Wages, Unemployment and Inequality with Heterogeneous Firms and Workers

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Introduction

Three prominent features of product and labor markets are:

1. substantial differences in size, performance and workforce composition across firms;
2. variation in wages and other labor market outcomes for workers with the same observed characteristics;
3. positive unemployment

A few facts about income inequality:

- residual inequality
- within-sector and between firms
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- We develop a general equilibrium model that captures these features
- We then use the model to study:
  - allocation of workers across firms
  - size and productivity distribution across firms
  - distribution of wages and income within and between sectors
  - sectoral and economy-wide unemployment rates
  - labor market outcomes for workers with similar ability
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  – residual inequality
  – within-sector and between firms
Ingredients of the model

1. Firm productivity heterogeneity
2. Unobserved worker ability heterogeneity
   - general worker ability
   - match-specific productivity
3. Random search and matching
4. Costly screening
5. Production technology with complementarities
Main Findings

• Labor market frictions:
  – Search friction increases sectoral unemployment, while screening friction reduces it
  – Labor market frictions have no affect on sectoral wage inequality, but they do affect economy-wide inequality
  – Both labor market frictions reduce welfare

• Interdependence between product and labor market:
  – Size distribution of firms is more dispersed in sectors with more productivity and ability dispersion
  – Wage inequality increases in the dispersion of firm productivity, but may increase or decrease in the dispersion of worker ability

• Labor market outcomes for workers with similar ability:
  – Workers with higher ability are less likely to end up unemployed
  – On average they receive higher wages and have higher dispersion of wages (but not necessarily of income)
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- Trade increases unemployment and wage inequality within sectors
Preferences and Demand

- **Utility function:**
  \[
  U = q_0 + \sum_{i=1}^{l} \frac{1}{\zeta_i} Q_i^\zeta_i, \quad \zeta_i > 0
  \]

- **Preferences for differentiated products in sector \( i \):**
  \[
  Q_i = \left[ \int_{\omega \in \Omega_i} q_i(\omega)^{\beta_i} d\omega \right]^{\frac{1}{\beta_i}}, \quad \zeta_i < \beta_i < 1
  \]

- **Demand functions:**
  \[
  q_i(\omega) = Q_i^{\frac{\beta_i - \zeta_i}{1 - \beta_i}} p_i(\omega)^{-\frac{1}{1 - \beta_i}}
  \]

- **Indirect utility function:**
  \[
  V = E + \sum_{i=1}^{l} \frac{1 - \zeta_i}{\zeta_i} Q_i^{\zeta_i},
  \]
  where \( Q_i^{-(1-\zeta_i)} = P_i \)
Market Structure

- All goods are produced with labor
- The homogeneous product requires one unit of labor per unit output and the market for this product is competitive: \( p_0 = w_0 = 1 \)
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- The market for brands of the differentiated product is monopolistically competitive:
  - Fixed entry cost $f_e$ in terms of the homogenous good
  - The firm then learns its productivity $\theta$, drawn from a Pareto distribution: $G_\theta(\theta) = 1 - (\theta_{\text{min}} / \theta)^z$, $z > 2$
  - Fixed production cost $f_d$ in terms of the homogeneous good
  - Revenue of the firm with output $y$
    \[
    r = Q^{-(\beta - \zeta)}y^\beta
    \]
  - If revenue is insufficient to cover all costs, the firm exits
Production Technology

• Output of a firm with productivity $\theta$ and $h$ employees with average ability $\bar{a}$:

$$y = \theta h^{\gamma} \bar{a} = \theta \left( \frac{1}{h} \right)^{1-\gamma} \int_0^h a_i \, di, \quad 0 < \gamma < 1$$

• Interpretation: human capital externalities or fixed managerial time at the level of the firm
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- Ability of workers $a$ is unobservable and distributed Pareto:

$$G_a(a) = 1 - \left( \frac{a_{\min}}{a} \right)^k, \quad k > 2$$

- By paying $bn$ a firm can match randomly with $n$ workers

- By paying $ca_c^\delta / \delta$ a firm can screen workers with ability below $a_c$
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$$\bar{a} = \frac{k}{k-1} a_c \quad \text{and} \quad h = n \cdot \left( \frac{a_{\min}}{a_c} \right)^k$$

$$y = \frac{ka_{\min}^{\gamma k}}{k-1} \theta n^{\gamma} a_c^{1-\gamma k}, \quad \gamma k < 1$$
Firm’s Problem

- Wage bargaining as in Stole and Zwiebel (1996): \( \frac{1}{1 + \beta \gamma} \) is the firm’s share of revenues
- Firm’s problem:

\[
\pi(\theta) \equiv \max_{n \geq 0, \quad a_c \geq a_{\min}} \left\{ \frac{1}{1 + \beta \gamma} Q^{-(\beta - \zeta)} \left[ \left( \frac{ka_{\min}^{\gamma k}}{k - 1} \right) \theta n^{\gamma} a_c^{1 - \gamma k} \right]^\beta - bn - \frac{c}{\delta} a_c^\delta - f_d \right\}
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$$

- Firms that sample more workers are also more selective:

$$
(1 - \gamma k) bn(\theta) = \gamma c a_c(\theta)^\delta
$$

- Assuming $\delta > k$, these firms also hire more workers:

$$
(1 - \gamma k) bh(\theta) = \gamma c a_{\min}^k a_c(\theta)^{\delta - k}
$$
Wages, Productivity and Profits

• As a result of bargaining, the wage rate is:

\[ w(\theta) = \frac{\beta \gamma}{1 + \beta \gamma} \frac{r(\theta)}{h(\theta)} = b \left[ \frac{a_c(\theta)}{a_{\text{min}}} \right]^k \]

• Size-wage premium in the model:

\[ \frac{\Delta \ln w(\theta)}{\Delta \ln h(\theta)} = \frac{k}{(\delta - k)} > 0 \]
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\[ t(\theta) = \frac{r(\theta)}{h(\theta)} \propto w(\theta) \]
Wages, Productivity and Profits

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• Productivity:

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• The profit of the firm is

\[ \pi(\theta) = \frac{\Gamma}{1 + \beta \gamma} r(\theta) - f_d, \quad \Gamma \equiv 1 - \beta \gamma - \frac{\beta}{\delta} (1 - \gamma k) \]

and revenue is

\[ r(\theta) = \kappa_r b^{\frac{\beta \gamma}{1 - \gamma k}} c^{\frac{\beta (1 - \gamma k)}{\delta r \Gamma}} Q^{-\frac{\beta - \xi}{\Gamma}} \theta^\beta \]
Free Entry and Zero Profit

- Zero-profit productivity cutoff:

\[ \pi(\theta_d) = \kappa \pi \left[ \beta \gamma c^{\beta(1-\gamma k)/\delta} Q^{-(\beta-\zeta)} \theta_d^\beta \right]^{1/\Gamma} - f_d = 0, \]

so that

\[ \pi(\theta) = f_d \left[ (\theta/\theta_d)^{\beta/\Gamma} - 1 \right] \]

- Free entry condition:

\[ f_e = \int_{\theta_d}^{\infty} \pi(\theta) \, dG_\theta(\theta) = f_d \int_{\theta_d}^{\infty} \left[ (\theta/\theta_d)^{\beta/\Gamma} - 1 \right] \, dG_\theta(\theta) \]
Labor Market

- Tightness of the labor market: \( x = N/L \)
- Hiring cost is increasing in the tightness of the labor market:
  \[
  b = \alpha_0 x^{\alpha_1}, \quad \alpha_0 > 1, \quad \alpha_1 > 0
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\frac{w(\theta) h(\theta)}{n(\theta)} = b
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\]

• The indifference condition for workers is therefore \( bx = 1 \) so that

\[
b = \alpha_0^{1/(1+\alpha_1)} > 1 \quad \text{and} \quad x = b^{-1} = \alpha_0^{-1/(1+\alpha_1)} < 1.
\]
Sectoral Equilibrium

- Productivity cutoff:

\[
\theta_d = \left[ \left( \frac{\beta}{z \Gamma - \beta} \right) \frac{f_d}{f_e} \right]^{1/z} \theta_{\text{min}}
\]

- Sectoral Output:

\[
Q^{\beta - \zeta} = \kappa_Q b^{-\beta \gamma} c^{-\beta (1 - \gamma k) / \delta}
\]

- Number of workers searching for job and number of firms:

\[
L = \frac{\beta \gamma}{1 + \beta \gamma} Q^{\zeta} \quad \text{and} \quad M = \frac{1}{\gamma z f_e} L
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Sectoral Equilibrium

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Proposition

Welfare is decreasing in search cost (b) and screening cost (c).
Variation Across Firms

- Revenue:
  \[ r(\theta) = \frac{1+\beta \gamma}{\Gamma} f_d \left( \frac{\theta}{\theta_d} \right)^{\beta/\Gamma} \]

- Employment:
  \[ h(\theta) = \frac{\beta \gamma}{b} f_d \left[ \frac{\beta(1-\gamma k)}{\Gamma} \frac{f_d}{ca^{\delta}_{\text{min}}} \right]^{-k/\delta} \left( \frac{\theta}{\theta_d} \right)^{\beta(1-k/\delta)/\Gamma} \]

- Screening cutoff:
  \[ a_c(\theta) = \left[ \frac{\beta(1-\gamma k)}{\Gamma} \frac{f_d}{c} \right]^{1/\delta} \left( \frac{\theta}{\theta_d} \right)^{\beta/\delta \Gamma} \]

- Wage rate:
  \[ w(\theta) = b \left[ \frac{\beta(1-\gamma k)}{\Gamma} \frac{f_d}{ca^{\delta}_{\text{min}}} \right]^{k/\delta} \left( \frac{\theta}{\theta_d} \right)^{\beta k/\delta \Gamma} \]
Distribution of Firm Size and Productivity

- Firm size (in terms of employment and revenue) and measured productivity are distributed Pareto:

\[ F_h(h) = 1 - \left( \frac{h_d}{h} \right)^{\frac{z \Gamma}{\beta (1-k/\delta)}} \quad \text{for} \quad h \geq h_d \equiv \frac{\beta \gamma}{\Gamma} \frac{f_d}{b} \left[ \frac{\beta (1-\gamma k)}{\Gamma} \frac{f_d}{ca_{min}^{\delta}} \right]^{-k/\delta}, \]

\[ F_r(r) = 1 - \left( \frac{r_d}{r} \right)^{\frac{z \Gamma}{\beta}} \quad \text{for} \quad r \geq r_d \equiv \frac{1+\beta \gamma}{\Gamma} f_d, \]

\[ F_t(t) = 1 - \left( \frac{t_d}{t} \right)^{\frac{z \delta \Gamma}{\beta k}} \quad \text{for} \quad t \geq t_d \equiv b \frac{1+\beta \gamma}{\beta \gamma} \left[ \frac{\beta (1-\gamma k)}{\Gamma} \frac{f_d}{ca_{min}^{\delta}} \right]^{k/\delta}. \]

- Recall that \( \Gamma = 1 - \beta \gamma - \beta (1-\gamma k) / \delta \) is increasing in \( k \) and \( \Gamma / k \) is decreasing in \( k \).

Proposition

Dispersion of firm size increases in dispersion of productivity (\( z \downarrow \)) and dispersion of ability (\( k \downarrow \)). Dispersion of measured productivity decreases in \( z \), but increases in \( k \).
Sectoral Unemployment

- Sectoral unemployment rate:

\[ u = \frac{L - H}{L} = 1 - \frac{H}{N} \frac{N}{L} = 1 - \sigma x, \quad x = 1/b \]

- Sectoral hiring rate:

\[ \sigma \equiv \frac{H}{N} = \left[ \frac{a_{\min}}{a_c(\theta_d)} \right]^k \frac{1}{1 + \mu} = \left[ \frac{\Gamma}{\beta (1 - \gamma k)} \frac{ca_{\min}^\delta}{f_d} \right]^{k/\delta} \frac{1}{1 + \mu} \]

where

\[ \mu \equiv \frac{\beta k / \delta}{z \Gamma - \beta} \in (0, 1) \]
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Proposition

*Unemployment is higher in sectors with higher search cost (b) and lower screening cost (c); unemployment is higher in sectors with more productivity dispersion (z ↓), but may be higher or lower in sectors with more ability dispersion (k ↓).*
Sectoral Wage Inequality

- Wage distribution:

\[ F_w(w) = 1 - \left( \frac{w_d}{w} \right)^{1+\mu^{-1}} \text{ for } w \geq w_d \equiv b \left[ \frac{\beta(1 - \gamma_k)}{\Gamma} \frac{f_d}{ca_{\min}^\delta} \right]^{k/\delta} > 1 \]
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- Lorenz curve and Gini coefficient:

\[ s_w = L_w(s_h) \equiv 1 - (1 - s_h)^{1/(1+\mu)}, \]

\[ G_w = 1 - 2 \int_0^1 L_w(s) \, ds = \frac{\mu}{2 + \mu} \]
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\[ s_w = \mathcal{L}_w (s_h) \equiv 1 - (1 - s_h)^{1/(1+\mu)}, \]

\[ G_w = 1 - 2 \int_0^1 \mathcal{L}_w (s) \, ds = \frac{\mu}{2 + \mu} \]

- Theil index:

\[ T_w = \int_{w_d}^\infty \frac{w}{\bar{w}} \ln \left( \frac{w}{\bar{w}} \right) \, dF_w (w) \]

\[ = \mu - \ln (1 + \mu) \]
Sectoral Wage Inequality

- A sufficient statistic for wage inequality:

\[ \mu = \frac{\beta k / \delta}{z \Gamma - \beta} \]

**Proposition**

*The sectoral distribution of wages is more equal in sectors with less productivity dispersion \((z \uparrow)\), but it can be more or less equal in sectors with less ability dispersion \((k \uparrow)\).*
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- Sectoral income inequality:

\[ T_i = T_w - \ln(1 - u) = \mu - \ln(1 + \mu) - \ln(1 - u) \]
Aggregate Rate of Unemployment

$$u = \sum_{i=1}^{l} \frac{L_i}{L} u_i = \sum_{i=1}^{l} \frac{L_i}{L} (1 - \sigma_i x_i).$$

- Search and screening costs ($b$ and $c$) reduce the size of the sector, $L_i$
- They have opposite effects on the sectoral unemployment rate, $u_i$
Aggregate Rate of Unemployment

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- Search and screening costs \((b\text{ and }c)\) reduce the size of the sector, \(L_i\)
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Proposition

An increase in a sector’s screening cost \((c)\) reduces aggregate unemployment while an increase in a sector’s search cost \((b)\) reduces aggregate unemployment if and only if

\[ \frac{u_i}{1 - u_i} > \frac{\beta - \zeta}{\gamma \beta \zeta}. \]
Aggregate Rate of Unemployment

![Graph showing the relationship between Aggregate Unemployment Rate and Search Cost, b with two lines: Low c and High c. The graph is labeled with axes for Aggregate Unemployment Rate on the y-axis and Search Cost, b on the x-axis. The y-axis ranges from 0.1 to 0.35, and the x-axis ranges from 1 to 6.]
Aggregate Wage and Income Inequality

- Theil index of aggregate income inequality:

\[ T_i = \sum_{i=1}^{l} \frac{L_i}{L} T_{ii} = \sum_{i=1}^{l} \frac{L_i}{L} \left[ \mu_i - \ln (1 + \mu_i) - \ln (1 - u_i) \right] \]

Proposition

An increase in screening cost \((c_i)\) reduces aggregate income inequality, while an increase in search cost \((b_i)\) reduces it if and only if

\[ T_{ii} > \frac{\beta - \zeta}{\gamma \beta \zeta}. \]

- Contrast this result with the previous result for aggregate unemployment rate
- Additionally, we show that both \(c\) and \(b\) can increase or decrease aggregate wage inequality
Aggregate Wage Inequality

Theil Index of Aggregate Wage Inequality

Screening cost, $c$

High $b$

Low $b$

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Unemployment and Wages Distribution by Ability

- Unemployment rate by ability:

\[ u(a) = 1 - x \cdot \sigma(a) = 1 - \frac{1}{b} \left[ 1 - \left( \frac{a_d}{a} \right)^{k/\mu} \right] \quad \text{for} \quad a \geq a_d \]
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• Wage distribution by ability:

\[ F_w(w \mid a) = \frac{1 - (w_d/w)^{1/\mu}}{1 - [w_d/w_c(a)]^{1/\mu}} \quad \text{for} \quad w_d \leq w \leq w_c(a) \equiv b \left( \frac{a}{a_{\text{min}}} \right)^k \]
Unemployment and Wages Distribution by Ability

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**Proposition**

Workers with higher ability have lower expected unemployment, higher average wages and higher wage inequality. Nevertheless, income inequality for workers with higher ability can be higher or lower.
Wage and Income Inequality by Ability

Thiel Indices, $T_w(a)$ and $T_ι(a)$

Abilities Relative to Cutoff, $a/a_d$
Inequality and Unemployment in a Global Economy

- We extend the analysis to an open two-country economy.
- Exports entail fixed export costs $f_x$ and variable costs $\tau$.
- Most productive firms export ($\theta \geq \theta_x$), less productive firms serve the domestic market ($\theta_d \leq \theta < \theta_x$) and least productive firms exit ($\theta < \theta_d$).
- In equilibrium with trade, all firm-specific variables jump discontinuously for exporting firms (at $\theta_x$).
Openness, $\rho \equiv \theta_d / \theta_x$

Sectoral Unemployment Rate, $u$

Unemployment Rate
Wage Profile

Productivity, $\theta$

Wage Rate, $w(\theta)$

- $w_d(\theta)$
- $w_c(\theta)$
- $w_x(\theta)$
Inequality of Wages

Theil Index of Wage Inequality

\[ T = T_W + T_B \]

Openness to Trade Index, \( \frac{\theta_d}{\theta_x} \)