and size, but also on its neighbors’.

Still, an extension of the model of section 3.4 to an arbitrary number of countries and types, general iceberg trade costs, type-specific Frechet heterogeneity parameter $\theta$, CRIE preferences, and country-type specific technology parameters $T_{ik}$ shares the quantitative predictions of Caron, Fally, and Markusen (2014, CFM) on bilateral trade flows by sector, demand, production and factor usage by country and sector. This extension is relevant because CFM replicate well these features of the data and because the empirical section 2 borrows specifications from the structural estimation in CFM. In future work, the extended model and new data sets may prove useful to disentangle exogenous differences in comparative advantage from endogenous innovation.

5  Conclusion

We develop a model to study three mechanisms through which home-market effects influence innovation: Firms in rich countries develop product varieties to suit the preferences of their consumers for high-quality and luxury goods. To the extent that geographically close firms share know-how, labor and input markets, the features of these income-elastic goods and the technologies to produce them will be better in rich countries. And to make use of local factors of production, technologies to produce income-elastic goods tend to complement skilled labor and substitute unskilled labor. The model rationalizes a series of systematic patterns of demand, production and trade that we document in the data. Quantifying these different mechanisms and separating them from exogenous sources of comparative advantage is a challenging and promising path for future work.

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40It is straightforward to define such an extended model. The difficulty would be to derive propositions equivalent to those in section 3.
References


A Appendix on the estimation of income elasticities

With the WIOD and GTAP data, the estimation method bellow follows Caron, Fally and Markusen (2014). We also extend the method to obtain estimates of income elasticities at the HS 4-digit level, at which expenditure data is unavailable. The goal is to infer expenditures and preferences from trade data, using gravity equations to identify demand and productivity shifters at the product level, and using more aggregated data to identify the border effect (i.e. relative size of domestic vs. international trade).

A.1 Assumptions

To guide our estimation of income elasticities, we adopt the same structure as in Fieler (2011) and Caron et al (2014). We assume that preferences take the form:

\[ U = \sum_k \alpha_k Q_k^{\delta_k-1} \delta_k \]

where \( Q_k \) is a CES aggregate of varieties within product category \( k \). This leads to the following expression for total expenditure \( E_{nk} \):

\[ E_{nk} = L_n \alpha_k \lambda_n^{-\delta_k} P_{nk}^{1-\delta_k} \]

where \( L_n \) is the number of consumers in country \( n \) and \( \lambda_n \) is the Lagrangian determined by the consumer’s budget constraint. Trade, in turn, is assumed to satisfy:

\[ X_{nik} = A_{ik}^\delta d_{ij}^{\theta_k} \frac{E_{nk}}{\Phi_{nk}} \quad (A.1) \]

where \( A_{ik} \) is interpreted as described in the text, and:

\[ \Phi_{nk} = \sum_i A_{ik} d_{ni}^{-\theta_k} \quad (A.2) \]

and where the price index satisfies:

\[ P_{nk} = \alpha_k^\prime \Phi_{nk}^{-\frac{1}{\delta_k}} \]
A.2 Strategy: first step

In a first step, we estimate gravity with both exporter fixed effects (to recover $A_{ik}$) and importer fixed effects $FM_{nk}$:

$$X_{nik} = A_{ik} d_{ni}^{-\theta_k} FM_{nk}$$

(with the normalization $A_{US,k} = 1$). As recommended by Santos Silva and Tenreyro (2006) and Fally (2015), we estimate this gravity equation with Poisson-PML. We estimate gravity for separately for each product category or sector for each dataset (BACI, WIOD and GTAP). Using exporter fixed effect and equation A.2, we can then reconstruct $\hat{\Phi}_{nk}$ and use it as a proxy for prices as in Caron et al (2014).

An important caveat, however, is that the home bias or “border effect” is not identified in the BACI trade data since these data do not provide internal trade flows. We use a mapping between HS 4-digit products (BACI) and the WIOD data to estimate of the border effect (assumed to be constant within each broad sector). Using the full set of exporter fixed effects $A_{ik}$ at the HS 4-digit level, we can then construct $\Phi_{nk}$ at a disaggregated level.

In addition, we construct an index $\Phi^*_{nk}$ solely based on foreign countries’ fixed effect for each market $n$, which can be interpreted as a foreign Market Access index:

$$\Phi^*_{nk} = \sum_{i \neq n} A_{ik} d_{ni}^{-\theta_k}$$

(excluding country $i = n$ from the sum). Note that $\Phi^*$ can be constructed in BACI without relying on estimates of the border effect from WIOD.

A.3 Strategy: second step

The next step is to estimate a final demand equation and income elasticities $\delta_k$. Across broad sectors, where we have data on total expenditures by sector and country, we can directly use expenditures as the left hand side variable. Using the approximation that the Lagrangian $\lambda_k$ is log-linear in per capita income: $\log \lambda_n \approx -\log y_n$, we obtain an equation that identifies income elasticity $\delta_k$ for each sector:

$$\log E_{nk} = \log \alpha_k + \delta_k \log y_n + \mu_k \log \hat{\Phi}_{nk}$$

using our estimates for $\hat{\Phi}_{nk}$, and observed real GDP per capita $y_n$. We treat $\log \alpha_k$ as an industry fixed effect.

Since total expenditure data are not available at the 4-digit HS level, we instead
suggest use importer fixed effect to estimate income elasticities. Following the structural gravity framework, the importer fixed effect should equal: \( FM_{nk} = \frac{E_{nk}}{\Phi_{nk}} \). Incorporating the expression above for final demand, the importer fixed effect \( FM_{nk} \) should then correspond to:

\[
FM_{nk} = b_k \alpha_k \lambda_n^{-\delta_k} \Phi_{nk}^{\frac{1-\sigma_k-\theta_k}{\delta_k}}
\]

Using again the approximation that the Lagrangian \( \lambda_k \) is log-linear in per capita income, we obtain our estimated equation for each distinct 4-digit product category:

\[
\log FM_{nk} = \log(\alpha_k b_k) + \delta_k \log y_n + \mu_k \log \Phi_{nk}
\]

using importer/industry fixed effects \( FM_{nk} \) as the left-hand side\(^{41}\), and industry fixed effects on the right-hand side to control for preference shifters \( \alpha_k \) and the border effect \( b_k \).

In both final demand specifications, we instrument \( \Phi_{nk} \) by international market access \( \Phi_{nk}^* \) to ensure that our results are not driven by spurious correlations between the importer fixed effects \( FM_{nk} \) and the exporter fixed effects \( A_{nk} \) that enter the calculation of \( \Phi_{nk} \) (but do not enter the construction of \( \Phi_{nk}^* \)).

**B Appendix for production I: Proof of prop. 2**

This appendix proves proposition 2. The proposition states *South consumes, produces and exports relatively more type 1, and North, type 2. Utility is lower in South than North.* We prove the first statement in subsection B.1.1 for the case of diversification and in section B.2 for the case of specialization. The welfare statement has the same proof in both cases, and it is in section B.3.

**B.1 Appendix for Production I: Full diversification**

From lemma 1, it suffices to show that South consumes relatively more type 1. We first express equilibrium mass of firms \( M_{i\tau} \) as a function of endogenous wage \( w \) and exogenous variables. Spending on type 1 in country \( i \) is \( L_i P_i \). Substituting in the pricing equation

\(^{41}\)This formulation implicitly equates expenditures with total absorption. While our data do not distinguish final good trade from trade in intermediates, we find that our results are robust to dropping products classified as Parts and accessories; Processed goods; Primary goods; or Investment goods in the Broad Economic Classification (BEC).
yields

\[ p_1 = \left[ \frac{L_S}{L_N} \left( \frac{w^{-\sigma} - d_1^{-\sigma}}{1 - d_1^{-\sigma} w^{-\sigma}} \right) \right]^{1/\sigma} \]  \hspace{1cm} \text{(B.1)}

Using equations (6) and (7), the equilibrium mass of firms is

\[ M_{S1} = \left\{ \frac{c_1}{(\sigma - 1)f_1 w^\sigma} \left[ d_1^{-\sigma} \left( m + (dw)^{1-\sigma} \right)^{\sigma \over 1-\sigma} L_N + (d_1^{-\sigma} m + w^{1-\sigma})^{\sigma \over 1-\sigma} L_S \right] \right\}^{\sigma - 1 \over \sigma} \]

where \[ m_1 = \frac{M_{N1}}{M_{S1}} = w^{1-\sigma} \left( \frac{p_1^{1-\sigma} - d_1^{1-\sigma}}{1 - p_1^{1-\sigma} d_1^{1-\sigma}} \right) \] \hspace{1cm} \text{(B.2)}

Although we do not use it in the proof, given \( P_{N1} \) and \( P_{S1} \), spending on type 2 is also a function of \( w \) and exogenous variables:

\[ X_{N2} = L_N (1 - P_{N1}) \]
\[ X_{S2} = L_N (ew - P_{S1}) \]

Rearranging equations (7) and (8) we get:

\[ M_{S2} = \frac{1}{\sigma f_{S2} w} \left[ \frac{X_{S2}}{m(w/d)^{\sigma - 1} + 1} + \frac{d_1^{-\sigma} X_{N2}}{m w^{\sigma - 1} + d_1^{-\sigma}} \right] \]

where \[ m_2 = w^{1-\sigma} \left[ x_2(w^{\sigma} - d_1^{1-\sigma}) - d_1^{1-\sigma} (1 - d_1^{1-\sigma} w^{\sigma}) \right] \]
\[ \left[ (1 - d_1^{1-\sigma} w^{\sigma}) - x_2 d_1^{1-\sigma} (w^{\sigma} - d_1^{1-\sigma}) \right] \] \hspace{1cm} \text{(B.3)}

\[ \text{B.1.1 Appendix for production I: Patterns of exports} \]

South is a net exporter of type 1 if and only if it consumes relatively more type 1:

\[ \frac{X_{S1}}{X_{N1}} = \frac{L_S P_{S1}}{L_N P_{N1}} > \frac{L_S (ew - P_{S1})}{L_N (1 - P_{N1})} = \frac{X_{S2}}{X_{N2}} \Leftrightarrow p_1 < (ew)^{-1} \] \hspace{1cm} \text{(B.4)}

There are now two cases. **Case 1:** \( w \geq 1 \) From equation (B.4), South is a net exporter of type 1 if and only if \( p_1 < 1/(ew) \Leftrightarrow p_1^{\sigma} > (ew)\sigma \). Assume by contradiction that this
inequality does not hold. Then, from equation (B.1),

\[(ew)\sigma \geq \left( \frac{1 - w^{-\sigma} d^{1-\sigma}}{w^{-\sigma} - d^{1-\sigma}} \right) \frac{L_N}{L_S} \]

\[\geq \left( \frac{1 - w^{-\sigma} d^{1-\sigma}}{w^{-\sigma} - d^{1-\sigma}} \right) e \]

\[\Leftrightarrow e^{\sigma-1} \geq w^{-\sigma} \left( \frac{1 - w^{-\sigma} d^{1-\sigma}}{w^{-\sigma} - d^{1-\sigma}} \right) \]

\[= w^{-\sigma} \left( \frac{w^{-\sigma} - d^{1-\sigma}}{1 - w^{-\sigma} d^{1-\sigma}} \right) \]

\[= \frac{1 - w^{-\sigma} d^{1-\sigma}}{1 - w^{-\sigma} d^{1-\sigma}} \geq 1 \quad \text{if } w \geq 1 \]

where the second line uses \(L_N \geq eL_S\) and the last inequality comes because the right-hand-side is strictly increasing in \(w\) and it is equal to one at \(w = 1\). This is a contradiction because \(e < 1\).

**Case 2: \(w < 1\)**

**step 2.1** Again by contradiction, suppose that the South is a net exporter of type 1 goods. Using the demand function for type 1, South’s imports from North are greater than or equal to its exports whenever

\[M_{N1} P_{S1}^\sigma L_S \geq M_{S1} w^{1-\sigma} P_{N1}^\sigma L_N \]

\[\equiv \frac{m_1}{w^{1-\sigma}} \geq p_1^\sigma \frac{L_N}{L_S} \geq p_1^\sigma e \]

where the last inequality follows from \(\frac{L_N}{L_S} \geq e\). Again by the contradiction hypothesis and by equation (B.4) above, \(p_1^\sigma \geq (ew)^{-\sigma}\). Then,

\[\frac{m_1}{w^{1-\sigma}} \geq w^{-\sigma} e^{1-\sigma} > 1 \]

where the last line comes from \(w < 1\) and \(e < 1\).

**step 2.2** From equation (B.2),

\[\frac{m_1}{w^{1-\sigma}} = \frac{p_1^{1-\sigma} - d^{1-\sigma}}{1 - (p_1 d)^{1-\sigma}} < 1 \]

where the inequality comes from the contradiction hypothesis, \(p_1 \geq (ew)^{-1} > 1\) whenever \(e < 1\) and \(w < 1\). Steps 2.1 and 2.2 contradict each other. Hence, South is a net exporter of type 1 goods. ■
B.2 Appendix for Production I: Specialization

We now prove the first statement of proposition 2 when either North or South produce only one type of good. The statement is that South is a net exporter of type-1 goods and the North is a net exporter of type-2 goods. We show here by contradiction that (i) the South cannot specialize in type 2 and (ii) the North cannot specialize in type 1.

**Case 1: South specializes in type 2 goods.** Then, using the ratio of the zero-profit conditions modifies equation (7) to:

\[
\begin{align*}
\frac{x_1}{x_2} &\geq \left( \frac{p_2}{p_1} \right)^{\sigma - 1} > 1 \\
\Rightarrow x_1 &\geq \left( \frac{p_2}{p_1} \right)^{\sigma - 1}
\end{align*}
\]

where the last inequality arises because \( \frac{p_2}{p_1} = dp_2 > 1 \). Then,

\[
\frac{X_{S1}}{X_{N1}} = \frac{L_SP_{S1}}{L_NP_{N1}} < \frac{L_S(ew - P_{S1})}{L_N(1 - P_{N1})} = \frac{X_{S2}}{X_{N2}} \Leftrightarrow p_1 > (ew)^{-1}
\]

Again, since the South specializes in type 2, \( p_1 = 1/d \). And hence \( w > d/e > d \). But then equation (6), allowing for incomplete specialization in North, is

\[
\frac{x_2}{p_2} \leq \left( \frac{w^{-\sigma} - d^{1-\sigma}}{1 - d^{1-\sigma}w^{-\sigma}} \right) < 0
\]

where the last inequality arises from \( (w^{-\sigma} - d^{1-\sigma}) < 0 \) when \( w > d \). This inequality obviously cannot hold, and hence South cannot specialize in type 2.

**Case 2: North specializes in type 1, and South diversifies** The proof here is the same as in appendix B.1.1. We only need to show that if North specializes in type 1, lemma 1 holds. That is, \( x_1 > x_2 \) and \( m_1 > m_2 \). With North’s specialization, \( m_1 > m_2 \) is trivial since \( m_2 = M_{N2} = 0 \) and \( M_{N1} > 0 \). The proof that \( x_1 > x_2 \) follows the derivation
of equation (B.5):

$$x_1 p_1^{\sigma-1} = \left( \frac{w-\sigma - d^{1-\sigma}}{1 - d^{1-\sigma} w^{-\sigma}} \right)$$

$$x_2 p_2^{\sigma-1} \leq \left( \frac{w-\sigma - d^{1-\sigma}}{1 - d^{1-\sigma} w^{-\sigma}} \right)$$

$$\Rightarrow \frac{x_1}{x_2} \geq \left( \frac{p_2}{p_1} \right)^{\sigma-1} > 1$$

(B.6)

where the last inequality arises because $\frac{p_2}{p_1} = \frac{d}{p_1} > 1$.

B.3 Appendix for Production I: Homebias and welfare

Given utility function, per capita aggregate quantity of type 1 must equal 1 in both countries:

$$1 = \frac{X_{1N}/(P_{1N}L_N)}{X_{1S}/(P_{1S}L_S)} = \frac{X_{1N}/L_N}{X_{1S}/(L_S)} \frac{M_{N1} + M_{S1}(dw)^{1-\sigma}}{M_{N1}d^{1-\sigma} + M_{S1}w^{1-\sigma}}^\frac{1}{\sigma-1}$$

$$= \frac{L_S}{L_N} \frac{m_1 + (dw)^{1-\sigma}}{m_1 d^{1-\sigma} + w^{1-\sigma}}^{\frac{1}{\sigma-1}}$$

$$< \frac{L_S}{L_N} \frac{m_2 + (dw)^{1-\sigma}}{m_2 d^{1-\sigma} + w^{1-\sigma}}^{\frac{1}{\sigma-1}}$$

(B.7)

$$< \frac{L_S}{L_N} \frac{m_2 + (dw)^{1-\sigma}}{m_2 d^{1-\sigma} + w^{1-\sigma}}^{\frac{1}{\sigma-1}}$$

(B.8)

$$= \frac{X_{2N}/(P_{2N}L_N)}{X_{2S}/(P_{2S}L_S)}$$

where the last line is the per capita utility in North relative to South. From lemma 1, $x_1 < x_2$ and $m_1 < m_2$. Inequality (B.7) follows from $x_1 < x_2$. Taking derivatives, one can show that expression $\left[ \frac{m + (dw)^{1-\sigma}}{m(d/w)^{1-\sigma}+1} \right]$ is increasing in $m$ if $d > 1$. Hence inequality (B.8) follows from $m_1 < m_2$.

C Appendix for production II

C.1 Appendix for production II: Proof of prop. 3

This appendix proves proposition 3: An equilibrium exists if $\alpha(\eta - 1) < 1$. If $\alpha \theta < 1$, then all countries produce varieties in all industries (diversification). If $\alpha \theta \geq 1$ then all production in an industry in one country (specialization). Consider the problem of a
representative firm in an industry of choosing labor allocation \((l_S, l_N)\) to maximize profits. The firm takes as given the price index and demand of type \(\tau\). To simplify notation, we omit industry and type subscripts, and assume \(T_S = T_N = d = 1\). Nothing conceptually changes if \(d > 1\). The firm’s problem is

\[
\max_l \left[ w^{-\theta} l_S^{\alpha \theta} + l_N^{\alpha \theta} \right]^{\frac{\eta-1}{\theta}} D - w l_S - l_N
\]

where \(D = P_N^{\eta-1} X_N + P_S^{\eta-1} X_S\) is aggregate type\(\tau\) demand. If \(l_S = 0\), then the problem becomes:

\[
\max_l D l^{\alpha (\eta - 1) - 1} - l
\]

If \(\alpha(\eta - 1) > 1\), then the problem has no solution. The firm attains infinite profits by setting \(l\) to infinity. If \(\alpha(\eta - 1) = 1\), then the problem is linear, and it only has finite solution(s) for very specific cases of \(a\) and \(b\). Now assuming \(\alpha(\eta - 1) < 1\), consider again problem (C.1). If \(\alpha \theta > 1\), then the term in brackets is maximized when all labor goes to North or all goes to South. If \(\alpha \theta < 1\), the term in brackets is maximized when labor is split between North and South. So, there is diversification in the first case and diversification in the second. An alternative way to see it is by taking first order conditions with respect to \(l_N\) (it is almost symmetric to \(l_S\)):

\[
\alpha(\eta - 1) l_N^{\alpha \theta - 1} \left[ w^{-\theta} l_S^{\alpha \theta} + l_N^{\alpha \theta} \right]^{\frac{\eta-1}{\theta}} - 1 \leq 0
\]

with equality if \(l_N = 0\). If \(\alpha \theta < 1\), then the right-hand-side tends to one as \(l_N\) tends to zero and \(l_S > 0\), which clearly violates the first order conditions. That is, if \(\alpha \theta < 1\) the solution is interior. If, \(\alpha \theta > 1\), then the first order conditions are satisfied at zero. Note further that problems (C.1) and (C.2) have a solution if \(\alpha(\eta - 1) \in (0, 1)\). So, to prove that an equilibrium exists we need to show that there exists a \(w\) that clears the market. But as \(w \to \infty\), the demand for Southern labor decrease at a rate \(-\theta < -1\) for a given aggregate demand \(D\), and aggregate demand \(D\) for any type increases at most at a rate of one-to-one with wages. The similar argument holds for demand for Northern labor as \(w\) tends to zero. So, we can constraint \(w\) to a compact set bounded away from zero. And a fixed point exists.
C.2 Appendix for production II: Symmetry

This appendix shows that all equilibria are symmetric if \( \alpha \theta < 1 \) and \( \eta < \theta + 1 \). That is, labor allocation is the same in all industries of the same type: \( \ell_i = \ell_{i'} \) for all \( i \in \{S, N\} \), all \( i, i' \in \tau \) and all \( \tau \in \{1, 2\} \). Let \( (\ell_N, \ell_S) \) be the average labor allocation of industries \( i \) and \( i' \) in North and South. Consider the problem of deviating from this average allocation by taking \( (\epsilon_N, \epsilon_S) \in \mathbb{R}^2 \) units labor from \( i \) to \( i' \). Since labor costs do not change and revenue is proportional to \( p_{i\tau}^{1-\eta} \), the problem takes the form:

\[
\max_{\ell} \left[ a(l_N + \epsilon_N)^{\alpha \theta} + b(l_S + \epsilon_S)^{\alpha \theta} \right]^{\frac{\eta-1}{\theta}} + \left[ a(l_N - \epsilon_N)^{\alpha \theta} + b(l_S - \epsilon_S)^{\alpha \theta} \right]^{\frac{\eta-1}{\theta}}
\]

where \( a > 0 \) and \( b > 0 \) are type-specific variables that the firm takes as constant. From symmetry, we take derivatives only with respect to \( \epsilon_N \). The first order condition is:

\[
a\alpha(\eta - 1) \left[ a(l_N + \epsilon_N)^{\alpha \theta} + (l_S + \epsilon_S)^{\alpha \theta} \right]^{\frac{n-1-\theta}{\theta}} (l_N + \epsilon_N)^{\alpha \theta - 1} - a\alpha(\eta - 1) \left[ a(l_N - \epsilon_N)^{\alpha \theta} + (l_S - \epsilon_S)^{\alpha \theta} \right]^{\frac{n-1-\theta}{\theta}} (l_N - \epsilon_N)^{\alpha \theta - 1} = 0
\]

Ignoring the positive constant \( a\alpha(\eta - 1) \), the second order condition is

\[
a\alpha(\eta - 1 - \theta) \left[ a(l_N + \epsilon_N)^{\alpha \theta} + (l_S + \epsilon_S)^{\alpha \theta} \right]^{\frac{n-1-2\theta}{\theta}} (l_N + \epsilon_N)^{2(\alpha \theta - 1)} + a\alpha(\eta - 1 - \theta) \left[ a(l_N - \epsilon_N)^{\alpha \theta} + (l_S - \epsilon_S)^{\alpha \theta} \right]^{\frac{n-1-2\theta}{\theta}} (l_N - \epsilon_N)^{2(\alpha \theta - 1)} + (\alpha \theta - 1) \left[ a(l_N + \epsilon_N)^{\alpha \theta} + (l_S + \epsilon_S)^{\alpha \theta} \right]^{\frac{n-1-\theta}{\theta}} (l_N + \epsilon_N)^{\alpha \theta - 2} + (\alpha \theta - 1) \left[ a(l_N - \epsilon_N)^{\alpha \theta} + (l_S - \epsilon_S)^{\alpha \theta} \right]^{\frac{n-1-\theta}{\theta}} (l_N - \epsilon_N)^{\alpha \theta - 2} < 0
\]

It is easy to see that the first order condition is satisfied at \( (\epsilon_N, \epsilon_S) = 0 \). Since \( (\eta - 1 - \theta) < 0 \) and \( (\alpha \theta - 1) < 0 \), the second order condition is satisfied everywhere. These conditions are sufficient for \( (\epsilon_N, \epsilon_S) = 0 \) to be the only maximum.

C.3 Appendix for production II: Specialization

This appendix discusses the case where \( \alpha \theta > 1 \) only one country produces goods in each industry. Given the choice of location \( i \), the representative firm’s profit-maximization problem takes the form:

\[
\pi_{i\tau} \max_{l \geq 0} p_{i\tau}^{1-\eta} D_{i\tau} - w_l
\]

where \( p_{i\tau} = \gamma_{i\tau} T_i^{1/\theta} w_l^{1-\alpha} D_{i\tau} \)

\[
= P_{i\tau}^{\eta-1} X_{i\tau} + d^{1-\eta} P_{j\tau}^{\eta-1} X_{j\tau}
\]

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where $j \neq i$. The unique optimal and resulting profit are, respectively:

$$
    l_{ui} = \left[ \alpha(\eta - 1) \gamma_{\tau}^{\eta - 1} w_i^{-\eta} D_{\tau} \right]^{1/(\alpha(\eta - 1))}
$$

$$
    \pi_{ui} = [1 - \alpha(\eta - 1)] [\alpha(\eta - 1)]^{1/(\alpha(\eta - 1))} \left[ \gamma_{\tau}^{\eta - 1} w_i^{(1-\eta)(1+\alpha)} D_{\tau} \right]^{1/(\alpha(\eta - 1))}
$$

Producing in North is more profitable than in South if $\pi_{Ni} \geq \pi_{Si}$ or equivalently,

$$
    \left[ \theta_N^{\eta - 1} - d^{1-\eta} \theta_S^{\eta - 1} w_{\eta(1-\eta)(1+\alpha)} \right] P_{N\tau}^{\eta - 1} X_{N\tau} \geq \left[ \theta_S^{\eta - 1} w_{(1-\eta)(1+\alpha)} - d^{1-\eta} \theta_N^{\eta - 1} \right] P_{S\tau}^{\eta - 1} X_{S\tau}
$$

We now show Lemma 7. If North disproportionately produces type $\tau$, then $P_{N\tau}^{\eta - 1} X_{N\tau} < P_{S\tau}^{\eta - 1} X_{S\tau}$ and $P_{S\tau}^{\eta - 1} X_{S\tau} \geq P_{N\tau}^{\eta - 1} X_{N\tau}^{\eta - 1}$ from zero-profit. Hence, $X_{N\tau} X_{S\tau} < X_{N\tau}^{\eta - 1} X_{S\tau}^{\eta - 1}$ as the proposition states. Proposition 8 is obtaining by observing that in order for production of any type to occur in South,

$$
    \theta_S^{\eta - 1} w_{(1-\eta)(1+\alpha)} - d^{1-\eta} \theta_N^{\eta - 1} > 0
$$

$$
    \equiv w < \left[ \frac{\theta_S}{\theta_N} \right]^{\frac{1}{1+\alpha}}
$$

From section 3.3.1, if South consumes relatively more type 2, then $w > d^{-1}$. Hence, if $\left[ d^{-1} \frac{\theta_S}{\theta_N} \right]^{\frac{1}{1+\alpha}} < d^{-1}$, then South is poorer than North. That is, it consumes relatively less type 2. ■

**D Appendix for production III**

**D.1 Appendix for production III: Proof of prop. 9**

We consider for simplicity only the closed economy. The problem for the open economy is qualitatively the same, but the notation is much heavier. Consider the problem of a representative firm in industry $i$ choosing the quantities of machinery, and skilled and unskilled labor for production. Given that spending on labor is proportional to spending on machinery,

$$
    w_{ih} h + w_{il} l = \frac{1}{\alpha} \left( A_h p_{ih}^\alpha + A_l p_{il}^\alpha \right)
$$
the problem is equivalent to choosing only machinery:

\[
\max_{(A_h, A_l) \geq 0} \left( \frac{w_h}{A_h^{\alpha}} \right)^{1-\rho} + \left( \frac{w_l}{A_l^{\alpha}} \right)^{1-\rho} D - (A_h F_h + A_l F_l) \left( \frac{w_h}{A_h^{\alpha}} \right)^{1-\rho} + \left( \frac{w_l}{A_l^{\alpha}} \right)^{1-\rho} \right]^{\frac{1}{1-\rho}}
\]

where \( D = \gamma T^{-1/\theta} P^{\eta-1}_r X_r \) is a type-specific constant that the firm takes as given. If \( \alpha \rho > 1 \) then the problem has no solution. To see this, set \( A_l = 0 \) and the problem becomes

\[
\max_{A_h \geq 0} \left( \frac{A_h^{\alpha}}{w_h} \right)^{\eta-1} D - A_h^{1-\alpha} F_h w_h
\]

If \( \alpha (\eta - 1) > 1 - \alpha \equiv \alpha \eta > 1 \), then setting \( A_h = \infty \) yields infinite profits. Back to the general problem (D.1), the first order conditions with respect to \( A_h \) are:

\[
\alpha (\eta - 1) D \left[ \left( \frac{w_h}{A_h^{\alpha}} \right)^{1-\rho} + \left( \frac{w_l}{A_l^{\alpha}} \right)^{1-\rho} \right]^{\frac{\rho-\eta}{1-\rho}} w_h^{1-\rho} A_h^{\alpha(\rho-1)-1}
\]

\[
- F_h \left[ \left( \frac{w_h}{A_h^{\alpha}} \right)^{1-\rho} + \left( \frac{w_l}{A_l^{\alpha}} \right)^{1-\rho} \right]^{\frac{1}{1-\rho}} + \alpha (A_h F_h + A_l F_l) \left[ \left( \frac{w_h}{A_h^{\alpha}} \right)^{1-\rho} + \left( \frac{w_l}{A_l^{\alpha}} \right)^{1-\rho} \right]^{\frac{\rho-\eta}{1-\rho}} w_h^{1-\rho} A_h^{\alpha(\rho-1)-1} = 0
\]

Dividing by \( \left[ \left( \frac{w_h}{A_h^{\alpha}} \right)^{1-\rho} + \left( \frac{w_l}{A_l^{\alpha}} \right)^{1-\rho} \right]^{\frac{\rho-\eta}{1-\rho}} > 0 \) and rearranging, the first order condition is:

\[
\alpha (\eta - 1) D \left[ \left( \frac{w_h}{A_h^{\alpha}} \right)^{1-\rho} + \left( \frac{w_l}{A_l^{\alpha}} \right)^{1-\rho} \right]^{\frac{\rho-\eta}{1-\rho}} w_h^{1-\rho} A_h^{\alpha(\rho-1)-1} + \alpha A_l F_l w_h^{1-\rho} A_h^{\alpha(\rho-1)-1} - (1 - \alpha) F_h w_h^{1-\rho} A_h^{\alpha(\rho-1)-1} - F_l w_l^{1-\rho} A_l^{\alpha(\rho-1)-1} = 0
\]

The right hand side converges to a negative number as \( A_h \) goes to infinity, because the exponents on \( A_h \) in the first two terms are smaller than \( \alpha (\rho - 1) \) if \( \eta \rho < 1 \). In addition, if \( \alpha (\rho - 1) < 1 \), then the right hand side tends to infinity as \( A_h \) goes to zero. Hence \( A_h = 1 \) and by symmetry \( A_l = 0 \) cannot be a solution. In sum, if \( \alpha \eta < 1 \), then a solution to the representative firm’s problem exists for any set of wages. From equations (11) and (18), the demand for a type of labor (country and skill level) goes to zero as a share of

---

\(^{42}\)The second term should be multiplied by \( \frac{1+\alpha}{\alpha} \), but the problem does not change if we drop it and incorporate it in the definition of \( D \).
aggregate demand at a rate $-\theta < -1$ as wages tends to infinity, while aggregate demand increases at most linearly. Then, we can constraint $w$ to a compact set bounded away from zero. And a fixed point exists.

D.2 Appendix for production III: Proof of prop. 10(iv)

Consider a planner who solves the following problem:

$$\min_{A_l, A_h, l, h} w_h h + w_l l$$

subject to

$$\left[ (A_h h)^{\frac{\rho - 1}{\rho}} + (A_l l)^{\frac{\rho - 1}{\rho}} \right]^{\frac{\rho}{\rho - 1}} \geq q_c + f_h A_h + f_l A_l$$

The market unit cost of production in country $i$ and type $\tau$ cannot be greater than this planner’s solution when wages are in country $i$ are $w_h$ and $w_l$ and the quantity of the input bundle produced for consumption-goods production is $q_c$. The solution to the planner’s problem is internal with first order conditions:

$$w_h - \lambda q^{1/\rho} A_h^{\frac{1 - \rho}{\rho}} h^{-1/\rho} = 0$$

$$w_l - \lambda q^{1/\rho} A_l^{\frac{1 - \rho}{\rho}} l^{-1/\rho} = 0$$

$$-q^{1/\rho} h^{\frac{1 - \rho}{\rho}} A_h^{-1/\rho} + f_h = 0$$

$$-q^{1/\rho} l^{\frac{1 - \rho}{\rho}} A_l^{-1/\rho} + f_l = 0$$

Taking ratios and rearranging, we get

$$\left( \frac{A_l}{A_h} \right)^{2 - \rho} = \frac{f_h}{f_l} \left( \frac{w_l}{w_h} \right)^{1 - \rho}$$

which is the same as the market solution when $y_{1\tau} = \infty$ for North and $y_{2\tau} = 0$ for South. Since the level of $A_h$ and $A_l$ is determined by $q$ in both the market and the planner’s solutions, this proves that the two solutions coincide. Moreover, the planner’s problem is convex so that a monotonic departure from the optimal $A_{l\tau}/A_{h\tau}$ results in a monotonic increase in prices $w_{\tau}$.

D.3 Appendix for production III: Proof of lemma 11

Lemma 11 states: In any equilibrium, if country $i$ consumes relatively more type $\tau$, $\frac{Q_{i\tau}}{Q_{j\tau}} > \frac{Q_{i\tau}}{Q_{j\tau}}$, then it also produces relatively more type $\tau$, $\frac{R_{i\tau}}{R_{j\tau}} > \frac{R_{i\tau}}{R_{j\tau}}$ and it is relatively more productive at industry $\tau$. 

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proof. Given that the equilibrium is symmetric, the proposition may be written at the industry level: if country $i$ consumes relatively more goods in industry $\iota$ than $\iota'$, $\frac{q_i}{q_{i'}} > \frac{q_{i'}}{q_i}$, then it also produces relatively more type $\tau_i' \frac{r_{i\iota}}{r_{i\iota'}} > \frac{r_{i\iota'}}{r_{i\iota}}$ and it is relatively more productive at industry $i$. We prove this statement. To simplify notation, let $(\iota, \iota') = (1, 2)$, and assume North consumes relatively more of industry 2, $\frac{q_{N2}}{q_{S2}} > \frac{q_{N1}}{q_{S1}}$.\footnote{The proof below only works for the strict inequality. I have not tried the proof for the equality case.} The proof consists of proposing an alternative allocation and showing that it constitutes a profitable deviation in one of the industries. Since industries have measure zero, the alternative deviation has no effect on the labor market.

**Step 1** Consider the following alternative labor allocations:

$$
\hat{h}_{i1} = \left( \frac{r_1}{r_2} \right) h_{i2}, \quad \hat{h}_{i2} = \left( \frac{r_2}{r_1} \right) h_{i1},
$$

$$
\hat{l}_{i1} = \left( \frac{r_1}{r_2} \right) l_{i2}, \quad \hat{l}_{i2} = \left( \frac{r_2}{r_1} \right) l_{i1} \quad \text{for } i = N, S
$$

$$
\frac{\hat{a}_{h1}}{a_{i1}} = \frac{a_{h2}}{a_{i2}}, \quad \frac{\hat{a}_{h2}}{a_{i2}} = \frac{a_{h1}}{a_{i1}} \quad \text{(D.2)}
$$

$$
\min\{\hat{c}_{S1}, \hat{c}_{N1}\} (F_h\hat{a}_{h1} + F_l\hat{a}_{l1}) = \frac{\alpha}{1 + \alpha} \frac{r_1}{r_1} \quad \text{(D.3)}
$$

$$
\min\{\hat{c}_{S2}, \hat{c}_{N2}\} (F_h\hat{a}_{h2} + F_l\hat{a}_{l2}) = \frac{\alpha}{1 + \alpha} \frac{r_2}{r_2}
$$

Under the new allocation, the *shares* of labor and machines in the two industries are switched, but *total* spending on labor and machines in each industry is maintained.

**Step 2** Total cost of new allocation the same as the original:

$$
w_{Nh}\hat{h}_{N1} + w_{Nh}\hat{h}_{S1} + w_{Sb}\hat{l}_{S1} = \left( \frac{r_1}{r_2} \right) \left( w_{Nh}h_{N2} + w_{Nh}l_{N2} + w_{Sb}h_{S2} + w_{Sb}l_{S2} \right) = r_1
$$

$$
w_{Nh}\hat{h}_{N2} + w_{Nh}\hat{h}_{S2} + w_{Sb}\hat{l}_{S2} = \left( \frac{r_2}{r_1} \right) \left( w_{Nh}h_{N1} + w_{Nh}l_{N1} + w_{Sb}h_{S1} + w_{Sb}l_{S1} \right) = r_2
$$

where the last equality in both lines comes from zero profits, $r_i = w_{Nh}h_{N_i} + w_{Nh}l_{N_i} + w_{Sb}h_{S_i} + w_{Sb}l_{S_i}$. By assumption spending on machines is a share $\frac{\alpha}{1+\alpha}$ of revenue.

**Step 3** We derive new price indices as functions of old indices. Since machines get a share $\frac{\alpha}{1+\alpha}$ of costs, original allocations satisfy:

$$
\min\{c_{S2}, c_{N2}\} (F_ha_{h2} + F_la_{l2}) = \frac{\alpha}{1 + \alpha} \frac{r_2}{r_2}
$$
Then, from equations (D.2) and (D.3), we have:

\[
\left( \frac{\alpha}{1+\alpha} \right) \frac{r_1}{\min\{\hat{c}_S, \hat{c}_N\}} = (F_h \hat{a}_{h1} + F_l \hat{a}_{l1}) = \frac{\hat{a}_{l1}}{a_{l2}}(F_h a_{h2} + F_l a_{l2}) = \frac{\hat{a}_{l1}}{a_{l2}} \left( \frac{\alpha}{1+\alpha} \right) \frac{r_2}{\min\{c_S, c_N\}}
\]

\[\Rightarrow \frac{\hat{a}_{l1}}{a_{l2}} = \frac{r_1}{r_2} \left( \frac{\min\{c_S, c_N\}}{\min\{\hat{c}_S, \hat{c}_N\}} \right).
\]

Switching the roles of skilled and unskilled labor, we have

\[
\frac{\hat{a}_{h1}}{a_{h2}} = \frac{\hat{a}_{l1}}{a_{l2}} = \frac{r_1}{r_2} \left( \frac{\min\{c_S, c_N\}}{\min\{\hat{c}_S, \hat{c}_N\}} \right).
\]

Then, input costs satisfies:

\[
\hat{c}_S = (1 + \alpha) \left[ \left( \frac{w_{Sh}}{a^\alpha_{h1}} \right)^{1-\rho} + \left( \frac{w_{Sl}}{a^\alpha_{l1}} \right)^{1-\rho} \right]^{1-\rho}
\]

\[
= (1 + \alpha) \left[ \frac{r_2}{r_1} \left( \frac{\min\{\hat{c}_S, \hat{c}_N\}}{\min\{c_S, c_N\}} \right) \right]^\alpha \left[ \left( \frac{w_{Sh}}{a^\alpha_{h2}} \right)^{1-\rho} + \left( \frac{w_{Sl}}{a^\alpha_{l2}} \right)^{1-\rho} \right]^{1-\rho}
\]

\[
= \left[ \frac{r_2}{r_1} \left( \frac{\min\{\hat{c}_S, \hat{c}_N\}}{\min\{c_S, c_N\}} \right) \right]^\alpha c_S,
\]

\[
\hat{c}_N = \left[ \frac{r_2}{r_1} \left( \frac{\min\{\hat{c}_S, \hat{c}_N\}}{\min\{c_S, c_N\}} \right) \right]^\alpha c_N;
\]

where the first line if the first order condition of final producers’ cost minimization problem (17) in the main text and the last line follows from symmetry. Suppose without loss of generality, \( S = \arg \min\{\hat{c}_S, \hat{c}_N\} \). Since labor and machines allocations are proportional to the original industry 2, \( S = \arg \min\{c_S, c_N\} \). Substituting in equation (D.4),

\[
\frac{\hat{c}_S}{c_S} = \left( \frac{\hat{c}_S}{c_S} \right)^\alpha \left( \frac{r_2}{r_1} \right)^\alpha = \left( \frac{r_2}{r_1} \right)^{\frac{\alpha}{1-\alpha}}
\]

\[
\frac{\hat{c}_N}{c_N} = \left( \frac{\hat{c}_S}{c_S} \right)^\alpha \left( \frac{r_2}{r_1} \right)^\alpha = \left( \frac{r_2}{r_1} \right)^{\frac{\alpha^2}{1-\alpha}} \left( \frac{r_2}{r_1} \right)^\alpha = \left( \frac{r_2}{r_1} \right)^{\frac{\alpha}{1-\alpha}}
\]


Finally, from equation (10) in main text, the new price index of industry 1 in South is

\[ \hat{p}_S = \gamma T_1^{-1/\theta} \left[ T_S(c_{S1})^{-\theta} + T_N(d_{N1})^{-\theta} \right]^{-1/\theta} = \gamma T_1^{-1/\theta} \left( \frac{r_2}{r_1} \right)^{\theta \tilde{\alpha}} \left[ T_S(c_{S2})^{-\theta} + T_N(d_{N2})^{-\theta} \right] = \left( \frac{T_2}{T_1} \right)^{1/\theta} \left( \frac{r_2}{r_1} \right)^{\theta \tilde{\alpha}} p_{S2}. \]

Analogously, \( \hat{p}_N = \left( \frac{T_N}{T_1} \right)^{1/\theta} \left( \frac{r_N}{r_1} \right)^{\theta \tilde{\alpha}} p_{N2}, \) \( \hat{p}_{S2} = \left( \frac{T_S}{T_2} \right)^{1/\theta} \left( \frac{r_S}{r_2} \right)^{\theta \tilde{\alpha}} p_{S1}, \) and \( \hat{p}_{N2} = \left( \frac{T_N}{T_2} \right)^{1/\theta} \left( \frac{r_N}{r_2} \right)^{\theta \tilde{\alpha}} p_{N1}. \)

**Step 4** By the contradiction hypothesis, alternative allocations cannot increase profits:

\[ q_{N1}(p_{N1} - \hat{p}_{N1}) + q_{S1}(p_{S1} - \hat{p}_{S1}) \leq 0 \quad \text{for} \quad i = 1, 2. \]

\[ \equiv q_{N1} \left[ \left( T_1^{1/\theta} r_1^{\theta \tilde{\alpha}} \right) p_{N1} - \left( T_2^{1/\theta} r_2^{\theta \tilde{\alpha}} \right) p_{N2} \right] + q_{S1} \left[ \left( T_1^{1/\theta} r_1^{\theta \tilde{\alpha}} \right) p_{S1} - \left( T_2^{1/\theta} r_2^{\theta \tilde{\alpha}} \right) p_{S2} \right] \leq 0, \quad (D.6) \]

\[ q_{N2} \left[ \left( T_2^{1/\theta} r_2^{\theta \tilde{\alpha}} \right) p_{N2} - \left( T_1^{1/\theta} r_1^{\theta \tilde{\alpha}} \right) p_{N1} \right] + q_{S2} \left[ \left( T_2^{1/\theta} r_2^{\theta \tilde{\alpha}} \right) p_{S2} - \left( T_1^{1/\theta} r_1^{\theta \tilde{\alpha}} \right) p_{S1} \right] \leq 0 \quad (D.7) \]

By proposition 10(iv), \( \frac{r_{N1}}{r_{N2}} > \frac{r_{S1}}{r_{S2}} \) implies \( \frac{c_{N1}}{c_{N2}} < \frac{c_{S1}}{c_{S2}} \), which in turn implies \( \frac{p_{N1}}{p_{N2}} < \frac{p_{S1}}{p_{S2}} \). So, if \( \frac{p_{S1}}{p_{S2}} \leq \left( \frac{T_N}{T_1} \right)^{1/\theta} \left( \frac{r_N}{r_1} \right)^{\theta \tilde{\alpha}}, \) then \( \frac{p_{N1}}{p_{N2}} < \left( \frac{T_N}{T_1} \right)^{1/\theta} \left( \frac{r_N}{r_1} \right)^{\theta \tilde{\alpha}} \) and inequality (D.7) is violated.

Similarly, \( \frac{p_{N1}}{p_{N2}} \geq \left( \frac{T_N}{T_1} \right)^{1/\theta} \left( \frac{r_N}{r_1} \right)^{\theta \tilde{\alpha}} \) implies \( \frac{p_{S1}}{p_{S2}} > \left( \frac{T_N}{T_1} \right)^{1/\theta} \left( \frac{r_N}{r_1} \right)^{\theta \tilde{\alpha}} \) and violates inequality (D.6). So, the only possibility is for \( \frac{p_{N1}}{p_{N2}} < \left( \frac{T_N}{T_1} \right)^{1/\theta} \left( \frac{r_N}{r_1} \right)^{\theta \tilde{\alpha}} < \frac{p_{S1}}{p_{S2}}. \) Then, rearranging (D.6) and (D.7), yields

\[ \frac{q_{N1}}{q_{N1}} \leq \frac{p_{N2} T_2^{1/\theta} r_2^{\theta \tilde{\alpha}} - p_{N1} T_1^{1/\theta} r_1^{\theta \tilde{\alpha}}}{p_{S1} T_1^{1/\theta} r_1^{\theta \tilde{\alpha}} - p_{S2} T_2^{1/\theta} r_2^{\theta \tilde{\alpha}}} \quad \text{and} \quad \frac{q_{N2}}{q_{N2}} \leq \frac{p_{N2} T_2^{1/\theta} r_2^{\theta \tilde{\alpha}} - p_{N1} T_1^{1/\theta} r_1^{\theta \tilde{\alpha}}}{p_{S1} T_1^{1/\theta} r_1^{\theta \tilde{\alpha}} - p_{S2} T_2^{1/\theta} r_2^{\theta \tilde{\alpha}}}, \]

\[ \Rightarrow \frac{q_{S1}}{q_{N1}} \leq \frac{q_{S2}}{q_{N2}}, \]

which contradicts the original hypothesis \( \frac{q_{S1}}{q_{N1}} > \frac{q_{S2}}{q_{N2}}. \)
Appendix for Extensions

E.1 Appendix for Extensions: Stone-Geary Preferences

We use the preference specification only in propositions 2, 5 and 8. This appendix shows that these propositions hold if preferences took the more general Stone-Geary form:

\[ U = (Q_1 - B)\gamma Q_2^{1-\gamma} \]

where \( Q_\tau \) is defined as in the main text, \( B \) is a minimum consumption requirement for basic goods \( \tau = 1 \) and \( \gamma \in (0,1) \). Preferences studied in the paper are the limiting case where \( B = 1 \) and \( \gamma \to 0 \). Like in the main text, we assume that both countries consume both goods. For propositions 5 and 8, it suffices to prove claim 1.

Claim 1: South spends relatively more of good 1 if and only if \( ew < \frac{P_{1S}}{P_{1N}} \).

For proposition 2, we still need to prove that (i) South consumes relatively more type 1 when \( w \geq 1 \) and (ii) welfare is higher in North. The original proofs are in appendix B.1.1 and B.3, respectively. All other proofs remain unchanged.

Claim 2: South consumes relatively more type 1 when \( w \geq 1 \). From the main text, relative price of good \( \tau \) satisfies:

\[ p_\tau^{1-\sigma} = \left( \frac{1-d^{1-\sigma}w^{-\sigma}}{w^{-\sigma} - d^{1-\sigma}} \right) x_\tau \]

Substituting \( x_\tau \) with demand for type 1, we have:

\[ p_1^{1-\sigma} = \left( \frac{1-d^{1-\sigma}w^{-\sigma}}{w^{-\sigma} - d^{1-\sigma}} \right) \left[ \frac{(1-\alpha)P_{N1B} + \alpha}{(1-\alpha)P_{S1B} + \alpha ew} \right] \frac{L_N}{L_S} \]

Suppose by contradiction that South consumes relatively more type 2. Then, from claim
1, \( ew \geq \frac{P_{N}}{P_{N1}} = (p_{1})^{-1} \) or \( ew^{\sigma - 1} \geq p_{1}^{1-\sigma} \) since \( \sigma > 1 \). Then, using \( \frac{L_{N}}{L_{S}} \geq e \), we have

\[
(ew)^{\sigma - 1} \geq \left( \frac{1 - d^{1-\sigma}w^{\sigma}}{w^{\sigma} - d^{1-\sigma}} \right) \left[ \frac{(1 - \alpha)P_{N1}B + \alpha}{(1 - \alpha)P_{S1}B + \alpha ew} \right] e
\]

\[
\Leftrightarrow e^{\sigma - 1} \geq w^{\sigma} \left( \frac{w^{\sigma} - d^{1-\sigma}}{1 - d^{1-\sigma}w^{\sigma}} \right) \left[ \frac{(1 - \alpha)P_{N1}Bw + \alpha w}{(1 - \alpha)P_{S1}B + \alpha ew} \right] e
\]

\[
\geq \left( \frac{1 - d^{1-\sigma}w^{\sigma}}{1 - d^{1-\sigma}w^{\sigma}} \right) \left[ \frac{(1 - \alpha)P_{S1}B/e + \alpha ew/e}{(1 - \alpha)P_{S1}B + \alpha ew} \right] e
\]

\[
= \left( \frac{1 - d^{1-\sigma}w^{\sigma}}{1 - d^{1-\sigma}w^{\sigma}} \right) \geq 1 \quad \text{if } w \geq 1
\]

where the third line comes from the contradiction hypothesis, \( P_{N1} \geq P_{S1}/(ew) \). The last line is a contradiction because \( e < 1 \) and \( \sigma > 1 \).]

Claim 3: Welfare is higher in North when North spends and produces relatively more of type 2.

\[
\frac{U_{S}}{U_{N}} = \frac{(ew - P_{S1}B)P_{S1}P_{S2}^{\alpha - 1}}{(1 - P_{N1}B)P_{N1}P_{N2}^{\alpha - 1}}
\]

\[
< ew \left( \frac{1 - P_{N1}B}{1 - P_{N1}B} \right) \frac{P_{S1}P_{S2}^{\alpha - 1}}{P_{N1}P_{N2}^{\alpha - 1}}
\]

\[
= ew \frac{P_{S1}P_{S2}^{\alpha - 1}}{P_{N1}P_{N2}^{\alpha - 1}}
\]

\[
< \left( \frac{P_{S1}/P_{S2}}{P_{N1}/P_{N2}} \right)^{1-\alpha} < 1
\]

where the second and fourth lines hold because South spends relatively more on type 1, which from claim 1 implies \( P_{S1} > ewP_{N1} \). The last inequality holds because South produces relatively more type 1, \( M_{S1}/M_{S2} > M_{N1}/M_{N2} \) and so \( P_{S1}/P_{S2} < P_{N1}/P_{N2} \).

E.2 Appendix for Extensions: CRIE Preferences

This appendix finds sufficient conditions for relative demand for type 1 in South to be larger than in North, \( \frac{X_{S1}}{X_{S2}} > \frac{X_{N1}}{X_{N2}} \), under CRIE preferences. Let the income per capita in South be \( ew \) where \( w \) is the wage rate in South, \( e < 1 \) in the monopolistic-competition set up and \( e = 1 \) in the Ricardian set up. The derivation consists of three steps. We first use the budget constraint to put a lower bound on the Lagrange multipliers, \( \frac{\lambda_{S}}{\lambda_{N}} \), as a function of wages. With this bound and bounds on prices \( p_{\tau} = \frac{P_{N\tau}}{P_{S\tau}} \in [d^{-1}, d] \), step 2 finds an upper bound on Southern income per capita \( ew \) such that South always consumes relatively more type 1, irrespective of the location of production. Step 3 uses technologies.
to ensure that equilibrium wages do not exceed this upper bound. Bounds on technologies are specific to the set up—monopolistic competition or Ricardian.

**Step 1.** The budget constraint is

\[
 ew = X_{S1} + X_{S2} \\
 1 = X_{N1} + X_{N2} \\
 \Rightarrow ew \geq \min \left\{ \frac{X_{S1}}{X_{N1}}, \frac{X_{S2}}{X_{N2}} \right\}
\]

Since there is nothing to prove if \( \frac{X_{S1}}{X_{S2}} > \frac{X_{N1}}{X_{N2}} \), we assume otherwise. Then, using the demand function,

\[
 ew \geq \frac{X_{S1}}{X_{N1}} = \left( \frac{\lambda_N}{\lambda_S} \right)^{\delta_1/p_1^{\delta_1-1}}
\]

\[
 \equiv \frac{\lambda_S}{\lambda_N} \geq \left( \frac{p_1^{\delta_1-1}}{ew} \right)^{\frac{1}{\delta_2}}.
\]

(E.1)

**Step 2.** Again using the demand function,

\[
 \frac{X_{S1}/X_{S2}}{X_{N1}/X_{N2}} = \left( \frac{\lambda_S}{\lambda_N} \right)^{\delta_2-\delta_1} p_1^{\delta_1-1} p_2^{1-\delta_2}
\]

\[
 \geq (ew)^{\frac{\delta_1-\delta_2}{\delta_1}} \frac{\delta_2(\delta_1-1)}{\delta_1} p_2^{1-\delta_2}
\]

\[
 \geq (ew)^{\frac{\delta_1-\delta_2}{\delta_1}} d^{\frac{\delta_2(1-\delta_1)}{\delta_1}} d^{1-\delta_2}
\]

\[
 \therefore ew < d^{\frac{\delta_1+\delta_2-2\delta_1\delta_2}{\delta_2-\delta_1}} \Rightarrow \frac{X_{S1}/X_{S2}}{X_{N1}/X_{N2}} > 1
\]

(E.2)

where the second line uses equation (E.1) and the third uses \( p_\tau \in [d^{-1}, d] \).

**Step 3.** In the monopolistic competition set up, \( w \leq d \) for zero-profit conditions in equation (7) to hold for at least one type. Hence, from statement (E.2), a sufficient condition for \( \frac{X_{S1}/X_{S2}}{X_{N1}/X_{N2}} > 1 \) is

\[
 e < d^{\frac{2\delta_1(1-\delta_2)}{\delta_2-\delta_1}}
\]
In the Ricardian case with diversification, the upper bound on wages as a function of technologies is given by equation (15):

\[
    w \leq \left[ d^\theta \frac{T_S}{T_N} \left( \frac{L_N}{L_S} \right)^{1-\theta \alpha} \right]^{\frac{1}{1+\theta}}
\]

Hence, from statement (E.2), a sufficient condition for \( \frac{X_{S1}}{X_{S2}} \frac{X_{N1}}{X_{N2}} > 1 \) is

\[
    \left[ d^\theta \frac{T_S}{T_N} \left( \frac{L_N}{L_S} \right)^{1-\theta \alpha} \right]^{\frac{1}{1+\theta}} < d^{\frac{\delta_1 + \delta_2 - 2\delta_1 \delta_2}{\delta_2 - \delta_1}}
\]

From appendix C.3, the upper bound on Southern wages in the specialization case (\( \alpha \theta > 1 \)) is \( w < \left[ d^\theta \frac{T_S}{T_N} \right]^{\frac{1}{1+\alpha}} \). Hence from step 2, a sufficient condition for \( \frac{X_{S1}}{X_{S2}} \frac{X_{N1}}{X_{N2}} > 1 \) is

\[
    \left[ d^\theta \frac{T_S}{T_N} \right]^{\frac{1}{1+\alpha}} < d^{\frac{\delta_1 + \delta_2 - 2\delta_1 \delta_2}{\delta_2 - \delta_1}}.
\]