Optimal Monetary Policy and Productivity Growth∗

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Abstract

This paper studies the optimal monetary policy response to unobserved changes in the growth rate of productivity. To this end, we formulate a sticky price model in which agents are uncertain about the persistence of observed productivity shocks. In this environment, trend shocks are initially perceived as transitory, causing wage demands to lag behind realized productivity. As a result, marginal costs and inflation fall after a positive shock, consistent with the data—and with conventional wisdom. From a positive standpoint, the model’s dynamics in response to a productivity shock are consistent with the main features of available VAR estimates. They also provide a remarkably accurate account of the observed evolution of output and inflation that followed the recent productivity pickup. From a normative perspective, we find that the real wage adjustment required by a positive productivity shock should be split between an increase in the nominal wage and a reduction in the price level, in proportions that depend on the relative stickiness of the two prices. Finally, the model provides formal support for the popular idea that the Federal Reserve reacted effectively to the unexpected productivity resurgence of the late 1990s.

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“By now, the story of the boom in information technology is well known, and nearly everyone perceives that the resulting more rapid growth of labor productivity is at least partly enduring. (...) With output per hour having accelerated, cost pressures have been patently contained. For the most part, the Federal Reserve generally recognized these changing fundamentals and calibrated American monetary policy accordingly. (Remarks by Chairman Alan Greenspan, Challenges for Monetary Policymakers, at the 18th Annual Monetary Conference: Monetary Policy in the New Economy, Cato Institute, Washington, D.C. October 19, 2000).

1 Introduction

The investigation of the role of technology and monetary policy shocks in economic fluctuations has been at the center of macroeconomic research for the last two decades, from the prototypical real business cycle models of Kydland and Prescott (1982) and Long and Plosser (1983) to the latest estimated structural models with nominal rigidities of Altig, Christiano, Eichenbaum and Linde (2002) and Smets and Wouters (2002), with the monetary vector autoregression literature and the New Neoclassical/Keynesian synthesis as intermediate empirical and theoretical steps respectively.1 Remarkably, given the historical evolution of this broad research agenda, the question of the interaction between monetary policy and technological progress, and in particular of the role that monetary policy might play in dampening economic fluctuations originating from productivity shocks, has received far less attention.2 This is all the more surprising if we recall that the first attempts at introducing money and prices in a dynamic stochastic general equilibrium model were conducted within an otherwise standard RBC model, with fluctuations driven exclusively by technology shocks.3

This paper attempts to fill this gap on three complementary levels. First, at the positive level, it proposes a sticky price model of the transmission of productivity shocks, whose specification is guided by the observed dynamic behavior of real and nominal variables in response to an identified technology shock. Besides reproducing the main features of the response of the U.S. economy to an “average” shock to productivity growth, the model provides a surprisingly accurate account of the puzzling evolution of inflation and real activity that accompanied the productivity revival of the second half of the 1990s (Ball and Moffitt, 2001; Staiger, Stock and Watson, 2001). It also provides some clues on the possible causes of the “great stagflation” of the 1970s.4 Second, shifting

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2 Ireland (1996), Galí (2001) and Galí, López-Salido and Vallés (2002) are among the most significant exceptions.

3 Among the early contributions were King and Plosser (1984) and Cooley and Hansen (1989). See also Cooley and Hansen (1995).

4 Blinder (1979) is a classic reference on the subject. De Long (1997) and Sargent (1999) are two modern and
onto normative grounds, we formulate a policy problem that is approximately equivalent to that of maximizing the utility of the representative agent in the model. By solving this problem, we are able to characterize the optimal path of the policy instrument following a growth rate shock, together with the associated equilibrium fluctuations of the endogenous variables. These fluctuations are then compared to those that would emerge from some simpler policy prescriptions, as well as to those that were observed after the major productivity shocks of the early seventies and mid-nineties. Finally, from a methodological perspective, the paper introduces a variant of the imperfect information story of Lucas (1972, 1975) into a New Keynesian model with nominal rigidities, thus mixing important aspects of the two predominant modern theories about the importance of money for business cycle fluctuations (Cooley and Hansen, 1995).

The first step in our analysis is to ask if the dynamics triggered by a shock to the growth rate of productivity in a standard sticky price model match, at least qualitatively, what is observed in the data. In conducting this comparison, we mainly rely on the VAR evidence presented by Altig et al. (2002) (henceforth ACEL), whose estimated impulse responses to a permanent technology shock are reported in figure 1.5 One of the most significant features of these impulse responses, in both economic and statistical terms, is the negative conditional correlation between output and inflation, just the opposite of the more familiar positive relation associated with movements along the Phillips curve. Inflation in particular declines significantly for at least two quarters, and possibly for as much as three years, after a positive shock to the growth rate of productivity. In a standard sticky price model on the contrary, a surge in productivity reduces unit labor costs, but with perfectly competitive labor markets, the ensuing increase in real wages exactly offsets the initial impact of the shock. This general equilibrium effect therefore severs the link between real marginal costs and productivity, insulating prices and inflation from technology shocks.6 The counterfactual behavior of the model along this particular dimension is especially troublesome for our analysis, since one of its main objectives is to derive a set of constraints for the optimal policy problem that reflect the nature of the trade-offs that central banks face in reality when trying to insulate the economy from the effects of a productivity shock.

To obviate this problem, we then formulate a model in which the response of wages to a productivity disturbance is delayed by a particular form of imperfect information, whereby private agents, upon observation of a shock, cannot conclusively infer its degree of persistence. As a result, whenever the shock is in fact persistent, workers come to this realization only gradually, causing wage demands to lag behind realized productivity. Decoupling the movements of real wages and productivity in the short run, this mechanism contributes in turn to a temporary fall in marginal costs, and therefore in inflation, following a positive productivity shock.

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5 ACEL identify this shock as the only source of the unit root in average labor productivity, as in Galí (1999). According to their estimates, the growth rate of the shock is distributed as an AR(1) process with an autoregressive coefficient of 0.8.

6 Section 1.1 illustrates this point more formally.
The idea that wage contracts might incorporate adjustments to slowly evolving wage aspirations, or to filtered estimates of the growth rate of productivity, and that the resulting inertia might cause wages to be decoupled from underlying productivity trends following significant changes in their rate of growth dates back at least to Jackman et al. (1982) and Braun (1984), and has witnessed a recent revival as a possible explanation of the unusual joint behavior of inflation and real activity in the second half of the nineties (Blinder, 2000; De Long, 2000; Ball and Moffitt, 2001). Our contribution in this respect is to take this idea from the realm of simple hypothesis, or of loosely specified regression models, and to incorporate it into a fully specified model of price and wage setting. We do this by relying on the version of the story based on the slow adjustment of trend-growth estimates (Blinder, 2000), first, because it does not require any departure from standard rational accounts of agents’ behavior and second, because uncertainty on the persistence of the recent productivity pickup has been one of the dominant notes in the debate spurred by this important event.7

As a possible alternative to the research strategy pursued in this paper, we could have indeed directly adopted the theoretical model proposed in ACEL, and shown there to provide a good match to the empirical impulse responses that we are also trying to replicate. Two main considerations kept us from following this approach. First, we wanted to limit the complexity of our model to concentrate on the transmission channels of productivity shocks that are likely to be more relevant for short term policy considerations. This meant in particular abstracting from capital accumulation and all the related real frictions. As an important by-product of this simplification, it is possible to derive a second order approximation to the utility function of the representative agent as a model-consistent welfare criterion. Second, even with its wide array of real frictions, ACEL’s model is not particularly successful at replicating the negative conditional correlation between inflation and real activity, which we consider a fundamental stylized fact of the transmission of productivity shocks. In this respect, imperfect information can be thought of as a complementary mechanism to the ones identified in ACEL to explain the observed behavior of aggregate variables in response to technology shocks.

The paper’s main results can be summarized as follows. First, following a persistent shock to the growth rate of productivity, our model can generate reductions in costs and inflation that are close to those observed in the data. However, other informational frictions besides partial information are quantitatively at least as important for this result as the latter. Second, monetary policy should maintain a procyclical stance in reaction to technology disturbances, decreasing the nominal interest rate below its flexible price counterpart following a positive growth rate shock. Such policy has the effect of inducing the desired long run adjustment of the real wage through increases in the nominal wage rate and reductions in the price level, in proportions that depend on the relative flexibility of the two prices. As a consequence, policies that strictly target the price

7 Jorgenson (2001) and Gordon (2002) are two prominent academic contributions to this debate; Greenspan (2000) provides the perspective of an important policymaker, while Woodward (2000) contains a journalistic account and a popular interpretation of the events, at least before the current recession...
or wage inflation rate are clearly undesirable. On the other hand, a policy that tries to stabilize the central bank’s forecast of the model consistent output gap is close to optimal. Finally, when perturbed by an appropriately scaled shock, the model’s optimal equilibrium displays a path of average productivity, interest rates, inflation and output growth, which is remarkably close to that observed in the second half of the 1990s. Hence the conclusion that in that period the Federal Reserve calibrated American monetary policy according to the model...

The rest of the paper is organized as follows. Section 1.1 offers some more detail on the “standard” sticky price model and on its behavior following a productivity shock. Section 2 lays down our positive model of the economy, starting with the assumptions on tastes, technology and information. In particular, section 2.6 contains the main steps in the derivation of the welfare criterion, with the details relegated to appendix A. Section 3 describes the dynamics of the model following a productivity shock, while section 5 is dedicated to the discussion of the optimal equilibrium, which is also contrasted with that which would emerge from some simple policy prescriptions. Sections 4 and 6 compare the macroeconomic developments that accompanied the two major post-war productivity shocks with simulations from the model, from a positive and normative perspective respectively. Section 7 concludes. Two appendices contain some details on the solution and filtering of the system of linearized first order conditions and on the computation of the optimal equilibrium in light of the assumed imperfect information.

1.1 The Neutrality of Productivity Shocks in a Standard Model

Defining a standard within a class of models that, like the sticky price models of the business cycle, is in continuous and precipitous expansion, is a hard and elusive task. Nevertheless, we think it useful to crystallize the received knowledge in the field in a paradigmatic model, not with the intent of denouncing its assumptions as flawed, but rather as a way of organizing our thoughts around a familiar and largely diffused benchmark. Three fundamental assumptions characterize in our judgment this prototypical model. First, output is produced with labor as the only input. Second, goods’ prices are sticky, and set in staggered fashion. Third, labor markets are perfectly competitive. For instance, these are the assumptions that underlie most of the contributions in the influential Taylor (1999) volume (see for example Rotemberg and Woodford, 1999), as well as the models in Woodford (1996), Rotemberg and Woodford (1997) and Clarida, Galí and Gertler (1999), just to name a few prominent examples.

A simple but fairly representative rendition of this model is that in Galí (2001), which we summarize here for expositional convenience.8 The representative household’s utility is separable between consumption and hours

\[ E_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{Y_t^{1-\sigma}}{1-\sigma} - H_t^{1+\varphi} \right) \]

8 We refer the reader to Galí (2001) for further details on the model and the derivation of its equilibrium.
and a continuum of monopolistically competitive firms indexed by \( z \) produces differentiated goods according to the log-linear production function

\[
y_t(z) = a_t + h_t(z)
\]

where \( a_t \) denotes the logarithm of productivity and \( H_t \equiv \int_0^1 \exp[h_t(z)] \, dz \). Given an economy-wide labor market, firms face a common real marginal cost

\[
s_t = w_t - p_t - a_t
\]

while workers’ optimization on the consumption-leisure margin implies

\[
w_t - p_t = \sigma y_t + \varphi h_t
\]

where \( w_t - p_t \) is the logarithm of the real wage and lower case letters denote the logarithms of their upper case counterparts. Making use of the approximation \( y_t = a_t + h_t \), we find that in equilibrium

\[
s_t = (\sigma - 1) a_t + (\sigma + \varphi) h_t
\]

The reason for focusing on this cost measure is that in this class of models real marginal costs are the only determinants of firms’ optimal pricing behavior, which, in the presence of price setting frictions à la Calvo (1983), results in a Phillips curve of the form

\[
\pi_t = \beta E_t \pi_{t+1} + \kappa \hat{s}_t
\]

where \( \pi_t \) is inflation, \( \hat{s}_t \) denotes the deviation of \( s_t \) from its steady state level and \( \kappa \) is a positive coefficient that depends on the frequency of price adjustment.

Two important considerations follow from this derivation. First, with the restriction \( \sigma = 1 \) imposed by the presence of secular growth, any shock to productivity will be fully reflected in the equilibrium real wage, perfectly compensating the partial equilibrium cost effect normally attributed to productivity changes. Evidently, under these conditions technology shocks cannot have any direct impact on prices and inflation. Intuitively, the assumption of perfectly competitive labor markets imposes the desired cointegration between productivity and real wages by forcing them to move together also in the short run. Introducing a wedge between the short run comovements of these two variables, imperfect information generates an environment in which increases in output per hour help to contain cost pressures, as observed by Chairman Greenspan in the paper’s opening quote. Note also that wage stickiness alone would not necessarily produce the desired result. This is because in a forward looking model, wage inflation would in fact overshoot the currently observed surge in productivity if agents anticipated further productivity increases in the future, as would be the case for the kind of persistent growth rate shocks considered here.\(^9\)

\(^9\) This is a consequence of the well-known front loading effect described by Christiano et al. (2001). Mankiw and Reis (2002) present a sticky information model that avoids this effect and that is very similar in spirit to the model developed in this paper.
A second important point is that, even in the case $\sigma \neq 1$, a monetary policy that stabilized the real marginal cost around its steady state level, or equivalently the level of output around its flexible price equilibrium, would also result in zero inflation, again severing the link between productivity and prices. This is precisely the case emphasized by Galí (2001), one in which productivity shocks do not pose any stabilization trade-off to the monetary authority. Interestingly, Galí et al. (2002) present some VAR evidence that, during the Volcker-Greenspan era, inflation did not significantly respond to identified productivity shocks, while this response was negative and significant in the pre-Volker period. This evidence would then seem to suggest that this paper’s normative question has a very simple answer, and that the Federal Reserve has behaved accordingly for at least twenty years. Several considerations make us doubt this conclusion. First, according to most accounts, the Federal Reserve spent a considerable amount of effort to “calibrate” its monetary policy in response to the productivity acceleration of the mid-nineties. Second, Galí et al. (2002) do not present any explicit empirical evidence on the statistical significance of the difference in the responses across the two sub-periods in their analysis. Third, as we will see in section 5, although in our model output gap stabilization closely approximates the optimal policy, it does not imply a stable inflation rate. Quite to the contrary, a policy that did stabilize inflation would result in very undesirable welfare consequences. We find these exceptions to provide enough ground for us to proceed with the analysis, which begins in the next section with the description of our structural model.

2 The Model

This section presents a dynamic stochastic general equilibrium model of the monetary transmission mechanism, whose microeconomic foundations derive from the work of Rotemberg and Woodford (1997) and Amato and Laubach (2002). As in Amato and Laubach (2002) and Erceg, Henderson and Levin (2000), we assume the existence of wage setting frictions in the labor market, along with price stickiness in the market for consumption goods. As argued in the previous section, abandoning the assumption of perfect competition in the labor market is a necessary condition for productivity shocks to play any role in this class of models. Assuming that labor and goods markets share the same kind of price frictions is a particularly convenient way of fulfilling this condition.

The section’s main contribution is twofold. First, it considers explicitly the possibility of secular growth. This requires to impose some restrictions on utility and production functions, of the kind that King, Plosser and Rebelo (1988, 2002) showed to be necessary to produce long run implications of a model economy in accordance with the basic growth facts. It also illustrates how these restrictions, together with a fairly natural assumption on wage indexation, are sufficient to maintain almost intact the basic structure of the standard models of pricing based on Calvo (1983). Second, it presents an environment in which productivity is perturbed by two shocks with differing degrees of persistence, but agents can only observe its overall level. As a consequence,
after an unexpected change in measured productivity, agents are uncertain about its persistence, which they are assumed to estimate through a Kalman filter. This in turn imparts some inertia to their reaction to the shock, which is critical to explain the observed negative conditional correlation between inflation and real activity.

The structural representation of the economy is completed with the derivation of a welfare criterion for policy evaluation, which is based on a second order expansion of the utility of the representative agent. Given appropriate assumptions on the magnitude of the distortions in the economy, this expansion, evaluated through a linear approximation to the structural equilibrium relations, provides an accurate gauge of the effect of alternative policies on the welfare of the representative agent, up to a residual that is of third order in the amplitude of the shocks.

2.1 Tastes, Technology and Information

We consider an economy populated by three classes of agents: a government, a continuum of households indexed by \( j \in [0, 1] \) and a continuum of firms indexed by \( z \in [0, 1] \), whose ownership is equally shared among the households.

**Government**  The government is composed of two branches, a fiscal and a monetary authority. The fiscal authority levies a proportional tax \( \tau \) on the sales of consumption goods and rebates the proceeds to the households through a lump-sum transfer. The monetary authority sets the level of the short term nominal interest rate, as further detailed in section 2.7 below.

**Firms**  Firm \( z \) is a monopolist on the market for its output, that is produced using the composite labor input \( h_t(z) \). As in Erceg *et al.* (2000), this is a Dixit-Stiglitz (1977) aggregate of a continuum of specialized “skills” distributed among the households

\[
h_t(z) \equiv \int_0^1 h_t^z (j) \theta w_{t-1}^1 \theta w_{t-1}^j dj \]

Each of these specialized “skills” trades at a wage \( w_t(j) \) on an economy-wide market, from which all firms draw their supply. They generate a total demand for skill \( j \)

\[
h_t(j) = H_t \left( \frac{w_t(j)}{W_t} \right)^{-\theta_w}
\]

where \( H_t \equiv \int_0^1 h_t(z)dz \) denotes aggregate hours and

\[
W_t \equiv \left[ \int_0^1 w_t(j)^{1-\theta_w} dj \right]^{1/\theta_w}
\]

is the wage index associated with the minimum expenditure purchase of one unit of \( H_t \).

Output \( y_t(z) \) is produced according to the production function

\[
y_t(z) = A_t f (h_t(z))
\]
where \( f \) is increasing and concave, and \( A_t \) represents a technology factor. Following a long-standing tradition in the New Keynesian analysis of the monetary transmission mechanism, we abstract from capital accumulation.

Our main point of departure from this literature is instead in the distributional assumptions on \( A_t \). In particular, we assume that the growth rate of productivity, \( \gamma_t^a \equiv \ln A_t - \ln A_{t-1} \), follows a stationary process, which is the sum of a persistent (“trend”) and an i.i.d. component, as in

\[
\gamma_t^a = \gamma_t + \varepsilon_t^a
\]

\[
\rho(L)(\gamma_t - \gamma) = \varepsilon_t^\gamma
\]

where \( \varepsilon_t^a \) and \( \varepsilon_t^\gamma \) are orthogonal i.i.d. shocks with mean zero and variances \( \sigma_a^2 \) and \( \sigma_\gamma^2 \) respectively, while \( \rho(L) \) is a stable polynomial in the lag operator. We furthermore assume that all agents in the economy, including the monetary authority, can observe the level of productivity \( A_t \), along with the vector of endogenous variables \( X_t \), but not the shocks \( \varepsilon_t^\gamma \) and \( \varepsilon_t^a \) separately. More formally, we denote agents’ information set by \( I_t \equiv \{ A_t; X_t \}_{\tau \leq t} \) and \( I_t^f \equiv \{ \varepsilon_t^\gamma, \varepsilon_t^a; X_t \}_{\tau \leq t} \), where \( I_t^f \) represents full information. Note that this form of limited information is not necessarily a “binding” constraint on agents’ decision making, at least as long as their actions do not require them to forecast future economic conditions. This would for instance be the case for a firm that can reoptimize its price every period. It is only when decision making involves expectations about the future evolution of productivity, as for a firm that is allowed to reset its price only at certain intervals, that partial information becomes relevant. In this case, we assume that agents would make the best possible use of all the available information and use a Kalman filter to forecast the future evolution of productivity, as detailed in appendix B.

**Households** Household \( j \) is endowed with one unit per period of the specialized labor \( h_t(j) \), on whose market it is a monopolist. The household sets the wage rate \( w_t(j) \) for period \( t \) and supplies the amount of hours demanded by firms at the posted price, as further discussed in section 2.4. It also chooses a sequence of consumption bundles \( \{ c^j_s(z), z \in [0,1] \}_{s=t}^\infty \) to maximize a time separable expected utility function of the form

\[
E_{t-1} \sum_{s=t}^\infty \beta^{s-t} [u(C_s(j)) - v(h_s(j))]
\]

where \( \beta \in (0,1) \) is the subjective discount factor and \( C_t(j) \) is an aggregator of the levels of consumption of different goods, with a constant elasticity of substitution \( \theta_p > 1 \)

\[
C_t(j) \equiv \left[ \int_0^1 c^j_s(z) \frac{\theta_p-1}{\theta_p} dz \right]^{\frac{\theta_p}{\theta_p-1}}
\]

The expectation in 3 is based on the information publicly available as of time \( t - 1 \), \( I_{t-1} \). This captures an information processing (or implementation) delay, as for example in Christiano et al.\(^\text{12}\) In what follows we refer to \( A_t \) indifferently as technology or (total factor) productivity. When the distinction is relevant, we will refer explicitly to (average) labor productivity.
(2001). In this context, besides preventing demand from jumping immediately in response to shocks, this assumption has the realistic feature of making the output gap unobservable, in a sense that will be made more precise in section 2.3. Note also that money does not enter the utility function, and in fact does not play any explicit role in our model. We are in other words assuming to be in a “cashless limiting economy”, in which the role of money balances in facilitating transactions is negligible (Woodford, 2002).

If we assume the existence of complete financial markets, and therefore of a unique kernel \( Q_{t,s} \) for the pricing of stochastic flows of nominal income, given a sequence of goods’ prices \( \{p_s(z), z \in [0, 1]\}^\infty_{s=t} \), we can write the household’s intertemporal budget constraint looking forward from time \( t \) as

\[
\sum_{s=t}^\infty E_{t-1} \left[ Q_{t-1,s} \left[ \int_0^1 p_s(z)c^*_t(z)dz \right] \right] \leq E_{t-1}Q_{t-1,t}B_t(j) + \sum_{s=t}^\infty E_{t-1}Q_{t-1,s} \left[ w_s(j)h_s(j) + \Pi_s(j) \right]
\]

(4)

where \( B_t(j) \) is initial wealth and \( \Pi_t(j) \) are the profits accruing to the household, net of any lump sum taxes.\(^\text{13}\) In general, depending on the realized frequency with which they are offered the chance to reset their wage, different agents will experience very different employment histories, and thus very different levels of human wealth. Nevertheless, if at any point in time \( \tau < t \) the distribution of wealth \( \{B_\tau(j)\}_j \) is such that the right hand side of (4) is the same for all \( j \), then, from then on, the optimal consumption profile will be identical for all households. In what follows we will assume that this is indeed the case, and consequently drop the \( j \) index that distinguishes individual consumption choices.

The solution to the household’s consumption problem yields the following optimality conditions. First, optimal intratemporal allocation of a given amount of expenditure across the differentiated goods implies

\[
c_t(z) = C_t \left( \frac{p_t(z)}{P_t} \right)^{-\theta_p}
\]

(5)

where \( c_t(z) \) is total demand for good \( z \), \( C_t \) is the index of aggregate consumption and \( P_t \) is defined analogously to (1). Second, the optimal choice of aggregate consumption satisfies

\[
\beta E_{t-1} [u_C(C_t)] = \Lambda_{t-1}E_{t-1} [Q_{t-1,t}P_t]
\]

(6)

where \( \Lambda_{t-1} \), the multiplier on the intertemporal budget constraint (4), measures the marginal utility of nominal income. Finally, the optimal allocation of resources across time and states achieved through complete financial markets implies

\[
\Lambda_t Q_{t,s} = \beta^{s-t} \Lambda_s
\]

(7)

and therefore

\[
\Lambda_t = \beta E_t [R_t\Lambda_{t+1}]
\]

(8)

\(^{13}\) We refer the reader to Woodford (2002) for a detailed derivation of the intertemporal budget constraint in a closely related model and for a general discussion of models in which current consumption decisions are based on lagged information.
where $R_{t}^{-1} = E_t [Q_{t,t+1}]$ is the price of a riskless bond held between $t$ and $t + 1$, so that $R_t$ is the gross short term nominal interest rate.

2.2 Flexible Price Equilibrium and the Balanced Growth Path

Having introduced the basic building blocks of the model, and before proceeding to the description of the price frictions that constitute the necessary premise to a well defined policy problem, it is useful to consider the model’s behavior with flexible prices. This exercise becomes even more crucial in the context of our non-stationary economy, since it will help us to characterize the balanced growth path that the economy would follow in the absence of shocks. It will then be around this path that we approximate the model’s first order conditions to provide a dynamic characterization of the equilibrium responses to shocks.

When prices are flexible, firms reset their price every period to maximize their instantaneous profits, taking as given the demand function for their output. From (5) and the fact that private consumption is the only source of demand, this function is simply

$$y_t(z) = Y_t \left( \frac{p_t(z)}{P_t} \right)^{-\theta_p}$$

(9)

The result of profit maximization is an optimal price (net of the sales tax) that is set as a fixed markup over marginal cost. In real terms

$$\frac{p_t(z)}{P_t} (1 - \tau) = \mu_p s_t(z)$$

where $\mu_p \equiv \frac{\theta_p}{\theta_p - 1} > 1$ is the optimal (gross) markup for a monopolist facing a demand curve with constant elasticity $-\theta_p$, and $s_t(z)$ is the real cost function for firm $z$

$$s_t(z) = \frac{W_t}{P_t} \left[ A_t f' \left( f^{-1} \left( \frac{Y_t}{A_t} \right) \right) \right]^{-1}$$

As for wages, they will similarly be set as a markup over the marginal rate of substitution between consumption and leisure

$$\frac{w_t(j)}{P_t} = \mu_w \frac{v_t(b_t(j))}{u_C(Y_t)}$$

where $\mu_w \equiv \frac{\theta_w}{\theta_w - 1} > 1$.

As a result, in a symmetric equilibrium with identical prices and wages across firms and households, the level of output is implicitly defined by the solution to

$$\frac{u_C(Y_t)}{v_t(f^{-1}(Y_{At}))} A_t f' \left( f^{-1} \left( \frac{Y_t}{A_t} \right) \right) = \frac{\mu_p \mu_w}{1 - \tau}$$

where $Y_{At} \equiv Y_t / A_t = f^{-1} (H_t)$ is the effective level of output. Note that, without some restrictions on the utility function $u$, we cannot guarantee the existence of a growth path with constant hours (and therefore a constant value of effective output), a very natural requirement to impose on any model economy. As shown by King at al. (1988), given separability between consumption and
leisure in the period utility function, the restriction $u(C) = \ln C$ is a necessary and sufficient condition for the existence of a balanced growth path, which we define here as a stationary equilibrium in which hours worked and the shares of income accruing to wages and profits are bounded away from zero and one. With this restriction, and in the absence of shocks,

$$\frac{f'(f^{-1}(\bar{Y}_A))}{\bar{Y}_A v_h(f^{-1}(\bar{Y}_A))} = \frac{\mu_p \mu_w}{1-\tau}$$

implicitly defines the constant level of effective output along the balanced growth path.

This expression also highlights how the presence of market power in the goods and labor markets, together with distortionary taxation, drives a wedge between the marginal rates of substitution and transformation of labor and consumption, whose equality characterizes the efficient equilibrium. To abstract from the role that monetary policy could (and optimally would) play in ameliorating this inefficiency, we then assume that the fiscal authority subsidizes the sales of consumption goods exactly enough to offset the monopoly distortions, setting $\tau = 1 - \mu_p \mu_w$ (Rotemberg and Woodford, 1999), from which we get

$$\bar{Y}_A^{-1} = \frac{v_h(f^{-1}(\bar{Y}_A))}{f'(f^{-1}(\bar{Y}_A))}$$  \hspace{1cm} (10)

By the second welfare theorem, as well as by simple inspection of (10), $\bar{Y}_t \equiv A_t \bar{Y}_A$, the efficient level of output, is seen to maximize the utility of the representative agent, given the resource constraint. As a result, a linear approximation to the structural equilibrium relations will be all what is needed to compute a second order accurate approximation to the utility of the representative agent, as further illustrated in section 2.6.

To complete the characterization of the balanced growth path we need only to remark that, when output is equal to its efficient level and $A_t$ grows at the deterministic rate $\gamma$, equation (6) implies that $\lambda_{A_t} \equiv \Lambda_t P_t A_t = \bar{Y}_A^{-1}$ so that from (8) we obtain

$$\ln \bar{R} = \gamma - \ln \beta + \pi$$

the value of the (continuously compounded) nominal interest rate along a growth path with steady inflation at rate $\pi$.\footnote{In what follows we will restrict our attention to the case $\pi = 0$.}

We now turn to the description of the equilibrium fluctuations induced by the productivity shocks, and of their interaction with the price setting frictions in the goods and labor markets. We begin with the analysis of the demand side of the economy.

### 2.3 Demand

The demand side of the economy is completely characterized by a dynamic IS equation, which links current expenditures to expectations of future real interest rates. This relation is obtained starting from a log-linear approximation to the Euler equation (8) around the balanced growth path with no inflation

$$\hat{\lambda}_{A_t} = E_t \left[ \hat{\pi}_{t+1} + \hat{\lambda}_{A_{t+1}} - \hat{\gamma}_{t+1}^a \right]$$  \hspace{1cm} (11)
where \( \hat{\lambda}_{At} \equiv \ln \left( \Lambda_t P_t A_t / \bar{Y}_A \right) \), \( \hat{i}_t \equiv \ln \left( R_t / \bar{R} \right) \) is the deviation of the nominal interest rate from its steady state value, \( \pi_t \equiv \ln \left( P_t / P_{t-1} \right) \) is the inflation rate and \( \hat{\gamma}^o_{it} \equiv \gamma^o_t - \gamma \). We can then translate this expression in terms of expenditures if we note that, with logarithmic utility, and defining the output gap variable 

\[
X_t \equiv \ln \left( \frac{Y_t}{A_t / \bar{Y}_A} - 1 \right)
\]

(6) and (7) imply the approximate relation

\[
x_t = -E_{t-1} \hat{\lambda}_{At} - \left( \ln A_t - E_{t-1} \ln A_t \right)
\]

which together with (11) yields

\[
x_t = E_{t-1} x_{t+1} - E_{t-1} \left[ \hat{i}_t - \pi_{t+1} - \hat{r}_t^o \right] - \left( \hat{\gamma}^o_t - E_{t-1} \hat{\gamma}^o_t \right)
\]

(12)

Here \( x_t \) denotes the deviation of output from its efficient level, our preferred notion of output gap, while \( \hat{r}_t^o \equiv E_t \hat{\gamma}^o_{t+1} \) is the real interest rate that would prevail under flexible prices and without informational delays, expressed in deviation from its value along the balanced growth path, \( \bar{i} \).

Note that if we employed an alternative notion of the output gap, as the deviation of output from its equilibrium level with flexible prices, but with informational delays, \( x_n^o \equiv \ln \left( \frac{Y_t}{E_{t-1} A_t \bar{Y}_A} \right) \), the IS equation could be written in the simpler form

\[
x_n^o = E_{t-1} x_{n+1}^o - \left( E_{t-1} \hat{i}_t - E_{t-1} \pi_{t+1} - \hat{r}_n^o \right)
\]

where now \( \hat{r}_n^o \equiv E_t \hat{\gamma}^o_{t+1} \) is the familiar natural rate of interest. The reason for choosing the more cumbersome formulation in (12) is that \( x_t \) is the notion of output gap that is relevant for welfare comparisons, as shown in section 2.6.

Another important characteristic of this demand formulation is that the output gap is not part of the information set \( I_{t-1} \) on which consumers base their decisions at time \( t \). In fact, even if expenditure demand is predetermined one period in advance, it is only with the realization of the productivity shock at time \( t \) that the amount of hours needed to satisfy that level of demand, and therefore the efficient level of output, become known. As a result, the model exhibits the realistic feature of an output gap whose exact realization is uncertain at the time at which agents are required to take their decisions. Moreover, since only forecastable movements in the policy instrument \( \hat{i}_t \) have an effect on the output gap at time \( t \), the goal of output stabilization, although desirable, cannot be perfectly achieved. The extent to which this influences the optimal policy is one of the objects of our analysis in section 5.

### 2.4 Wage Setting

The wage setting mechanism is based on the random staggering device of Calvo (1983), modified to accommodate indexation to productivity growth and decision lags. In particular, we assume that every period a fraction \( 1 - \alpha_w \) of outstanding labor contracts is randomly selected for renegotiation. A fraction \( \tilde{\psi}_w \) of these new contracts is set on the basis of \( I_{t-1} \) information and carries an initial wage \( w^1_t \). The remaining contracts are set on the basis of \( I_{t-2} \) information, with wage \( w^2_t \). In general,
this structure of delays has the objective of moderating the response of wages to shocks in the short run, to bring it more in line with the available empirical evidence. Here, it has the added benefit of attributing a consistent information set to members of the same household operating in the goods and labor markets.

We complete the characterization of the wage setting process describing the behavior of the fraction \( \alpha_w \) of wages excluded from renegotiation. Similarly to Altig et al. (2002), we assume that these wages are indexed to past price inflation and to productivity growth, so that a wage contract \( j \) that is not renegotiated between \( t-1 \) and \( t \) carries a wage \( w_t(j) \) such that

\[
\ln w_t(j) = \ln w_{t-1}(j) + \Omega_t
\]

where \( \Omega_t \) is defined as

\[
\Omega_t = \lambda_w \pi_{t-1} + \lambda_\gamma E_{t-1} \gamma^a_t + (1 - \lambda_\gamma) \gamma^a_{t-1}
\]

The reasons for assuming this particular form of indexation are twofold. First, in a stochastic equilibrium that exhibits stationary fluctuations around a balanced growth path, real wages and productivity need to be cointegrated. Given that under Calvo wage setting a positive fraction of wages is not drawn for renegotiation for an arbitrarily large number of periods, contractual indexation guarantees that those wages do not fall excessively out of line with the general level of productivity. Second, consistency with the informational assumptions laid down so far requires that wages be indexed to a measure of productivity included in the information set \( I_{t-1} \). Our indexation scheme considers two such measures, the expectation of the future value of productivity, \( E_{t-1} \hat{\gamma}_t \), and its latest available measurement, \( \hat{\gamma}_t \). As we will see, some of our results are sensitive to the prevalent form of indexation, which explains why we propose a specification that nests both measures.

The expression for the wage index

\[
W_t = \left[ (1 - \alpha_w) \left( \tilde{\psi}_w \left( w_t^1 \right)^{1-\theta_w} + \left( 1 - \tilde{\psi}_w \right) \left( w_t^2 \right)^{1-\theta_w} \right) + \alpha_w \left( W_{t-1} \Omega_t \right)^{1-\theta_w} \right]^{1/\alpha_w}
\]

provides a useful summary of the assumptions introduced so far. To determine the equilibrium value of its components, we first note that wages reoptimized in period \( t \) on the basis of \( t-1 \) information, \( w^1_t \), are chosen to maximize the expected present discounted value of wage income, expressed in terms of utils and net of the disutility of labor

\[
E_{t-1} \left\{ \sum_{s=t}^{\infty} (\alpha_w \beta)^{s-t} \left[ \Lambda_s w^1_{t,s} h^1_s - v \left( h^1_s \right) \right] \right\}
\]

where \( w^1_{t,s} \equiv w^1_t \left( \prod_{k=t+1}^{s} \Omega_k \right) \) is the wage rate at time \( s \), conditional on no renegotiation having occurred between \( t \) and \( s \), and \( h^1_s \equiv H_s \left( w^1_{t,s} \right)^{-\theta_w} \) is the amount of labor demanded at that wage.

---

15 The forward looking nature of standard optimizing models of pricing implies a front loaded response of prices to forecastable movements in the fundamentals, as observed for example by Christiano et al. (2001). Information delays help to moderate this effect.
The optimal choice of \( w^1_t \) therefore satisfies

\[
0 = E_{t-1} \left\{ \sum_{s=t}^{\infty} (\alpha_w \beta)^{s-t} h_s^1 \left[ w_{1,s} A_s - \mu_w v_h (h_s^1) \right] \right\}
\]

which we log-linearize around the balanced growth path with no inflation to obtain

\[
E_{t-1} \left\{ \sum_{s=t}^{\infty} (\alpha_w \beta)^{s-t} \left[ (1 + \nu \theta_w) \dot{w}^1_t - (1 + \nu \phi) \dot{x}_s - (1 + \nu \theta_w) \sum_{k=t+1}^{\infty} \left( \dot{\pi}_k^w - \dot{\Omega}_k + \dot{\omega}_A \right) \right] \right\} = 0
\]

where \( \nu = \frac{v_w h_k}{v_h} \) measures the curvature of the disutility of labor, \( \phi^{-1} \equiv \frac{f' h}{f} \) is the elasticity of the production function, \( \dot{w}^1_t \equiv \ln \left( \frac{w^1_t}{W_t} \right) \), \( \dot{\pi}_k^w \equiv \ln \left( \frac{W_t}{W_{t-1}} \right) - \gamma \) is the deviation of nominal wage inflation from its steady state value, \( \dot{\Omega}_t \equiv \ln \Omega_t - \gamma \) and \( \dot{\omega}_A_t \equiv \ln \left( \frac{W_t}{A_t} P_t \dot{\omega}_A \right) \) is the percentage deviation of the effective real wage from its value on the balanced growth path

\[
\dot{\omega}_A = \mu_w Y_A v_h (f^{-1} (Y_A))
\]

The first order condition can then be solved for \( \dot{w}^1_t \) and, together with the fact that \( w^2_t = E_{t-2} w^1_{t-1} \), and after some algebra, this is enough to characterize the evolution of the wage index. This is usefully summarized by a wage Phillips curve of the form

\[
\ddot{\pi}_t^w = (1 - \psi_w) E_{t-2} \ddot{\pi}_t^w + \psi_w E_{t-1} \left[ \kappa_w \dot{x}_t - \xi_w \dot{\omega}_A + \beta \ddot{\pi}_{t+1}^w \right] + \zeta_w \Phi_t \tag{13}
\]

where \( \psi_w \equiv \frac{\alpha_w \beta}{1 - \psi_w (1 - \alpha_w)} \), \( \kappa_w \), \( \xi_w \) and \( \zeta_w \) are positive coefficients, \( \ddot{\pi}_t^w \equiv \ddot{\pi}_t^w - E_{t-2} \dot{\Omega}_t \) is the deviation of wage inflation from the forecastable component of its index and \( \Phi_t \equiv \dot{\Omega}_t - E_{t-2} \dot{\Omega}_t \) is its forecast error.\(^\text{16}\) According to this equation, deviations of wage inflation from the value that would be predicted on the basis of indexation alone are a function of the expected future values of the effective wage level and of the output gap. The latter serves as a proxy for the amount of hours that workers expect to supply at the posted wage, and for the marginal disutility thereof.

The fact that in this class of models with indexation the resulting Phillips curve is conveniently expressed in terms of deviations of inflation from its index is familiar from Woodford (2002). In this particular example, we used instead the deviation \( \ddot{\pi}_t^w \) from the forecast because this is the notion of wage inflation that is relevant for welfare comparisons, as shown in section 2.6. Note however that even if we adopted the more conventional deviation \( \ddot{\pi}_t^w - \dot{\Omega}_t \), we would still need to append a “shock” term similar to \( \Phi_t \) to the resulting wage setting relation.

### 2.5 Price Setting

The price setting process in the goods’ market has the same basic structure described above, except that contractually fixed prices are now assumed to be partially indexed only to past observed

\(^{16}\) The details of the calculations, which follow very closely the steps illustrated in Rotemberg and Woodford (1998), are collected in an Appendix that is available from the author upon request. The values of the coefficients that appear in the log-linearized supply block, equations 13 and 15, are collected in table 4.
inflation. Therefore, a price \( p_t(z) \) that is not drawn for reoptimization at time \( t \) is updated as

\[
\ln p_t(z) = \ln p_{t-1}(z) + \lambda_p \pi_{t-1}
\]

Moreover, we maintain the assumption that a fraction \( \bar{\psi}_p \) of the prices to be reoptimized each period is chosen on the basis of time \( t-1 \) information, while the remaining \( 1 - \bar{\psi}_p \) prices are based on information dated \( t-2 \). The price index is now

\[
P_t = \left[ (1 - \alpha_p) (\bar{\psi}_p (p_t^1)^{1-\theta_p} + (1 - \bar{\psi}_p) (p_t^2)^{1-\theta_p}) + \alpha_p \left( P_{t-1} \left( \frac{P_{t-1}}{P_{t-2}} \right) \right)^{1-\theta_p} \right]^{1/1-\theta_p}
\]

Prices reoptimized on the basis of \( I_{t-1} \) information, \( p_t^1 \), are chosen to maximize the expected present discounted value of future profits

\[
E_{t-1} \left\{ \sum_{s=t}^{\infty} \alpha_p^{s-t} Q_{t,s} \left[ (1 - \tau) p_t^1 \left( \frac{P_{t-1}}{P_{t-2}} \right) \lambda_p \right]^{A_s} y_{As}^1 - W_s f^{-1} (y_{As}^1) \right\}
\]

where

\[
y_{As}^1 = Y_{As} \left[ \frac{p_t^1}{p_s^1} \frac{P_{t-1}}{P_{t-2}} \lambda_p \right]^{\theta_p}
\]

is the level of demand for a good whose price has not been reoptimized between times \( t \) and \( s \). The first order condition for the maximization of (14) is

\[
0 = E_{t-1} \left\{ \sum_{s=t}^{\infty} \left( \alpha_p \beta \right)^{s-t} \frac{\lambda_p A_s}{1 - \theta_p} \left[ \frac{p_t^1}{p_s^1} \frac{P_{t-1}}{P_{t-2}} \lambda_p \right]^{\theta_p} - \frac{\mu_s}{1 - \theta_p} W_s f^{-1} (y_{As}^1) \right\}
\]

which we again log-linearize and solve to obtain an expression for \( \bar{p}_t^1 \equiv \ln (p_t^1/P_t) \)

\[
\frac{1 + \omega_p \theta_p}{1 - \alpha_p \beta} \bar{p}_t^1 = E_{t-1} \left\{ \sum_{s=t}^{\infty} \left( \alpha_p \beta \right)^{s-t} \left[ \omega_p x_s + \bar{\omega}_s + (1 + \omega_p \theta_p) \sum_{k=t+1}^{s} \tilde{\pi}_k \right] \right\}
\]

where \( \omega_p \equiv -\frac{f''}{f'} \) is the elasticity of the real marginal cost function with respect to output, holding wages fixed, and \( \tilde{\pi}_t \equiv \pi_t - \lambda_p \pi_{t-1} \) is the deviation of price inflation from its index.

As before, manipulation of the first order condition and of the expression for the price index results in a price Phillips curve of the form

\[
\tilde{\pi}_t = (1 - \psi_p) E_{t-2} \tilde{\pi}_t + \psi_p \left[ \kappa_p x_t + \xi_p \omega_{At} + \beta \tilde{\pi}_{t+1} \right]
\]

(15)

where \( \psi_p \) is defined analogously to \( \psi_w \) and \( \xi_p \) and \( \kappa_p \) are positive coefficients. This equation relates the amount of inflation in excess of automatic indexation to the expected future values of the output gap and the effective real wage. These terms reflect in turn the two basic components of the marginal cost, one capturing the effect of higher output demand on the efficiency of the marginal input, the other measuring its unit cost. Differently from the wage equation 13, this relation is not perturbed by terms like \( \Phi_t \) because the amount of the automatic indexation for prices is perfectly forecastable on the basis of information dated \( t-2 \). This also accounts for the different definition of the inflation deviation \( \tilde{\pi}_t \).
2.6 The Welfare Criterion

As already pointed out in section 2.1, the availability of a complete set of financial markets, providing households with perfect insulation from idiosyncratic income fluctuations, implies the existence of a representative consumer. Nevertheless, since the source of those fluctuations is the realized frequency with which wages can be reoptimized, different households still experience heterogeneous employment histories, and therefore different levels of *ex-post* utility. It seems then natural to compare the performance of different monetary policies on the basis of their effect on the utility of the “average” household, defined as

$$E_{-1} \sum_{t=0}^{\infty} \beta^t U_t \equiv E_{-1} \sum_{t=0}^{\infty} \beta^t \left\{ \ln C_t - \int_0^1 v(h_t(j)) \, dj \right\}$$

Following Woodford (2002), this section sketches the main steps in the derivation of a second order expansion to this utility function. Given first order accuracy of the log-linearized constraints under which monetary policy is conducted, this approximation guarantees that, given a sequence of economies ordered by progressively tighter bounds ||$\varepsilon$|| on the amplitude of the exogenous shocks, policies that appear to be better under the approximate criterion, indeed deliver higher levels of utility for the representative agent in economies far enough along the sequence.\(^{17}\)

Starting from the consumption term, we see that no approximation is actually needed, since

$$\ln Y_t = \ln \left( \frac{Y_t}{A_t \hat{Y}_A} \right) + \ln \hat{Y}_A A_t = \hat{Y}_A + t.i.p.$$  \hspace{1cm} (16)

where we used the fact that consumption is the only source of demand in the model (i.e. $C_t = Y_t$) and that the exogenous term $\ln \hat{Y}_A A_t$ does not alter the preference ranking of alternative paths for the endogenous variables, hence the notation *t.i.p.* (term independent of policy). As for the disutility of labor, we have

$$\int_0^1 v(h_t(j)) \, dj = v_h \hat{h} \left\{ E_j \hat{h}_t(j) + \frac{1}{2} (1 + \nu) \left[ \text{Var}_j \hat{h}_t(j) + \left( E_j \hat{h}_t(j) \right)^2 \right] \right\} + O \left( ||\varepsilon||^3 \right)$$

where $\hat{h}_t(j) \equiv \ln \left( h_t(j) / \hat{h} \right)$ is the percentage deviation of total demand for “skill” $j$ from its steady state value. With a fair amount of algebra, this expression can be turned into

$$\int_0^1 v(h_t(j)) \, dj = v_h \hat{h} \left\{ \hat{Y}_A + \frac{1}{2} (1 + \omega) \hat{Y}_A^2 + \frac{1}{2} (1 + \omega_p \theta_p) \theta_p \text{Var}_z \hat{p}_t(z) + \frac{1}{2} (1 + \nu \theta_w) \theta_w \phi^{-1} \text{Var}_j \hat{w}_t(j) \right\}$$

where $\omega \equiv \omega_p + \nu \phi$ and the reminder is omitted for notational simplicity. Together with (16) this yields

$$U_t = \left( 1 - v_h \hat{Y}_A \right) \hat{Y}_A - v_h \hat{Y}_A \left\{ \frac{1}{2} (1 + \omega) \hat{Y}_A^2 + \frac{1}{2} (1 + \omega_p \theta_p) \theta_p \text{Var}_z \hat{p}_t(z) + \frac{1}{2} (1 + \nu \theta_w) \theta_w \phi^{-1} \text{Var}_j \hat{w}_t(j) \right\}$$

Three important points emerge from this approximation. First, even though we wrote the first order term in the expansion of $U_t$ as $\left( 1 - v_h \hat{Y}_A \right) \hat{Y}_A$, the efficiency of the balanced growth

\(^{17}\) More precisely, $||\varepsilon||$ is a uniform bound on the elements of the vector of stationary shocks $\varepsilon_t \equiv (\varepsilon_t^e, \varepsilon_t^f)$.
path maintained by fiscal policy implies the equality of the marginal rates of substitution and transformation, \( v_h \tilde{Y}_A = f' \), so that in fact this term is equal to zero. In other words, movements away from the optimum—which coincides with the efficient equilibrium by the second welfare theorem—do not change the level of utility of the representative agent, up to first order. This is the reason why a first order approximation to the equilibrium fluctuations in the endogenous variables is sufficient to compute a second order approximation to \( U_t \). If the first order term in the expansion of \( U_t \) received a positive weight, second order terms in the approximation of the equilibrium dynamics would generate terms of the same order in the expansion, and could therefore not be omitted. Assuming that fiscal policy eliminates the first order inefficiency generated by monopolistic competition avoids this further complication.

Second, utility depends on the deviations of output \( Y_t \) from its efficient level \( A_t \tilde{Y}_A \), the notion of output gap that was proposed in section 2.3 and that appears in the pricing relations derived above. The reason for this is simple. Agent’s utility is decreasing in the amount of hours worked, which in turn depends on the level of effective demand \( Y_t/A_t \). It is then fluctuations in this variable that are going to reduce households’ welfare.

Third, the other fundamental source of welfare losses is the distortion associated with price and wage dispersion. As shown in appendix A, this dispersion can be expressed as a function of inflation in the relevant price indexes, which results in

\[
E_{-1} \sum_{t=0}^{\infty} \beta^t U_t = -2\Omega E_{-1} \sum_{t=0}^{\infty} \beta^t L_t + O \left( ||\varepsilon||^3 \right)
\]

(17)

where

\[
L_t \equiv \Lambda_p \left\{ (\tilde{\pi}_t - E_{t-2}\tilde{\pi}_t)^2 + \psi_p (E_{t-2}\tilde{\pi}_t)^2 \right\} + \Lambda_w \left\{ (\tilde{\pi}_t^w - E_{t-2}\tilde{\pi}_t^w)^2 + \psi_w (E_{t-2}\tilde{\pi}_t^w)^2 \right\} + \Lambda_x x_t^2
\]

(18)

and the weights are defined in the appendix. Note that in the loss function \( L_t \) the level of price dispersion is related to the deviations of price and wage inflation from their indexes, as defined in sections 2.5 and 2.4. This should come as no surprise, since if indexation were the only source of price changes, no price dispersion would result.

2.7 Monetary Policy

This section completes the description of the macroeconomic environment with a discussion of the operations of the monetary authority. A first important point in this respect is that, given the negligible role of money balances in this economy, monetary policy is effective only through the impact of interest rate changes on agents’ willingness to substitute their consumption over time. Moreover, because of the informational delays that characterize demand behavior, only the forecastable component of the nominal interest rate has an impact on their spending decisions, as it is evident from Euler equation (12). This implies that, even if the policy authority could collect information with no delay, incorporating this information in its decisions would simply increase the volatility of its instrument, with no other discernible effect on the economy. We will therefore
assume that the central bank refrains from exploiting this information, or equivalently, and more realistically, that it shares the same information processing deficiencies as the private sector. As already pointed out, this includes in any case imperfect information on the state of productivity.

As for the authority’s procedures to set its instrument, we consider three possible alternatives. The first is that the central bank targets one of three endogenous variables, price inflation, wage inflation or (its forecast of) the output gap. Rather than for their prescriptive content, we consider these policies because they represent useful benchmarks to which to compare the optimal equilibrium. Note also that strict output gap targeting is not within the central bank’s menu of available choices due to the one period delay with which demand reacts to changes in the policy instrument. The second alternative is that interest rate policy is set according to a feedback rule of the form

\[ \hat{\pi}_t = \phi_1 \hat{\pi}_{t-1} + (1 - \phi_1) \left[ \phi_\pi \pi_t + \frac{1}{\bar{\xi}} \phi_\pi E_{t-1} \hat{x}_t \right] \]

with parameters fixed at the values estimated by Clarida et al. (2000) in a closely related model and summarized in table 6. We take this rule to represent a reasonable approximation to actual U.S. monetary policy and use it to generate the simulated impulse responses that will be compared to those estimated from the data. Finally, we consider the state contingent policy that maximizes the unconditional expectation of the intertemporal loss function (17), under the constraint represented by the model’s dynamics.\(^\text{18}\)

Note also that, in all our experiments, we are going to report the values of the losses associated with different policies and/or different models in terms of an “inflation equivalent” (Dennis and Söderström, 2002), defined as the amount of steady inflation that agents would be willing to accept in exchange for the elimination of all fluctuations. This is computed as

\[ \pi^\ast \equiv \sqrt{\frac{(1-\beta) \text{Loss}}{\bar{\Lambda}_p \psi_p}} \]

and is reported in our figures as an annualized percentage.

2.8 Calibration

The calibration of the model’s parameters, summarized in tables 1 to 5, is based on the estimates of Amato and Laubach (2002), with two important exceptions. The first regards the elasticity of intertemporal substitution in consumption. This coefficient, estimated at (.26)\(^{-1}\) by Amato and Laubach (2002), is fixed here at one, the standard value in the real business cycle literature, because of the restriction on utility imposed by the presence of long run growth. This discrepancy is due to the fact that, in this class of models, aggregate consumption is usually interpreted as overall private expenditure, a composite with a much higher sensitivity to interest rate changes than simple non-durable consumption (Rotemberg and Woodford, 1997). As a consequence, a given adjustment

\(^{18}\) The exact formulation of the problem and the fundamental steps in its solution can be found in appendix C. Giannoni and Woodford (2002) contains a detailed discussion of the reasons for and implications of considering the unconditional expectation in the optimal policy problem.
of the policy instrument has a smaller impact on the level of demand in our calibration than what is more commonly assumed, a difference that should certainly be taken into consideration when interpreting the simulated impulse responses. The second exception regards the assumed degree of stickiness of prices and wages, which our benchmark calibration fixes at the values estimated by ACEL.\textsuperscript{19}

To calibrate the productivity process, we start by noting that the Wold representation of (2) can be written as

\[(1 - \rho L) \gamma_t = (1 - \theta L) \varepsilon_t \tag{19}\]

with \(\theta = \rho - K\) and \(\operatorname{Var}(\varepsilon_t) = \sigma_{\varepsilon}^2 + P\), where \(K\) and \(P\) are the steady state gain and MSE of the Kalman filter associated with (2).\textsuperscript{20} The fundamental disturbance \(\varepsilon_t\) would then be the structural shock identified by ACEL’s long run restriction if (2) were the true data generating process, as this is the shock inducing the observed unit root in average productivity. Since one of the aims of our positive analysis is to compare the model’s impulse responses to those estimated by ACEL, we start by restricting the parameters of (19) to match the moments of their productivity process. Unfortunately though, the latter is assumed to follow a simple AR(1), providing us with only two valid restrictions. For the third one, which we can think of as providing information on the “signal to noise ratio” \(\eta \equiv \sigma_\gamma / \sigma_a\), we then turn to direct estimates of hidden state models of productivity, as for example those in Roberts (2001) and French (2001).

More specifically, we proceed as follows. First, we match ACEL’s estimated autocorrelation coefficient of 0.8 with the one that would be obtained by fitting an AR(1) process to data generated by (19). This produces a value for \(\rho\) of 0.93. Second, we choose the scale of \(\varepsilon_t\) so that the long run effect on output of a one standard deviation shock to \(\varepsilon_t\) matches the VAR estimate of 0.6\% (see figure 1). This results in an overall volatility of the growth rate of productivity of 0.735\%, extremely close to the 0.763\% found by Prescott (1986) in estimates based on the Solow residual and that is often used as a benchmark in the real business cycle literature (see for example Cooley and Prescott, 1995). Finally, we calibrate \(\eta\) so that our model reproduces the Kalman gain found by Roberts (2001), who estimates a model of labor productivity of the same form as (2), but in which \(\gamma_t\) is assumed to follow a random walk. This results in a value of \(\eta\) of 14\%. The reason for following this procedure, rather than for example directly matching the estimated signal to noise ratio, is that the gain provides more direct evidence on agents’ “learning” speed. In fact, in Roberts’ (2001) I(2) model, \(K\) measures exactly the fraction of each observed change in productivity that is optimally attributed to trend, rather than transitory, shocks. Moreover, everything else being equal, reducing the persistence of \(\gamma_t\) reduces the updating speed, while the opposite effect follows from an increase in the signal to noise ratio. With \(\rho = 0.93\) then, our parametrization requires a higher \(\eta\) than Roberts’ (2001) 8\% to reproduce the same gain. To the extent that a lower signal to noise ratio

\textsuperscript{19} The main reason for favoring the estimates of ACEL is that Amato and Laubach (2002) restrict the two coefficients to be the same in their estimation procedure. This restriction is particularly troublesome given the evidence in Christiano et al. (2001) that points to wage stickiness as the key nominal friction.

\textsuperscript{20} See appendix B for more details.
helps the model to generate a decline in inflation then, this procedure yields a more conservative estimate of this parameter, in the sense of making it less likely to spuriously produce the desired result. In any event, given the importance of this parameter in the transmission of productivity shocks to costs and inflation, we also checked the robustness of our simulations across a fairly wide range of alternative values. According to these experiments, our main results do not significantly change for values of $\eta$ of up to 50%, which we consider well above the range of reasonable values for this parameter.

To provide a visual illustration of the effect of this calibration on the persistence of the trend shocks and on the learning speed, figure 2 displays impulse responses to the shock of the actual and forecasted values of $\gamma_t$, under full and partial information. As we can see, the relatively low value of $\eta$ results in a fairly gradual learning. However, given the decay of the initial shock, forecast errors disappear approximately five years after the initial shock.

3 The Transmission of Productivity Shocks

Having introduced the central features of the model, we now turn to the analysis of its equilibrium behavior. In this section, we begin by considering the case of a monetary authority that sets its instrument according to a deterministic policy rule. This allows us to study the transmission mechanism of the productivity shocks that is built into the behavior of the private sector, and to highlight the role of imperfect information in this mechanism. Then, in section 5, we analyze the optimal policy problem and compare its solution to the prescriptions of some simple targeting rules. We will show that, even if in the optimum the output gap is nearly stabilized, this policy does not result in a stable inflation rate. Quite to the contrary, inflation stabilization produces undesirable welfare outcomes. Finally, in sections 4 and 6, we compare the model’s predictions to the observed evolution of some key macroeconomic indicators during two controversial episodes of the recent past, the seventies’ productivity slowdown and its speedup during the last decade.

In what follows, we will mainly focus our attention on the model’s dynamic responses to persistent shifts in the growth rate of productivity. We do this for two reasons. First, from a theoretical perspective, these changes generate the kind of misperception of actual production possibilities that, according to our reasoning, should moderate wage demands and cause them to trail realized productivity. We argued that it is through this channel that a surge in productivity growth might result in lower inflation. Second, from the empirical point of view, these shocks closely fit the description of the productivity disturbances that hit the U.S. economy in the 1970s and in the 1990s. An important element of our analysis will be to investigate whether shocks of this kind are capable of reproducing the “puzzling” behavior of inflation and real activity observed in those periods. However, when comparing the simulated impulse responses to those identified with the VAR long run restriction, we need to take into account that that restriction can only recover a linear combination of the two structural shocks, the “unconditional” disturbance $\varepsilon_t$ in the Wold repre-
sentation (19). Therefore, for the purpose of that comparison, the model will be perturbed with this particular disturbance, rather than with the theoretically more interesting, but unobservable, trend shock $\varepsilon_t$.

Ideally, to isolate the role of our assumptions on the behavior of the private sector in the transmission of shocks, we would like to describe their dynamic effects controlling for any systematic response of policy to the observed economic conditions. Given the general equilibrium nature of our model though, in which today’s choices depend not only on past conditions, but also on future expectations, this is clearly impossible. In other words, there is no obvious “neutral” policy stance whereby the monetary authority can be usefully thought of as “doing nothing”, since defining a rational expectation equilibrium requires to specify some (possibly state contingent) rule of conduct for the central bank. This task is further complicated by the well-known indeterminacy issues that are typical of this class of models (Woodford, 2002). In particular, a peg of the interest rate to any exogenous quantity, like its steady state level, or the exogenously fluctuating natural rate of interest, results in indeterminacy of the rational expectations equilibrium, therefore ruling out these policies as candidate implementations of a neutral policy stance. As a second best alternative then, we consider a simple class of targeting rules, whose objective is simply to stabilize one of the endogenous variables in the system. As such, these policies do not involve explicit feedback from the economy, which might in turn obscure the direct impact of the productivity shock on private behavior.

To introduce the basic features of the model’s equilibrium behavior, figure 3 displays impulse responses of the key endogenous variables to a positive shock to the trend growth rate of productivity, given a policy that stabilizes the one period ahead forecast of the output gap. The shock is realized at time $t = 1$ and is assumed to increase the growth rate of productivity by one percentage point. The impulse responses of the actual and forecasted productivity growth to this particular shock are depicted in figure 2, under full and partial information. Starting from the upper right corner of the figure, we observe that the response of the output gap is a properly scaled mirror image of the persistent forecast error incurred by agents trying to predict the future growth rate, the difference between the continuous and dashed lines in the right panel of figure 2. This follows immediately from equation (12), which implies $x_t - E_{t-1}x_t = - (\hat{\gamma} - E_{t-1}\hat{\gamma})$ and therefore $x_t = - (\hat{\gamma} - E_{t-1}\hat{\gamma})$, given that policy maintains $E_{t-1}x_t = 0$. In economic terms, with demand predetermined at time $t - 1$, an unexpected increase in productivity decreases the amount of labor that firms need to hire to satisfy that demand, therefore decreasing the output gap. Lower labor demand and wage setters’ conservative estimates of the persistence of the shock restrain the response of wage inflation, which translates in turn into a fairly pronounced deflation in the price of

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21 At this stage, the choice of the shock’s size in only a normalization, since we are not yet in the position of quantitatively comparing the simulated to the VAR impulse responses. This comparison is undertaken in figure 5, in which the model is perturbed by the “unconditional” shock $\varepsilon_t$.

22 Recall that in this simple model, with labor as the only input and no other shocks besides those to productivity, the output gap is a monotonic transformation of hours. In the log-linear approximation considered here this relation is $\mathcal{H}_t = \phi x_t$. 

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consumption. Finally, this equilibrium is accompanied by a decline in the gap between the nominal interest rate and the natural rate of interest, which is our measure of the nominal stance of policy. Note however from the approximate Euler equation that stabilization of the expected output gap implies that the forecastable component of the real interest rate, \( E_{t-1} [i_t - \pi_{t+1}] \), is equal to the natural rate, implying a zero (expected) real interest rate gap. This also confirms that output gap targeting can in some sense be considered a reasonable approximation to “neutral” policy.

To further illustrate the role of the assumed informational frictions in the model’s equilibrium dynamics, figure 4 displays impulse responses to the trend shock across several different models, which incorporate various combinations of those frictions. More specifically, the thick dashed line (denoted by ALp-F-s in the legend) represents an extremely simplified version of our model, without any decision delay, full information and indexation of contractual wages to the observed contemporaneous growth rate of productivity. This model is included in the simulations as a normative benchmark, since under this specification a policy that targets the (now observable) output gap can attain the unconstrained optimum. It also represents the most direct extension of the “standard” model of section 1.1 to our non stationary environment. The dashed-dotted line (ALp-F) reintroduces the decision delays, with wages indexed to past productivity, but maintains the observability of all productivity shocks, which is instead dropped in the model represented with the continuous thin line. Finally, the thick continuous line represents our benchmark model, with partial information and wages indexed to forecasted productivity growth. Note also that the simulations are carried out under the assumption that policy follows the Volcker-Greenspan interest rate rule, which we argued is an empirically satisfactory representation of actual policy in the last twenty years.

As it is evident from the figure, the combination of our assumptions produces a model in which the response of nominal wage inflation to a surge in productivity is significantly dampened with respect to all the other specifications. This implies in turn that, while in those models the response of inflation is either close to zero, or significantly positive, in our benchmark specification inflation takes a fairly pronounced and persistent dip. Note in particular how the “standard” model predicts a sizable positive jump in inflation, which is caused by a loose monetary policy and accompanied by an initial response of nominal wage inflation that amplifies the initial 1% impulse coming from the increase in productivity growth.

Moreover, the figure illustrates that the assumption that informational delays also affect the wage indexation process, once interacted with partial information, provides the key added value in explaining the decline in inflation. It is in fact only moving from the thin to the thick continuous line that inflation is seen to turn negative after the positive shock. This is because indexing wages to actual productivity, as in the thin line, would undo most of the sluggishness in newly set wages generated by workers’ errors in forecasting future productivity growth. This consideration is especially relevant in quantitative terms under the baseline calibration, in which 78% of wages are subject to indexation every period. Ultimately, what assumption represents a more accurate
depiction of reality remains an empirical question, which is not the objective of this paper to investigate. Nevertheless, at least when judged in terms of its macroeconomic implications, indexation to forecasted productivity produces results that are clearly more in line with the evidence, which justifies our preference for this specification.

Further clues on the effect of the model’s informational frictions on the transmission of productivity shocks comes from the evolution of the variables depicted in the last line of figure 4. These represent the fundamental determinants of price and wage inflation identified by the Phillips curves (13) and (15), namely the “long horizon forecasts” of unit labor costs ($\omega_{At}$) and the output gap ($x_t$). They are defined as the expected present discounted value of the ratio of the real wage to productivity, and of the output gap respectively, or more formally as

$$E_{t-1} \sum_{s=0}^{\infty} \beta^s z_{t+s}$$

(20)

$\forall t$ along the horizon of the impulse response simulation, with $z_t = \ln \frac{W_t}{A_tP_t}$ or $z_t = x_t$. Note that these forecasts are scaled by different coefficients in the two Phillips curves, but their dynamic evolution is fully captured by (20). The responses of these two variables, which reflect the unfolding of agents’ uncertainty about the productivity shock, depict a slightly different picture than the ex-post real wage evolution described above. First of all, we observe that partial information does have a sizable effect on the forecasts of future labor costs, as initially suggested by our intuition. This effect does not translate into a more pronounced deflation because in the benchmark model agents also expect a pronounced and persistent surge in the output gap, which is indeed confirmed by the ex-post evolution of that variable. The expectation of positive gaps compensates at least in part for the expected decline in labor costs, contributing to restrain the deflationary effects of the productivity shock. This suggests that a richer demand specification, including for example habits in consumption (Fuhrer, 2000; Christiano et al., 2001), by further smoothing the response of the output gap, might induce a more pronounced dip in inflation, bringing it even closer to the data. We leave the quantitative exploration of this effect for future research.

Our analysis so far has been limited to considering the model’s impulse responses to a trend productivity shock, since we argued that this is the theoretically more interesting case. As we already had the chance to remark though, these responses cannot be compared with those estimated by ACEL and reported in figure 1, because their VAR identification procedure, under the null represented by our model, would only isolate a particular linear combination of the model’s two structural shocks. Of course, given the assumed distribution of those shocks, we can simulate the effect of an impulse to that particular linear combination, which will then be consistent with ACEL’s experiment on the data. Note also that, differently from before, the scale of the simulated shocks now also matters for a meaningful comparison of the model to the data. Following our calibration, we then simulate an unconditional shock that produces a long run increase in the level of output that matches the 0.6% estimated in the VAR (see figure 1). As indicated in section 2.8, this implies a standard deviation of the growth rate of productivity that is in line with standard
values.

The findings of this experiments are illustrated in figure 4, which again compares the four models presented above under the empirically realistic assumption that monetary policy follows the Volcker-Greenspan interest rate rule. Starting with the upper row of the table, we immediately note that our preferred specification is the only one to produce the negative correlation between output and inflation responses that is a clear feature of the VAR estimates. Moreover, the profile of the two responses is very similar to that depicted in figure 1. Output jumps by approximately 0.4% in the period immediately following the shock (recall that output is predetermined in the model), with only a small adjustment from there to its new long run level. Inflation reaches its trough response of slightly more than −0.2% two quarters after the shock, and adjusts back from there in less than two years. In the VAR point estimates, the maximum response happens on impact and is closer to −0.6%, but the adjustment back towards steady state is similarly sharp. Moreover, the simulated impulse responses fit easily within the VAR 95% confidence bands represented by the shaded area, with the only exception of the impact response.

As for the rest of the variables, the model is also fairly successful in replicating the behavior of real wages, that exhibit a smoother profile than the other real variables, and of consumption, that in our model is of course just equal to output. The federal funds rate on the contrary is predicted to fall in the model, while the VAR point estimates show it rising, but the model’s prediction is comfortably inside the estimated confidence intervals. Finally, looking at the response of average hours in the last panel, we see that in the model they decline on impact, but jump immediately after towards a positive level, from which they then adjust (almost) monotonically towards their steady state. Note that this is broadly in line with the estimated profile, except that there the peak response is only reached after one year, and hours actually never turn negative, at least according to the point estimates. We find this discrepancy between the model and ACEL’s estimates very interesting, especially in light of the observation, common to several VAR studies, that hours seem to decline in response to an identified technology shock (see for example Galí, 1999 and Francis and Ramey, 2001), an observation to which ACEL take exception on the basis of their own estimates.

As already pointed out, the initial dip in hours depends on the unexpected increase in labor productivity, coupled with a predetermined level of demand. This points in turn to what is perhaps the main weakness of the model, at least when observed through the lens of its responses to productivity shocks. One notable characteristic of the empirical impulse responses is in fact that most variables jump immediately in response to the identified technology shock. Some of them, like inflation, return monotonically to their steady state value, while others, like output, consumption and the real wage are either constant after an initial jump, or slowly increase towards their new steady state level. In the model on the other hand, even if the overall profile of the responses is very close to the VAR’s, the initial response is constrained to be zero. This depends in turn on the fact that this class of models was originally devised to match the estimated responses to monetary policy shocks. In particular, the informational delays that prevent the endogenous variables from
jumping on impact are at least in part justified by the identifying assumption, common to recursive monetary VARs, that decision variables do not react to changes in policy within a quarter. The estimated responses to technology shocks show that this assumption is not quite compatible with the behavior of those variables under all circumstances and that more research is needed to reconcile these observations.23

4 The New Economy and The Productivity Slowdown

The extraordinary performance of the U.S. economy in the second half of the 1990s, with its unprecedented combination of rapid GDP growth, low unemployment and moderate inflation, and the contemporaneous surge in labor productivity to levels not seen for more than two decades, have attracted a great deal of attention among scholars, policy makers and the public.24 In the midst of the excitement about these developments, some observers hailed the arrival of a “New Economy”, whose most thaumaturgic virtues were promptly disposed of by the recession of 2001. In the meantime though, the acceleration of productivity, the fundamental underpinning of most moderate views of the New Economy, has become a well documented fact, whose quantitative scope has survived virtually untouched the recent recession.25 As extensively documented above, the model proposed in this paper has the potential to generate falling inflation and high growth rates against the backdrop of an increase in the growth rate of labor productivity, precisely the combination of macroeconomic conditions registered in the U.S. in the nineties. It is still an open question though if the model can quantitatively account for the evolution of these variables during those episodes, once perturbed with an appropriately scaled technology shock.

Answering this question requires first to take a stance on a reasonable chronology regarding the unfolding of the shock in the data. This task is considerably simplified for the period under study by the fact that 1995 is generally recognized as a year in which the US economy was close to a “steady state” (see for example Meyer, 2001), while it was in 1996 that the first signs of a possible productivity revival started to emerge. We therefore isolate the twenty quarters from 1996:I to 2000:IV as the relevant period for our “case study”, under the assumption that the “New Economy shock” hit the U.S. in the first quarter of 1996 and unfolded over the subsequent five years.

This reconstruction is corroborated for instance by the sequence of testimonies by Chairman Alan Greenspan accompanying the Federal Reserve Board’s semiannual monetary policy reports to the Congress.26 The first cautious references to a surge in productivity can be found in the testimony of February 1997, in which we read that “faster productivity growth last year meant

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23 Woodford (2002a) presents a model that can in principle accomodate these different patterns of reaction to different shocks.
25 The average growth rate of labor productivity from 1948 to the second quarter of 2002 was 2.25%. It was 2.8% from 1948 to 1973, 1.45% from 1973 to 1995 and 2.4% from 1995 to 2002:II (data are from the latest available release (September 5th, 2002) by the Bureau of Labor Statistics).
26 Testimonies and Reports from 1996 to 2002 can be found at http://www.federalreserve.gov/boarddocs/hh/.
that rising compensation gains did not cause labor costs per unit of output to increase any more rapidly”, but it is not until July 1997 that the Fed starts referring to this faster growth as potentially persistent: “although the anecdotal evidence is ample and manufacturing productivity has picked up, a change in the underlying trend is not yet reflected in our conventional data for the whole economy.” A further step in the updating of the Fed’s trend estimates can be detected in July 1998, when Greenspan reports that “evidence continues to mount that the trend of productivity has accelerated, even if the extent of that pickup is as yet unclear.” Finally, in July 1999 the Chairman declares that “to date, 1999 has been an exceptional year for the American economy (...). At the root of this impressive expansion of economic activity has been a marked acceleration in the productivity of our nation’s workforce.”

Note that this sequence of progressively more sanguine statements about the nature of the measured changes in productivity is in accordance with the informational assumptions of our model, according to which policy makers, as well as the public, slowly update over time their estimates of the persistent component of productivity. Whether the assumed distribution of the productivity factor in the model is also a reasonable characterization of the stochastic properties of the “true” New Economy shock is indeed impossible to ascertain, either from these statements, or from the data, if we assume, as it seems realistic, that the econometrician shares the same information as the agents in the model. The main source of concern in this respect is indeed the degree of persistence of the growth rate shock. The fact that the surge in labor productivity growth that we traced to the beginning of 1996 has so far survived virtually unscathed suggests that the initial shock might have been significantly more persistent than what indicated by the “average” estimates of ACEL. But then again, another persistent shock, or maybe a sequence of transitory shocks, might have hit productivity in the meantime. Trying to discriminate among these alternative scenarios is beyond the scope of the present case study, which therefore simply retains the model’s distributional assumptions.

Another key ingredient of this case study is the magnitude of the productivity shock to be simulated. To calibrate this quantity, we followed the same procedure used to calibrate the “average” shock considered in section 3. First, we computed the long run impact of the New Economy shock in the data. The average growth rate of output between 1948:I and 1995:IV was 3.39%, but increased to 3.95% between 1996:I and 2000:IV, the 20 quarters that represent the object of our “case study”. Compounded over these five years, the extra 56 basis points of annual growth result in a level of output at the end of the period that is 3.36% higher than what it would have been under average growth.\(^{27}\) Assuming that this extra income can be entirely attributed to faster productivity growth, and that twenty quarters represent the relevant notion of “long run”, this leaves us with an initial shock to productivity growth of approximately 1.2% at an annual rate. Note that even if this number is many times the estimated standard deviation of the permanent shock, the growth rate of labor productivity in the U.S. in fact jumped from an average of 1.5% in

\(^{27}\) Recall that the shock considered in the simulations of section 3 resulted in a long run effect on the level of output of 0.6%.
the period 1973-1995 to an average of 2.5% between 1996 and 2000.

The dynamics of labor productivity, output growth and inflation predicted by the model in response to the technology shock described above are reported in figure 6, along with their observed counterparts, as dashed and continuous lines respectively, under the assumption that monetary policy follows the Volcker-Greenspan estimated interest rate rule. Considering that the continuous lines simply represent the raw data, and that no special effort has been directed at specifying and calibrating the model to match those data, we find the model’s rendition of the broad characteristics of this episode truly remarkable. In particular, the model matches extremely well the dynamics of average labor productivity, with the possible exception of the last few quarters, in which the slow decay of the dashed line is outsripped by what looks like a further resurgence of productivity, similar to that observed at the beginning of the sample. This discrepancy is reflected in turn in the behavior of output growth, that is observed to be even more persistent than what implied by the AR(1) decay built into the model. Similar considerations also apply to inflation, which declines by a full percentage point in the data, but by only half that much according to the model.

We should probably remark at this point that it is far from the spirit of the exercise to assess the “fit” of the model according to any formal criterion. It is therefore quite useful, at least to provide some perspective on the results above, to ask how the model would fare when compared with observations on the period following the seventies’ productivity slowdown, which we date to the third quarter of 1973.28 As we can see from figure 7, although the model predicts inflation and real activity to move in the right direction, the amplitude of their fluctuations is orders of magnitude wider than that produced by the model. Interestingly however, the simulation correctly identifies the dates of the peak responses in both inflation and output growth, and is fairly close to the observations in the last part of the sample. Moreover, we can note an asymmetry in the responses of the two variables, with the fall in real growth much closer to the data than the increase in inflation.

We conclude from this exercise that an explanation of the macroeconomic events of the second half of the nineties based exclusively on the effects of the productivity speedup appears to be reasonable in light of the model. On the other hand, the productivity slowdown emerges as a possible, but only very partial explanation of the stagflation of the early seventies. The extent to which these results are influenced by our representation of monetary policy, and in particular how the model’s optimal equilibrium compares to the observed fluctuations are questions that need to wait until after our discussion of optimal policy. They will be taken up in turn in section 6.

5 Optimal Policy

Having established that our model provides a reasonably accurate depiction of the transmission of productivity shocks, we are now ready to tackle the normative question posed by the paper’s

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28 French (2001) conducts Andrews tests on the growth rate of TFP and finds a significant break in 1973:III. As before, our qualitative results are not overly sensitive to choosing another break date around this period.
title. In particular, we wish to investigate the nature of the equilibrium fluctuations that minimize the welfare loss of the representative agent and the path of the policy instrument that implements them. We are also going to compare this optimal equilibrium with the ones that would emerge under some of the simple policy rules introduced in section 2.7, to gain further insights into the propagation of productivity shocks and the properties of the optimal policy. We will instead leave for further research the question of implementation of the optimal policy and of the choice of simple rules that might approximate the optimal equilibrium.

We begin the normative analysis from figure 8, which considers impulse responses of the endogenous variables to the “unconditional” shock in the Wold representation (19). This allows us to compare the optimal responses to those obtained under the Volcker-Greenspan interest rate rule and to the VAR estimates in figures 5 and 1. In our preferred model (the thick line), a drop in the nominal interest rate that more than compensates for the increase in the natural rate of interest is accompanied by a surge in demand, but by deflation in the price of consumption goods and virtually no nominal wage inflation. The comparison with the other specifications illustrates how the deflationary effect of the productivity shock, which is one of the defining characteristics of the model, and of the data, facilitates monetary policy’s task of balancing the fluctuations in price and wage inflation. With the exception of the simplest model (the dashed line), in which optimal policy actually achieves the unconstrained optimum, the monetary authority responds to the shock by turning distinctly contractionary. This results in a drop in demand, which together with the higher labor productivity produces a steep and persistent reduction in hours. These adjustments in the real variables are indeed just an instrument through which the monetary authority reaches its objective of minimizing a combination of price and wage dispersion. All models display in fact similar degrees of price deflation and a reduction in wage inflation with respect for example to the corresponding equilibria illustrated in figure 4. This figure clearly illustrates how the transmission mechanism embedded in the benchmark model, which we argued is a better representation of reality than more standard sticky price models, has also very distinctive implications for the desirable monetary policy response to productivity shocks. This in turn has significant effects on the resulting equilibrium fluctuations of the endogenous variables.

Turning now our attention to the optimal policy responses to the trend shock $\varepsilon_t$, figure 9 presents the simulated dynamics of the endogenous variables, together with those of the target variables identified by the approximate welfare criterion (18), namely the forecastable and unforecastable components of wage and price inflation, expressed in deviation from their respective indexes.\footnote{The second line of the figure reports the forecastable components of $\tilde{\pi}_t$ and $\tilde{\pi}_t^w$, and not of the measured inflation rates $\pi_t$ and $\pi_t^w$, whose values are instead reported in the first line. This explains why the sum of the responses in the second and third line is not equal to those in the first.} What we note immediately is that the main source of inefficiency in the models in which wages are assumed to be indexed to past productivity growth (the thin continuous and dashed-dotted lines) comes from the behavior of wage inflation, which compounds high volatility of both its forecastable and unforecastable components. In the attempt to curb this volatility, optimal policy
turns contractionary, inducing a recession that has the effect of moderating workers’ wage demands. As a by-product, the recession also induces a drop in price inflation. This explains why it is not desirable to reach for the monetary breaks even more decisively. Such a policy would in fact reduce the welfare losses stemming from wage fluctuations, but cause an even deeper drop in price inflation, increasing the losses through that channel. The contrast with the optimal dynamics just described helps to highlight the role of imperfect information in shaping the optimal dynamics of the benchmark model. Under this specification, policy responds to the productivity shock by lowering the nominal interest rate below the natural rate in order to limit the contractionary impact of the surge in productivity on labor demand and the output gap. Differently from what would happen under the other specifications though, this milder contraction in hours worked does not result in a jump in wage demands because of the slow adjustment of workers’ estimates of future productivity. The net result is a price deflation of very similar propositions to those registered under the alternative specifications, but with very different welfare consequences.

One notable implication of the welfare ranking of the optimal policies across the models illustrated in figure 9 is that indexation of wages to observed productivity growth results in higher losses than indexation to its forecast. This might seem surprising at first, since in a perfectly competitive labor market real wages should efficiently incorporate all shocks to productivity, which is just the opposite of what happens in our model. The solution to this apparent paradox is in the nature of the approximation to the “true” loss function adopted here, and in particular in the assumption that the economy is close to being efficient. This assumption implies that, up to first order, real wages are in fact indexed to productivity growth, or more precisely that fluctuations in productivity are small enough that the fact that wages are not indexed to its growth rate (but only to its steady state value) does not have first order welfare consequences. As a result, the only sources of welfare losses left in our approximation are the second order distortions emerging from price dispersion. But a high degree of price dispersion, and high losses, are exactly what will result from a model in which newly set wages are based on filtered estimates of future productivity, while contractual wages are indexed to its much more volatile realized value.

5.1 Simple Policy Rules

Our last look at the issue of optimal monetary policy design in response to growth rate shocks is through the window of the targeting rules introduced in section 2.7. Figure 10 compares equilibrium fluctuations and level of welfare, as measured by the “inflation equivalent”, under the optimal policy and three simple policies that target price inflation, wage inflation and the forecast of the output gap.

The first notable aspect of this comparison is that strict inflation targeting is a particularly undesirable policy to insulate the economy from the effect of trend productivity shocks.30 Differently

30 Blanchard (1997) first pointed out the undesirable consequences of strict inflation targeting in a stylized model with wage stickiness. This point was further elaborated by Erceg et al. (2000).
from what is commonly argued though, this is not because, in the attempt of stabilizing inflation, the central bank might end up repressing real growth. On the contrary, according to the model, to prevent inflation from falling in response to the shock, monetary policy needs to turn expansionary. This expansion will then result in a boom in demand, which leads workers to set higher wages, increasing their volatility. Of course, from a welfare perspective, the higher volatility in wage inflation is compensated by the complete stabilization of price inflation. Quantitatively, these opposing effects actually appear roughly equivalent, if we compare the fluctuations in the dotted line with what happens under wage stabilization (the dashed-dotted line), leaving us with a significant unexplained welfare gap between the two policies. We can reconcile these apparently contradictory observations if we note that, under the benchmark calibration, the variability of wages receives nine times as much weight in the loss function as inflation fluctuations, due to the lower frequency with which wages are expected to adjust (see table 5). This asymmetry reflects in turn the intuitive observation that, for a given amount of volatility in the growth rate of a price index, higher stickiness gives rise to more dispersion in the underlying price distribution (Aoki, 2001; Benigno, 2000). If we are willing to accept this simple intuition as the basis of our welfare criterion, and we trust the empirical evidence of more stickiness in the labor rather than in the goods’ market, the result follows immediately. We find that this example offers an especially compelling illustration of the misleading welfare implications that might be drawn from a welfare criterion not carefully grounded in the microeconomic details of the model economy.

In light of the preceding observations, it is then not surprising that a policy that stabilizes nominal wage inflation around its steady state growth rate $\gamma$, would produce better outcomes than those obtained under inflation stabilization. Note also that wage inflation targeting is close to a mirror image of strict inflation targeting in that it requires a contraction in monetary policy to reduce the demand for goods and labor services and workers' wage demands. This avoids the surge in wage inflation that would otherwise follow the positive productivity shock, but also causes a fairly pronounced disinflation. As already pointed out though, this leaves the representative consumer better off because the undesirable fluctuations in inflation can be more easily absorbed, thanks to the relative flexibility of goods' prices. Finally, the role of relative price stickiness is confirmed by figure 11, in which we report impulse responses for the Amato and Laubach (2002) calibration, $\alpha_p = \alpha_w = .66$, which also implies almost equal weights for price and wage inflation in the loss function. In this case optimal policy tries to balance the opposite fluctuations in price and wage inflation, reflecting their similar weights in the welfare function. Note however that, due to wages’ higher flexibility, wage inflation grows higher than in the benchmark case, even if the policy contraction is accompanied by a slightly deeper and more prolonged recession. Similarly, inflation fluctuations are at least in part dampened by the higher price stickiness, resulting in higher inflation (i.e. less deflation) regardless of the more pronounced recession.

Returning now to figure 10, we observe that the policy that stabilizes the forecast of the output gap produces intermediate results between the inflation targeting policies considered above. Under
this policy, a modest drop in the interest rate limits the negative impact of the surge in productivity on the output gap, therefore avoiding the boom that characterizes inflation targeting as well as the recession that accompanies the stabilization of wage inflation. What is more remarkable though is the fact that output gap forecast targeting closely approximates the optimal policy, almost exactly reproducing its equilibrium fluctuations. This does not at all depend on the direct welfare effect of a stable output gap on the volatility of hours, since this component of the policy objective receives a negligible share of the weight in the utility approximation (see table 5). This result stems rather from the fact that stabilization of the forecast of the output gap achieves (almost) the right balance between fluctuations in the two price indexes that dominate the welfare criterion (Woodford, 2002). Furthermore, this conclusion is robust to the alternative calibration considered in figure 11, even though in this case a slightly more pronounced contraction than that produced by the targeting policy would be desirable. Finally, we should point out that the desirability of output gap targeting, already noted for example by Erceg et al. (2000) in their related model with price and wage rigidities, carries over to our framework, in which the output gap is in fact unobservable, under the form of output gap forecast targeting. In other words, even in an economy like ours, in which the monetary authority lacks an accurate measure of the output gap within the quarter, almost optimal results can be obtained by simply targeting the available estimates of the output gap.31

In conclusion, we can summarize the findings in this section by noting that a positive shock to the growth rate of productivity, though temporary, has a permanent effect of the level of productivity, therefore requiring an equivalent increase in the real wage in the long run. According to our analysis, this should be achieved by a combination of positive nominal wage inflation and deflation in the price of consumption. The optimal combination of these two changes depends on the relative stickiness of the two prices, with the more flexible price carrying the bulk of the adjustment.

6 Optimal Policy, Stagflation and the New Economy

The attempt to bring our model to bear on some specific historical episodes, what we referred to as case studies, has been so far only positive in nature. In section 4, we simulated the model’s responses to a technology shock assuming that monetary policy followed the interest rate rule estimated by Clarida et al. (2000) for the Volcker-Greenspan era, which we argued is a reasonable characterization of actual policy for the post-1979 period. This approach to the case studies left open at least two important questions. First, how close is the Volcker-Greenspan interest rate rule to the optimal policy prescribed by the model for the second half of the 1990s? Second, does the failure of the model to reproduce the dynamic properties of output and inflation in the 1970s depend on the assumed interest rate rule? We take these questions up in turn in the rest of the

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31 Interestingly, Amato and Laubach (2002), in a model much closer to ours than that of Erceg et al. (2000), find that “a strong interest rate response to the output gap can lead to severely sub-optimal outcomes.” They however do not explicitly consider output gap targeting rules.
Figure 12 provides a clear answer to the first question: actual policy in the 1990s was very close to optimal according to our model. In fact, optimal fluctuations in output, and especially in inflation, are significantly closer to the data than those implied by the interest rate rule underlying figure 6, and the same is true for the Federal Funds rate, especially in the first part of the sample.\textsuperscript{32} Interestingly, the policy loosening of 1998:IV corresponds to the first significant departure of measured growth from the model’s prediction, while the tightening of the last three quarters of the sample, which exceeds the model’s prescription, is associated with a pronounced decline in the growth rate of real activity. In conclusion, our analysis corroborates the widespread opinion that the Federal Reserve skillfully steered the U.S. economy of the second half of the nineties in the face of the non trivial challenges posed by a significant shift in the growth rate of productivity.

Turning now to the period following the 1973 productivity slowdown, we recall from figure 7 that the model falls significantly short of replicating the extreme volatility found in the data, especially with respect to inflation. This is indeed not too surprising, especially if we contrast the model’s extremely restricted set of shocks with the turbulence in the fundamentals that is one of the period’s most salient features (Primiceri, 2002; Stock and Watson, 2002). One popular explanation for this turbulence has been advanced by Clarida \textit{et al.} (2000), who argue that monetary policy in the pre-Volcker period did not fulfill the “Taylor principle”, which would have caused indeterminacy of the rational expectations equilibrium and sunspot fluctuations. Since our solution algorithm cannot simulate an indeterminate system, we cannot directly test this theory within our model. Nevertheless, we can investigate the role of different parametrizations of the interest rate rule on the model’s equilibrium, at least as long as they guarantee its existence and uniqueness. Figure 13 then displays simulated and actual data on GDP growth and inflation under the pre-Volcker rule in table 6. This is the rule estimated by Clarida \textit{et al.} (2000) for the period 1960:I to 1979:II, except for the coefficient on inflation, which we assume takes the value closest to the estimate, but that is still compatible with equilibrium determinacy.\textsuperscript{33} Surprisingly, this rule dampens the predicted fluctuations in inflation when compared to the Volcker-Greenspan rule (figure 7), even leading to a fall in inflation in the second part of the sample. This evidence cannot of course rule out the explanation of the data proposed by Clarida \textit{et al.} (2000), since our model is silent on what would happen if the coefficient on inflation in the policy reaction function were in fact lower than one. It does suggest however that the failure of the model to account for the quantitative extent of the observed fluctuations does not depend on the kind of interest rate rule assumed in figure 7, at least as long as we restrict our analysis to the class of determinate rules.

There is however something more that we can learn from the model about the role of monetary policy in the economic landscape of the seventies. Figure 14 contains a fairly detailed comparison

\textsuperscript{32} The dip in the interest rate in 1998:IV corresponds with the loosening of policy that followed the Russian debt crisis and the default of LTCM in the summer of 1998.

\textsuperscript{33} Clarida \textit{et al.} (2000) estimate a value for the reaction coefficient to inflation of 0.83, with a standard deviation of 0.07.
between the data and the model’s optimal equilibrium. This picture confirms that the observed fluctuations in real activity and inflation were indeed much wider than those produced by the model, when perturbed by only one technology shock. Interestingly though, if we concentrate only on the first few quarters of the period, up to the beginning of 1975, we observe a combination of tight monetary policy, depressed output, a real wage inflation that is broadly in line with the model’s prediction, but still high inflation. This particular macroeconomic configuration is at least in principle compatible with the traditional idea that the run up in inflation was the product of the oil price shock of 1973, to which monetary policy clearly appears to have responded fairly vigorously at the outset. High real interest rates in the face of a “cost-push” shock could in turn explain the precipitous drop in demand and the high level of inflation, in the absence of significant cost pressures originating from the labor markets. On the other hand, the subsidence of the commodity price shock in 1975, and the concomitant loosening of policy, could be compatible with a decline in inflation, the steep recovery of output and the subsequent build up of cost pressures in the labor market.\textsuperscript{34} Note also that policy did not turn loose in comparison to the model’s prescription until the end of 1976, at which point inflation starts inching up again. We hasten to stress that this reconstruction of the events is highly speculative, even though roughly in line with the chronology of the commodity price shocks emphasized for example by Blinder (1979). Whether augmenting the model with an exogenous source of cost pressure, like a commodity price shock, could help to reconcile quantitatively its predictions with the data is a question that we leave for future research.

7 Conclusions

In this paper, we investigated the relationship between monetary policy and productivity growth, from both a positive and a normative perspective. We argued that standard New Keynesian models, built around the assumption of a perfectly competitive labor market, do not capture a key feature of the data, the negative correlation between inflation and real activity conditional on shocks to the growth rate of productivity. To solve this problem, we proposed a model in which wages respond to a trend shock to productivity with a delay. This is due to a particular form of imperfect information, whereby agents, after observing such shock, cannot conclusively establish its degree of persistence. As a result, in the case in which the shock is in fact persistent, workers come to this realization only gradually, so that wage demands lag behind realized productivity, moderating the reaction of marginal costs and inflation. An extensive investigation of its equilibrium behavior, documented that the model represents a significant improvement over the existing literature. The model’s dynamics following a productivity shock, calibrated to match the shock identified by Altig \textit{et al.} (2002) within a structural VAR, are qualitatively and quantitatively in line with the estimated

\textsuperscript{34} This tentative reconstruction is roughly in line with DeLong (1997), who writes: “Thus I would tentatively conclude that the supply shocks of the 1970s were in large part sound and fury. They came. They caused headlines, jumps in the price level, and recessions. And when they had passed the inflation situation was much the same as before their impact.”
responses. In particular, the model is able to reproduce the conditionally negative correlation between inflation and output that is one of the salient features of the data. Moreover, it provides a remarkably close account of the evolution of labor productivity, inflation, output and interest rates that followed the “New Economy shock” of the mid-nineties. All these considerations lead us to conclude that the model safely qualifies as an adequate laboratory to study the optimal policy response to technology shocks.

In the normative part of the analysis then, we found that this response is characterized by deflation in the goods’ market and by a moderate amount of nominal wage inflation, at least when compared with the predictions of the standard model. The economic nature of this response can be summarized by noting that cointegration of income and real wages implies that a permanent increase in productivity must be followed by an equivalent movement in real wages. In the optimum, this is achieved by an increase in the nominal wage, accompanied by a fall in the price of consumption, with the less sticky of the two price indexes carrying the bulk of the adjustment. In our benchmark model, this adjustment is engineered by the monetary authority through a reduction in the nominal interest rate. In other words, the popular argument that monetary policy should “accommodate” a surge in the growth rate of productivity survives the scrutiny of our formal analysis. Nevertheless, a reduction in nominal rates is optimal here not so much because it allows higher productivity to be turned into higher output, but rather because it contributes to balance wage and price adjustments, minimizing the distortions associated with price dispersion. Not surprisingly then, a policy of strict inflation targeting, forcing the adjustment of real wages to fall entirely on nominal wage changes, results in undesirable welfare consequences. Finally, we argued that the pickup in productivity was the predominant influence on the observed fluctuations of output and inflation in the second half of the 1990s, and that the stance of American monetary policy in that same period was very close to the optimal prescription of the model.
A Derivation of the Welfare Criterion

This appendix derives expressions for the price and wage dispersion measures $\text{Var}_z \tilde{p}_t (z)$ and $\text{Var}_j \tilde{w}_t (j)$ as a function of inflation in the associated price indexes, following the steps described in Rotemberg and Woodford (1998).

Starting from the price dispersion measure, we define $\bar{p}_t \equiv E_z \ln p_t (z)$, $\bar{p}_{t-1} \equiv \bar{p}_{t-1} + \lambda_p \pi_{t-1}$ and $\Delta^P_t \equiv \text{Var}_z \ln p_t (z) = \text{Var}_z \tilde{p}_t (z)$ and write

$$
\Delta^P_t = \text{Var}_z \left[ \ln p_t (z) - \bar{p}_{t-1} \right] = E_z \left[ \left( \ln p_t (z) - \bar{p}_{t-1} \right)^2 \right] - \left( E_z \ln p_t (z) - \bar{p}_{t-1} \right)^2 
= \alpha_p E_z \left[ \left( \ln p_{t-1} (z) + \lambda_p \pi_{t-1} - \bar{p}_{t-1} \right)^2 \right] + (1 - \alpha_p) \tilde{\psi}_p \left[ \ln p_t^1 - \bar{p}_{t-1} \right]^2 
+ (1 - \alpha_p) \left( 1 - \tilde{\psi}_p \right) \left[ \ln p_t^2 - \bar{p}_{t-1} \right]^2 - (\Delta \tilde{p}_t - \lambda_p \pi_{t-1})^2 
= \alpha_p \Delta^P_{t-1} + (1 - \alpha_p) \left\{ \tilde{\psi}_p \left[ \ln p_t^1 - \bar{p}_{t-1} \right]^2 + \left( 1 - \tilde{\psi}_p \right) \left[ \ln p_t^2 - \bar{p}_{t-1} \right]^2 \right\} - (\Delta \tilde{p}_t - \lambda_p \pi_{t-1})^2 
$$

(21)

and

$$
\bar{p}_t - \bar{p}_{t-1} = E_z \left[ \ln p_t (z) - \bar{p}_{t-1} \right] 
= \alpha_p E_z \left[ \ln p_{t-1} (z) - \bar{p}_{t-1} \right] + (1 - \alpha_p) \tilde{\psi}_p \left[ \ln p_t^1 - \bar{p}_{t-1} \right] 
+ (1 - \alpha_p) \left( 1 - \tilde{\psi}_p \right) \left[ \ln p_t^2 - \bar{p}_{t-1} \right] 
= (1 - \alpha_p) \left\{ \tilde{\psi}_p \left[ \ln p_t^1 - \bar{p}_{t-1} \right] + \left( 1 - \tilde{\psi}_p \right) \left[ \ln p_t^2 - \bar{p}_{t-1} \right] \right\} 
$$

(22)

Taking expectations at $t - 2$, and remembering that (up to our log-linear approximation), $\ln p_t^2 = E_{t-2} \ln p_t^1$ this becomes

$$
E_{t-2} \left[ \bar{p}_t - \bar{p}_{t-1} \right] = (1 - \alpha_p) \left[ \ln p_t^2 - \bar{p}_{t-1} \right] + \mathcal{O}(\|\varepsilon\|^2)
$$

which substituted back into (22) yields

$$
\bar{p}_t - \bar{p}_{t-1} - \left(1 - \tilde{\psi}_p \right) E_{t-2} \left[ \bar{p}_t - \bar{p}_{t-1} \right] = (1 - \alpha_p) \tilde{\psi}_p \left[ \ln p_t^1 - \bar{p}_{t-1} \right] + \mathcal{O} \left( \|\varepsilon\|^2 \right)
$$

Squaring these last two expression and using $\Delta \tilde{p}_t = \pi_t + \mathcal{O} \left( \|\varepsilon\|^2 \right)$ we then obtain

$$
\left[ \ln p_t^2 - \bar{p}_{t-1} \right]^2 = (1 - \alpha_p)^{-2} (E_{t-2} \bar{\pi}_t)^2 
$$

and

$$
\left[ \ln p_t^1 - \bar{p}_{t-1} \right]^2 = (1 - \alpha_p)^{-2} \tilde{\psi}_p^{-2} \left( \bar{\pi}_t - E_{t-2} \bar{\pi}_t + \tilde{\psi}_p E_{t-2} \bar{\pi}_t \right)^2 
$$

where we are suppressing terms of third order or higher. Substituting this expression into (21) yields an AR(1) representation for the dispersion measure

$$
\Delta^P_t - \alpha_p \Delta^P_{t-1} = (1 - \alpha_p)^{-1} \tilde{\psi}_p^{-1} \left( \bar{\pi}_t - E_{t-2} \bar{\pi}_t + \tilde{\psi}_p E_{t-2} \bar{\pi}_t \right)^2 + \frac{1 - \tilde{\psi}_p}{1 - \alpha_p \tilde{\psi}_p} (E_{t-2} \bar{\pi}_t)^2 - \bar{\pi}_t^2 
= \left( \frac{1 - \tilde{\psi}_p (1 - \alpha_p)}{\tilde{\psi}_p (1 - \alpha_p)} + 1 \right) \left( \bar{\pi}_t - E_{t-2} \bar{\pi}_t \right)^2 + \frac{1}{1 - \alpha_p \tilde{\psi}_p} (E_{t-2} \bar{\pi}_t)^2 - \bar{\pi}_t^2 + un \text{f} 
= \frac{\alpha_p \tilde{\psi}_p}{(1 - \alpha_p \tilde{\psi}_p)} \left( \bar{\pi}_t^2 - (1 - \tilde{\psi}_p) (E_{t-2} \bar{\pi}_t)^2 \right) + un \text{f}
$$

36
where \( unf \) collects terms that cannot be forecasted on the basis of information available at time \(-1\).

As for wage dispersion, we proceed analogously and after defining \( \bar{w}_t \equiv E_j \ln w_t (j) \), \( \bar{w}^i_{t-1} \equiv \bar{w}_{t-1} + E_{t-2} \ln \Omega_t \) and \( \Delta^w_t \equiv \text{Var}_j \ln w_t (j) = \text{Var}_j \bar{w}_t(j) \) we write

\[
\Delta^w_t = \text{Var}_z [\ln w_t (z) - \bar{w}^i_{t-1}]
= \alpha_w \Delta^w_{t-1} + (1 - \alpha_w) \left\{ \tilde{\psi}_w [\ln w^i_t - \bar{w}^i_{t-1}]^2 + \left(1 - \tilde{\psi}_w \right) \left[ \ln w^2_t - \bar{w}^i_{t-1} \right]^2 \right\} 
- \left( \Delta \bar{w}_t - E_{t-2} \ln \Omega_t \right)^2
\] (23)

and

\[
\tilde{w}_t - \bar{w}^i_{t-1} = (1 - \alpha_w) \left\{ \tilde{\psi}_w [\ln w^i_t - \bar{w}^i_{t-1}] + \left(1 - \tilde{\psi}_w \right) \left[ \ln w^2_t - \bar{w}^i_{t-1} \right] \right\}
\]

Taking expectations at \( t - 2 \)

\[
E_{t-2} \tilde{\pi}^w_t = (1 - \alpha_w) \left[ \ln w^2_t - \bar{w}^i_{t-1} \right] + \mathcal{O} \left( ||\varepsilon||^2 \right)
\]

which we again substitute back to obtain

\[
\tilde{\pi}^w_t - (1 - \tilde{\psi}_w) E_{t-2} \tilde{\pi}^w_t = (1 - \alpha_w) \tilde{\psi}_w [\ln w^i_t - \bar{w}^i_{t-1}] + \mathcal{O} \left( ||\varepsilon||^2 \right)
\]

where we now used \( \ln w^2_t = E_{t-2} \ln w^1_t + \mathcal{O} \left( ||\varepsilon||^2 \right) \) and \( \Delta \bar{w}_t = \tilde{\pi}^w_t + \mathcal{O} \left( ||\varepsilon||^2 \right) \). Squaring these last two expressions and plugging the results into (23) we then find

\[
\frac{1 - \alpha_w}{\alpha_w} \tilde{\psi}_w \left( \Delta^w_t - \alpha_w \Delta^w_{t-1} \right) = (\tilde{\pi}^w_t)^2 - (1 - \tilde{\psi}_w) \left( E_{t-2} \tilde{\pi}^w_t \right)^2 + unf
\]

which, following Woodford (2002, chapter 6), can be solved backwards to yield

\[
\frac{1 - \alpha_w}{\alpha_w} \tilde{\psi}_w (1 - \alpha_w \beta) E_{t-1} \sum_{t=0}^{\infty} \beta^t \Delta^w_t = E_{t-1} \sum_{t=0}^{\infty} \beta^t \left\{ (\tilde{\pi}^w_t)^2 - (1 - \tilde{\psi}_w) \left( E_{t-2} \tilde{\pi}^w_t \right)^2 \right\}
\]

with an analogous expression for \( \Delta^p_t \)

\[
\frac{1 - \alpha_p}{\alpha_p} \tilde{\psi}_p (1 - \alpha_p \beta) E_{t-1} \sum_{t=0}^{\infty} \beta^t \Delta^p_t = E_{t-1} \sum_{t=0}^{\infty} \beta^t \left\{ (\tilde{\pi}^w_t)^2 - (1 - \tilde{\psi}_w) \left( E_{t-2} \tilde{\pi}^w_t \right)^2 \right\}
\]

Putting everything together produces the period loss function

\[
L_t \equiv \Lambda_p \left\{ \tilde{\pi}^2_t - (1 - \psi_p) \left( E_{t-2} \tilde{\pi}^2_t \right)^2 \right\} + \Lambda_w \left\{ (\tilde{\pi}^w_t)^2 - (1 - \psi_w) \left( E_{t-2} \tilde{\pi}^w_t \right)^2 \right\} + \Lambda_x x^2_t
\]

where the weights, normalized so that \( \Lambda_p + \Lambda_w = 1 \), are

\[
\Lambda_p \equiv \frac{\theta_p (\xi_p \psi_p)^{-1}}{\Theta} \quad ; \quad \Lambda_w \equiv \frac{\theta_w (\xi_w \psi_w \phi)^{-1}}{\Theta} \quad ; \quad \Lambda_x \equiv \frac{1 + \omega}{\Theta}
\]
B Solution and Filtering

Even though the paper reports results only for the parametrization of the productivity process discussed in section 2.8, our solution procedure can accommodate a much more general class of processes. This section discusses the outline of that procedure under these more general assumptions.

We consider a stochastic process for the growth rate of productivity of the form

\[ \hat{\gamma}_t^a = \hat{\gamma}_t + \mu^a (L) \varepsilon_t^a \]

\[ \hat{\gamma}_{t+1} = \rho (L) \hat{\gamma}_t + \mu^\gamma (L) \varepsilon_{t+1}^\gamma \]

where \( \varepsilon_t^a \) and \( \varepsilon_t^\gamma \) are i.i.d. shocks with \( E[\varepsilon_t^a] = E[\varepsilon_t^\gamma] = 0 \), \( \text{Var}[\varepsilon_t^a] = \sigma_c^2 \) and \( \text{Var}[\varepsilon_t^\gamma] = \sigma_\gamma^2 \), while \( \rho (L) \), \( \mu^a (L) \) and \( \mu^\gamma (L) \) are polynomials in the lag operator. Following Hamilton (1994, section 13.1), this process can be represented in state space form as

\[ \hat{\gamma}_t^a = \mu^0 \xi_t \]

\[ \xi_{t+1} = \Xi \xi_t + \Theta \epsilon_{t+1} \]

where \( \mu, \Xi \) and \( \Theta \) are appropriately defined matrices of parameters and \( \epsilon_t = \begin{bmatrix} \varepsilon_t^a & \varepsilon_t^\gamma \end{bmatrix} \).

Given this representation, we can run a standard Kalman filter to forecast the growth rate of productivity as

\[ \hat{\gamma}_{t|t-1} = \mu^0 \xi_{t|t-1} \]

\[ \xi_{t|t} = \xi_{t|t-1} + K \mu^0 (\xi_t - \xi_{t|t-1}) \]

\[ \xi_{t+1|t} = \Xi \xi_{t|t} \]

where the Kalman gain is

\[ K = P \mu (\mu^0 P \mu)^{-1} \]

and \( P \equiv E \left[ (\xi_{t+1} - \xi_{t+1|t}) (\xi_{t+1} - \xi_{t+1|t})' \right] \) solves the usual Riccati equation

\[ P = \Xi \left[ P - P \mu (\mu^0 P \mu)^{-1} \mu^0 P \right] \Xi' + \Theta \Sigma \Theta' \]

with \( \Sigma \equiv E[\epsilon_t \epsilon_t'] \). Note that in the full information case the state vector would simply be forecasted as

\[ \xi_{t+1|t} = \Xi \xi_t \]

Defining now an appropriate partition of the vector of endogenous variables \( y_t \) between jumps \( (x_t) \) and states \( (X_t) \), we express the system of linearized first order conditions in the canonical form

\[ \begin{bmatrix} A^0 E_t y_{t+1} \\ \xi_{t+1} \end{bmatrix} = \begin{bmatrix} B^1 \\ B^2 \end{bmatrix} \begin{bmatrix} \varepsilon_t \\ \xi_t \end{bmatrix} + \begin{bmatrix} B^3 \\ 0_{n_x \times n_\xi} \end{bmatrix} \xi_{t|t} + \begin{bmatrix} 0_{n_y \times n_\xi} \\ \Theta \end{bmatrix} \epsilon_{t+1} \]
Taking expectations at time $t$, this becomes

$$AE_t \begin{bmatrix} y_{t+1} \\ \xi_{t+1|t} \end{bmatrix} = B \begin{bmatrix} y_t \\ \xi_{t|t} \end{bmatrix}$$

This system can now be solved with standard methods to produce a state space representation

$$\begin{bmatrix} y_t \\ \xi_{t|t} \\ X_{t+1} \\ \xi_{t+1|t} \end{bmatrix} = \begin{bmatrix} D^1 & D^2 \\ 0_{n_{\xi}\times n_X} & I_{n_{\xi}} \\ G^1 & G^2 \\ 0_{n_{\xi}\times n_X} & \Xi \end{bmatrix} \begin{bmatrix} X_t \\ \xi_{t|t} \\ X_t \\ \xi_{t|t} \end{bmatrix}$$

which, together with

$$\xi_{t|t} = \xi_{t|t-1} + K\mu' (\xi_t - \xi_{t|t-1}) = K\mu' \xi_t + (I - K\mu') \xi_{t|t-1}$$

and (24), we write recursively as

$$y_t = \left[ \begin{array}{ccc} D^1 & D^2 & D^2 K\mu' \\ 0_{n_{\xi}\times n_X} & I_{n_{\xi}} & D^2 \\ & & 0_{n_{\xi}\times n_{\xi}} \\ X_{t+1} \\ \xi_{t+1|t} \\ \xi_{t+1} \end{array} \right] + \left[ \begin{array}{c} X_t \\ \xi_{t|t-1} \\ \xi_t \end{array} \right] \left[ \begin{array}{c} 0_{n_{X}\times n_{\xi}} \\ 0_{n_{\xi}\times n_{\xi}} \\ \Theta \end{array} \right] \epsilon_{t+1}$$

or, more compactly

$$y_t = DX^\xi$$
$$X_{t+1}^\xi = GX^\xi_t + H\epsilon_{t+1}$$

with the obvious definition of the extended state $X^\xi_t$.

Note also that under full information we simply have $K\mu' = I_{n_\xi}$, which in turn simplifies the matrices above to

$$D \equiv \begin{bmatrix} D^1 & 0_{n_{\gamma}\times n_\xi} & D^2 \\ 0_{n_{\xi}\times n_{\gamma}} & 0_{n_{\xi}\times n_\xi} & G^2 \end{bmatrix}; \quad G \equiv \begin{bmatrix} G^1 & 0_{n_{X}\times n_\xi} \\ 0_{n_{\xi}\times n_X} & 0_{n_{\xi}\times n_\xi} & \Xi \end{bmatrix}$$
C Computing the Optimal Policy

Given the particularly simple form of the objective function $L_t$, the optimal policy problem can be formulated as

$$
\min_{\{i_t, \xi_t\}_{t=0}^\infty} \mathbb{E} \sum_{t=0}^\infty \beta^t \{ x'_t W_{xx} x_t + X'_t W_{XX} X_t \}
$$

subject to

$$
\begin{align*}
A_{xx}^0 E_t x_{t+1} + A_{xx}^0 X_{t+1} &= B_{xx}^1 x_t + B_{xx}^1 X_t + B_{xx}^1 \xi_t + B_{xx}^1 \xi_{t|t} \\
A_{XX}^0 X_{t+1} &= B_{xx}^1 x_t + B_{XX}^1 X_t + B_{XX}^2 \xi_t + B_{XX}^3 \xi_{t|t} \\
\xi_{t+1} &= \Xi \xi_t + \Theta \xi_{t+1}
\end{align*}
$$

The Lagrangian for this problem is therefore

$$
\mathcal{L} = \mathbb{E} \sum_{t=0}^\infty \beta^t \{ x'_t W_{xx} x_t + X'_t W_{XX} X_t \}
$$

$$
\begin{align*}
&+ 2 \varphi_{\tau t+1}^{x} \left( A_{xx}^0 x_{t+1} + A_{xx}^0 X_{t+1} - B_{xx}^1 x_t - B_{xx}^1 X_t - B_{xx}^1 \xi_t - B_{xx}^1 \xi_{t|t} \right) \\
&+ 2 \varphi_{\tau t+1}^{X} \left( A_{XX}^0 X_{t+1} - B_{xx}^1 x_t - B_{XX}^1 X_t - B_{XX}^2 \xi_t - B_{XX}^3 \xi_{t|t} \right) \\
&+ 2 \varphi_{\tau t+1}^{\xi} \left( \xi_{t+1} - \Xi \xi_t - \Theta \xi_{t+1} \right)
\end{align*}
$$

where it should be noted that (differently from the more general case treated in Svensson and Woodford, 2002), $\varphi_{\tau t+1}^{x}$ and $\varphi_{\tau t+1}^{X}$ are both measurable with respect to $I_t$ (the information set including the endogenous variables and $\{A_t\}_{t \leq \tau}$), but not $I_t^f$ (the full information set also including $\{e^f_t, \xi^f_t\}_{t \leq \tau}$), while $\varphi_{\tau t+1}^{\xi}$ is measurable with respect to $I_t^f$ information.

The first order conditions for this problem are

$$
\begin{align*}
\dot{t}'_{t+1} : & \quad 0 = B_{xx}^1 x_{t+1} \\
x'_t : & \quad 0 = W_{xx} x_t + \beta^{-1} A_{xx}^0 x_{t+1} - B_{xx}^1 x_t - B_{xx}^1 X_t - B_{xx}^1 \xi_t - B_{xx}^1 \xi_{t|t} \\
X'_t : & \quad 0 = W_{XX} X_t + \beta^{-1} A_{xx}^0 x_{t+1} - B_{xx}^1 x_t - B_{xx}^1 X_t - \beta^{-1} A_{XX}^0 X_{t+1} - B_{xx}^1 x_t - B_{xx}^1 X_t - B_{XX}^2 \xi_t - B_{XX}^3 \xi_{t|t} \\
\dot{\xi}'_t : & \quad 0 = - \beta^{-1} \varphi_{\tau t+1}^{x} - (B_{XX}^2 \xi_t + B_{XX}^3 \xi_{t|t}) + \beta^{-1} A_{XX}^0 x_{t+1} - B_{xx}^1 x_t - B_{xx}^1 X_t - B_{XX}^2 \xi_t - B_{XX}^3 \xi_{t|t} \\
\varphi_{\tau t+1}^{x} : & \quad 0 = A_{xx}^0 E_t x_{t+1} + A_{xx}^0 X_{t+1} - B_{xx}^1 x_t - B_{xx}^1 X_t - B_{xx}^1 \xi_t - B_{xx}^1 \xi_{t|t} \\
\varphi_{\tau t+1}^{X} : & \quad 0 = A_{XX}^0 X_{t+1} - B_{xx}^1 x_t - B_{XX}^1 X_t - B_{XX}^2 \xi_t - B_{XX}^3 \xi_{t|t} \\
\varphi_{\tau t+1}^{\xi} : & \quad 0 = \xi_{t+1} - \Xi \xi_t - \Theta \xi_{t+1}
\end{align*}
$$

which, after taking expectations with respect to $I_t$ information, can be cast in first order form as

$$
\begin{bmatrix}
\tilde{y}_{t+1} \\
\tilde{\xi}_{t+1|t}
\end{bmatrix}
= \begin{bmatrix}
\tilde{A}_{E_t} \\
\tilde{B}_{\xi_{t+1|t}}
\end{bmatrix}
\begin{bmatrix}
\tilde{y}_t \\
\tilde{\xi}_{t|t}
\end{bmatrix}
$$

where the vector $\tilde{y}_t$ now also contains the multipliers. The solution to this system is again of the form

$$
\begin{align*}
y_t & = \tilde{D} \tilde{X}^\xi_t \\
\tilde{X}^\xi_{t+1} & = \tilde{G} \tilde{X}^\xi_t + \tilde{H} \tilde{e}_{t+1}
\end{align*}
$$
where $\bar{X}_t^\xi \equiv \begin{bmatrix} X_t & \varphi_t^\xi & \xi_{t|t-1} & \xi_t \end{bmatrix}$.

From this state space representation we can then derive

$$\bar{X}_t^\xi = \sum_{\tau=0}^{t-1} \bar{G}^\tau \bar{H} \xi_{t-\tau} + \bar{G}^t \bar{X}_0^\xi$$

and

$$\Sigma_{\bar{X}_t} \equiv E\left[ \bar{X}_t^\xi \bar{X}_t^\xi \right] = \bar{G} \Sigma_{\bar{X}_t} \bar{G}' + \bar{H} \Sigma \bar{H}'$$

which we substitute into the loss function to obtain the value of the optimal plan

$$\text{Loss} = E \sum_{t=0}^\infty \beta^t y_t' W y_t$$

$$= E \sum_{t=0}^\infty \beta^t \left[ \bar{D} \left( \sum_{\tau=0}^{t-1} \bar{G}^\tau \bar{H} \xi_{t-\tau} + \bar{G}^t \bar{X}_0^\xi \right) \right]' \bar{W} \left[ \bar{D} \left( \sum_{\tau=0}^{t-1} \bar{G}^\tau \bar{H} \xi_{t-\tau} + \bar{G}^t \bar{X}_0^\xi \right) \right]$$

$$= \sum_{t=0}^\infty \beta^t \left\{ \sum_{\tau=0}^{t-1} \text{tr} \left[ \bar{G}^\tau \bar{D}' \bar{W} \bar{D} \bar{G}^\tau \bar{H} \Sigma \bar{H}' \right] + \text{tr} \left[ \bar{G}^t \bar{D}' \bar{W} \bar{D} \bar{G}^t \Sigma_{\bar{X}_t} \right] \right\}$$

$$= \frac{1}{1-\beta} \sum_{t=0}^\infty \text{tr} \left[ \beta \bar{G}' \bar{D}' \bar{W} \bar{D} \bar{G}' \bar{H} \Sigma \bar{H}' \right] + \sum_{t=0}^\infty \text{tr} \left[ \beta^t \bar{G}' \bar{D}' \bar{W} \bar{D} \bar{G}' \Sigma_{\bar{X}_t} \right]$$

where $\bar{L}$ solves the Sylvester equation

$$\bar{L} = \beta \bar{G}' \bar{L} \bar{G} + \bar{D}' \bar{W} \bar{D}$$

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References


Amato, Jeffrey D. and Thomas Laubach (2002), “Estimation and Control of an Optimization-Based Model with Sticky Prices and Wages,” Journal of Economic Dynamics and Control, forthcoming


Figures and Tables

<table>
<thead>
<tr>
<th>Productivity</th>
<th></th>
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</thead>
<tbody>
<tr>
<td>$\rho$</td>
<td>.93</td>
</tr>
<tr>
<td>$\sigma_a$</td>
<td>.27%</td>
</tr>
<tr>
<td>$\sigma_\gamma$</td>
<td>.039%</td>
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Table 1: Calibrated parameter values: productivity process.

<table>
<thead>
<tr>
<th>Tastes</th>
<th>Technology</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>$\phi^{-1}$</td>
</tr>
<tr>
<td>$\nu$</td>
<td>$\omega_p$</td>
</tr>
<tr>
<td>$(\theta_p - 1)^{-1}$</td>
<td>$(\theta_w - 1)^{-1}$</td>
</tr>
<tr>
<td>.99</td>
<td>.75</td>
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<td>.2</td>
<td>.33</td>
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<tr>
<td>.19</td>
<td>.13</td>
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</table>

Table 2: Calibrated parameter values: tastes and technology.

<table>
<thead>
<tr>
<th>Stickiness</th>
<th>CE</th>
<th>AL</th>
<th>Indexation</th>
<th>ALf</th>
<th>ALp</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_p$</td>
<td>.42</td>
<td>.66</td>
<td>$\lambda_\gamma$</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$\alpha_w$</td>
<td>.78</td>
<td>.66</td>
<td>$\lambda_p$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\tilde{\psi}_p$</td>
<td>.56</td>
<td>.56</td>
<td>$\lambda_w$</td>
<td>0</td>
<td>0</td>
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<tr>
<td>$\tilde{\psi}_w$</td>
<td>.56</td>
<td>.56</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3: Calibrated parameter values. **Stickiness**: CE refers to the benchmark calibration; AL refers to the estimates of Amato and Laubach (2002). **Indexation**: ALf represents indexation of wages to the forecast of productivity; ALp represents indexation of wages to past productivity.
Phillips Curves Parameters

\[ \omega_w = \nu \phi \]
\[ \omega = \omega_w + \omega_p \]
\[ \zeta_w = \frac{\alpha_w}{1 - \psi_w (1 - \alpha_w)} \]
\[ \psi_p = \frac{\alpha_p \psi_p}{1 - \psi_p (1 - \alpha_p)} \]
\[ \xi_p = \frac{1 - \alpha_p}{\alpha_p} \frac{1 - \alpha_p \beta}{1 + \omega_p \theta_p} \]
\[ \kappa_p = \xi_p \omega_p \]
\[ \psi_w = \frac{\alpha_w \psi_w}{1 - \psi_w (1 - \alpha_w)} \]
\[ \xi_w = \frac{1 - \alpha_w}{\alpha_w} \frac{1 - \alpha_w \beta}{1 + \nu \theta_w} \]
\[ \kappa_w = \xi_w (1 + \omega_w) \]

Table 4: Parameters of the wage and price Phillips curves, expressed as functions of the structural parameters.

<table>
<thead>
<tr>
<th>Loss Function</th>
<th>CE</th>
<th>AL</th>
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<tbody>
<tr>
<td>( \Lambda_p )</td>
<td>.11</td>
<td>.52</td>
</tr>
<tr>
<td>( \Lambda_w )</td>
<td>.89</td>
<td>.48</td>
</tr>
<tr>
<td>( \Lambda_x )</td>
<td>.0026</td>
<td>.0035</td>
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Table 5: Calibrated parameter values: loss function. CE and AL denote the values of the welfare weights derived under the benchmark calibration and the estimates of Amato and Laubach (2002) respectively.

<table>
<thead>
<tr>
<th>Interest Rate Rule</th>
<th>Volcker-Greenspan</th>
<th>Pre-Volcker</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_i )</td>
<td>.79</td>
<td>.68</td>
</tr>
<tr>
<td>( \phi_{\pi} )</td>
<td>2.15</td>
<td>1</td>
</tr>
<tr>
<td>( \phi_x )</td>
<td>.93</td>
<td>.27</td>
</tr>
</tbody>
</table>

Table 6: Calibrated parameter values: interest rate rule. The Volcker-Greenspan column refers to estimates in Galí et al. (2000) for the period 1979:III to 1996:IV, while the Pre-Volcker column refers to estimates for the period 1960:I to 1979:II.
Empirical Impulse Responses

Figure 1: Impulse responses to an innovation in technology (from Altig et al., 2002). The shaded area denotes 95% confidence bands.
Growth Rate of Productivity, Actual and Forecasted

Figure 2: Impulse responses of the actual ($\gamma_a^t$) and forecasted ($\gamma_{a,t-1}^o$) growth rate of productivity to a 1% persistent technology shock ($\varepsilon_t$), under full and partial information. Productivity growth is expressed as an annualized percentage rate.
Figure 3: Simulated impulse responses to a 1% $\epsilon^t$ shock for the benchmark model; monetary policy is assumed to target the one period ahead forecast of the output gap. Inflations and the interest rate gap are expressed as annualized percentage rates, while the output gap is expressed in percentage points per quarter.
Figure 4: Simulated impulse responses to a 1% $\varepsilon_t^\gamma$ shock under the Volcker-Greenspan interest rate rule (table 6). ALp and ALf refer to the type of wage indexation (table 3); F and P refer to Full and Partial information respectively; s refers to the simple model with no information delays. The thick line (ALf-P) is the benchmark model. Inflations and the interest rate gap are expressed as annualized percentage rates, while the level variables are expressed as percentage deviations from their steady state value; the long horizon forecasts are expressed in percentage points per quarter.
Empirical Comparison: Interest Rate Rule

Figure 5: Simulated impulse responses to the “VAR shock” ($\varepsilon_t$) under the Volcker-Greenspan interest rate rule (table 6). ALp and ALf refer to the type of wage indexation (table 3); F and P refer to Full and Partial information respectively; s refers to the simple model with no information delays. The thick line (ALf-P) is the benchmark model. Inflation and the interest rate are expressed as annualized percentage rates, while the level variables are expressed as percentage deviations from their steady state value.
Figure 6: Simulated (dotted line) and actual (solid line) evolution of productivity growth, inflation and GDP growth for the period 1996:I to 2000:IV, under the Volcker-Greenspan interest rate rule. The model is hit in 1996:I with a productivity shock that decays like an AR(1) process with the benchmark parameter 0.8. All variables are expressed as growth rates over the previous year, in deviation from their average over the year 1995.
Figure 7: Simulated (dotted line) and actual (solid line) evolution of productivity growth, inflation and GDP growth for the period 1973:III to 1978:II, under the Volcker-Greenspan interest rate rule. The model is hit in 1973:III with a productivity shock that decays like an AR(1) process with the benchmark parameter 0.8. All variables are expressed as growth rates over the previous year, in deviation from their average over the entire business cycle 1971:I-1973:II.
Figure 8: Simulated impulse responses to the “VAR shock” ($\varepsilon_t$) under the Volcker-Greenspan interest rate rule (table 6). ALp and ALf refer to the type of wage indexation (table 3); F and P refer to Full and Partial information respectively; s refers to the simple model with no information delays. The thick line (ALf-P) is the benchmark model. Inflation and the interest rate are expressed as annualized percentage rates, while the level variables are expressed as percentage deviations from their steady state value.
Figure 9: Simulated impulse responses to a 1% $\varepsilon_t$ shock under the optimal policy. ALp and ALf refer to the type of wage indexation (table 3); F and P refer to Full and Partial information respectively; s refers to the simple model with no information delays. The thick line (ALf-P) is the benchmark model. $\pi^*$ is the “inflation equivalent”. Inflations and the interest rate gap are expressed as annualized percentage rates, while the output gap is expressed in percentage points per quarter.
Policy Comparison: Targeting Rules

Figure 10: Simulated impulse responses to a 1% $\varepsilon_t^\gamma$ shock under the optimal policy; benchmark calibration. The different line styles refer to inflation targeting, nominal wage inflation targeting, output gap forecast targeting and the optimal policy, as indicated by the legend. $\pi^*$ is the “inflation equivalent”. Inflations and the interest rate gap are expressed as annualized percentage rates, while the output gap is expressed in percentage points per quarter.
Figure 11: Simulated impulse responses to a 1% $\varepsilon_t^\gamma$ shock under the optimal policy, with $\alpha_w = \alpha_p = 0.66$. The different line styles refer to inflation targeting, nominal wage inflation targeting, output gap forecast targeting and the optimal policy, as indicated by the legend. $\pi^*$ is the “inflation equivalent”. Inflations and the interest rate gap are expressed as annualized percentage rates, while the output gap is expressed in percentage points per quarter.
Figure 12: Simulated (dotted line) and actual (solid line) evolution of Federal Funds rate, inflation and GDP growth for the period 1996:I to 2000:IV, under the optimal policy. The model is hit in 1996:I with a productivity shock that decays like an AR(1) process with parameter 0.9. Inflation and GDP growth are expressed as percentage growth rates over the previous year, while the Federal Funds rate is expressed in annualized percentage points per quarter. All variables are in deviation from their average over the year 1995.
Figure 13: Simulated (dotted line) and actual (solid line) evolution of productivity growth, inflation and GDP growth for the period 1973:III to 1978:II, under the pre-Volcker interest rate rule. The model is hit in 1973:III with a productivity shock that decays like an AR(1) process with the benchmark parameter 0.8. All variables are expressed as growth rates over the previous year, in deviation from their average over the entire business cycle 1971:I-1973:II.
Optimal Policy: The Productivity Slowdown

Figure 14: Simulated (dotted line) and actual (solid line) evolution of GDP growth, inflation, real wage inflation and the *ex-post* real interest rate for the period 1973:III to 1978:II, under the optimal policy. The model is hit in 1973:III with a productivity shock that decays like an AR(1) process with the benchmark parameter 0.8. All variables are expressed as growth rates over the previous year, with the exception of the interest rate, which is a four quarters moving average of annualized quarterly rates.