Bubbles and Crashes
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Systemic risk – a broad definition

- Systemic risk build-up during (credit) bubble ... and materializes in a crisis
  - “Volatility Paradox” → contemp. measures inappropriate
- Spillovers/contagion – externalities
  - Direct contractual: domino effect (interconnectedness)
  - Indirect: price effect (fire-sale externalities)
    - credit crunch, liquidity spirals

- Adverse GE response
  - amplification, persistence
Credit bubble 2004-2006

ABX 7-1 Prices

- AAA
- AA
- A
- BBB
- BBB−
US House price index – Case-Shiller

Housing Price Index (Jan 2000=100, ratio scale)
Why do bubbles persist?

Do professional traders ride the bubble or attack the bubble (go short)?

What happened in March 2000?

- Loss of ca. 60 % from high of $ 5,132
- Loss of ca. 85 % from high of Euro 8,583
Do (rational) professionals ride the bubble?

- South Sea Bubble (1710-1720)
  - Issac Newton
    - 04/20/1720 sold shares at £ 7,000 profiting £3,500
    - Re-entered the market later – ending up losing £20,000
    - “I can calculate the motions of the heavenly bodies, but not the madness of people”

  - Druckenmiller of Soros’ Quantum Fund didn’t think that he party would end so quickly.
    - “We thought it was the eighth inning, and it was the ninth.”
  - Julian Robertson of Tiger Fund refused internet stocks.

- Housing bubble (2007)
  - Chuck Prince “Dance as long as the music is playing”
Minsky moment – Wile E. Coyote Effect
Stylized facts

- Initial innovation justifies some price increase
- Momentum leads to price overshooting
  - Extrapolative expectations
- Many market participants seem to be aware that the “price is too high” but keep on holding the asset
  - “Play as long as the music plays”
- Resell-option is crucial for speculative bubbles
- Credit bubbles lead to extra amplification effects in downturn (since they can impair financial sector)
  - subprime borrowing was only 4% of US mortgage market
  - Amplification focus of next lecture
Classical Question

- Suppose behavioral trading leads to mispricing

- Can mispricing or bubbles persist in the presence of rational arbitrageurs?

- What type of information can lead to the bursting of bubbles?
  - Why does classic backwards induction argument fail?
  - Recall centipede game
On Market Efficiency

- Keynes (1936) \(\Rightarrow\) bubble can emerge
  - “It might have been supposed that competition between expert professionals, possessing judgment and knowledge beyond that of the average private investor, would correct the vagaries of the ignorant individual left to himself.”

- Friedman (1953), Fama (1965)

  Efficient Market Hypothesis \(\Rightarrow\) no bubbles emerge
  - “If there are many sophisticated traders in the market, they may cause these “bubbles” to burst before they really get under way.”
Main Bubble Literature

- Limits to Arbitrage
  - Noise trader risk: Shleifer & Vishny (1997), DSSW (1990a&b)
  - Synchronization risk

- Rational bubble symmetric info
  - Santos & Woodford (1997)
  - More later ...

- Bubbles and asymmetric info
  - Asymmetric information: Tirole (1982), Allen et al. (1993)

- Bubbles and heterogeneous (prior) beliefs
  - Harrison & Kreps (1978), Hong & Stein, Scheinkman & Xiong (2003),
Timing Game - Synchronization

- (When) will behavioral traders be overwhelmed by rational arbitrageurs?
- Collective selling pressure of arbitrageurs *more than suffices* to burst the bubble.
- Rational arbitrageurs understand that an eventual collapse is inevitable. But when?
- Delicate, difficult, dangerous *TIMING GAME*!
Elements of the Timing Game

- **Coordination** at least $\kappa > 0$ arbs have to be ‘out of the market’
- **Competition** only \( \text{first } \kappa < 1 \) arbs receive pre-crash price.
- **Profitable ride** ride bubble as long as possible.
- **Sequential Awareness**

A *Synchronization Problem* arises!

- Absent of sequential awareness competitive element dominates $\Rightarrow$ and bubble burst immediately.
- With sequential awareness incentive to TIME THE MARKET $\Rightarrow$ “delayed arbitrage” $\Rightarrow$ persistence of bubble
Overview

- Introduction
- Model setup
- Preliminary analysis
- Persistence of bubbles
- Public events
- Price cascades and rebounds
- Empirical evidence & Hedge funds
- Common action of $\kappa$ arbs
- Sequential awareness
  - Random $t_0$ with $F(t_0) = 1 - e^{-\lambda t_0}$

$$p_t = e^{gt}$$

$$\beta p_t$$

$$1 - \beta(\cdot)p_t$$

$t_0$ random starting point
$t_0 + \eta \kappa$ $\kappa$ traders are aware of the bubble
$t_0 + \eta$ all traders are aware of the bubble
$t_0 + \bar{\tau}$ bubble bursts for exogenous reasons

maximum life-span of the bubble $\bar{\tau}$
Payoff structure

Focus: “when does bubble burst”

$t_0$ is only random variables, all others are CK

Cash payoff (difference)

- Sell one share at $t - \Delta$ instead of at $t$

$$p_{t-\Delta}e^{r\Delta} - p_t$$

where $p_t = \begin{cases} e^{gt} & \text{prior to crash} \\ ((1 - \beta(t - t_0))e^{gt} & \text{after the crash} \end{cases}$

Price at the time of bursting (tie breaking rule)

- Pre crash price for first random orders up to $\kappa$
Payoff structure, Trading

- Small transaction costs $ce^{rt}$
- Risk-neutrality but max/min stock position
  - Max long position
  - Max short position
  - Due to capital constraints, margin requirements etc.

- Definition 1: trading equilibrium
  - Perfect Bayesian Nash Equilibrium
  - Belief restriction: trader who attacks at time $t$ believes that all traders who became aware of the bubble prior to her also attack at $t$. 
Sell out condition for $\Delta \to 0$ periods

- Sell out at $t$ if:
  \[ \Delta h(t|t_i) E_t [\beta p_t | \cdot] \geq (1 - \Delta h(t|t_i) (g - r) p_t \Delta) \]
  appreciation rate
  benefit of attacking
  cost of attacking

- $h(t|t_i) \geq \frac{g-r}{\beta^*}$

- RHS $\to (g - r)$ as $t \to \infty$
  - Bursting date: $T^*(t_0) = \min\{T(t_0 + \eta \kappa), t_0 + \bar{t}\}$
Sequential awareness

Distribution of $t_0$

Distribution of $t_0 + \bar{\tau}$
(bursting if nobody attacks)

trader $t_i$

$t_i - \eta$ since $t_i \leq t_0 + \eta$

$t_i$ since $t_i \geq t_0$

$t_0$

$t_0 + \bar{\tau}$
Sequential awareness

Distribution of $t_0$

- $t_i - \eta$ since $t_i \leq t_0 + \eta$
- $t_i$ since $t_i \geq t_0$

Distribution of $t_0 + \bar{\tau}$ (bursting if nobody attacks)

- $t_j - \eta$
- $t_j$
- $t_0$
- $t_0 + \bar{\tau}$
Sequential awareness

Distribution of $t_0$

since $t_i \leq t_0 + \eta$

t since $t_i \geq t_0$

Distribution of $t_0 + \bar{\tau}$
(bursting if nobody attacks)

trader $t_i$

$\eta$

trader $t_j$

$\eta$

trader $t_k$

$t_0$

$t_k$

$t_0 + \bar{\tau}$

Brunnermeier
Conjecture 1: Immediate attack

⇒ Bubble bursts at $t_0 + \eta \kappa$

when $\kappa$ traders are aware of the bubble
Conjecture 1: Immediate attack

⇒ Bubble bursts at $t_0 + \eta \kappa$

when $\kappa$ traders are aware of the bubble

If $t_0 < t_i - \eta \kappa$, the bubble would have burst already.
**Conjecture 1: Immediate attack**

→ **Bubble bursts at** $t_0 + \eta \kappa$

when $\kappa$ traders are aware of the bubble

Distribution of $t_0$

Distribution of $t_0 + \eta \kappa$

If $t_0 < t_i - \eta \kappa$, the bubble would have burst already.
**Conjecture 1: Immediate attack**

\[ \Rightarrow \text{Bubble bursts at } t_0 + \eta \kappa \]

when \( \kappa \) traders are aware of the bubble
Conjecture 1: Immediate attack

⇒ Bubble bursts at $t_0 + \eta \kappa$

Distribution of $t_0$
Conjecture 1: Immediate attack

⇒ Bubble bursts at $t_0 + \eta \kappa$

Distribution of $t_0$

$\lambda / (1 - e^{\lambda \eta \kappa})$

Bubble bursts for sure

$t_i - \eta$

$t_i - \eta \kappa$

$t_i$

$t_i + \eta \kappa$
Conjecture 1: Immediate attack

⇒ Bubble bursts at $t_0 + \eta\kappa$
Conjecture 1: Immediate attack

⇒ Bubble bursts at $t_0 + \eta \kappa$

hazard rate of the bubble

$$h = \frac{\lambda}{1 - e^{-\lambda(t_i + \eta \kappa - t)}}$$

Distribution of $t_0$
**Conjecture 1: Immediate attack**

⇒ Bubble bursts at $t_0 + \eta \kappa$

Recall the sell out condition:

$$h(t|t_i) \geq \frac{g - r}{\beta^*}$$

hazard rate of the bubble

$$h = \frac{\lambda}{1 - e^{-\lambda(t_i + \eta \kappa - t)}}$$

Distribution of $t_0$

$$\frac{\lambda}{1 - e^{\lambda \eta \kappa}}$$

$t_i - \eta, t_i - \eta \kappa, t_i, t_i + \eta \kappa$
Conjecture 1: Immediate attack

⇒ Bubble bursts at \( t_0 + \eta \kappa \)

Recall the sell out condition:

\[
h(t | t_i) \geq \frac{g - r}{\beta^*}
\]

hazard rate of the bubble

\[
h = \frac{\lambda}{1 - e^{-\lambda(t + \eta \kappa - t)}}
\]

distribution of \( t_0 \)

optimal time to attack \( t_i + \tau_i \)

⇒ “delayed attack is optimal”
Proposition 3: Suppose $\frac{\lambda}{1-e^{-\lambda \eta \kappa}} > \frac{g-r}{\bar{\beta}}$

- Unique trading equilibrium
- Traders begin attacking after a delay of $\tau^*$ periods
- Bubble bursts due to endogenous selling pressure at a size of $p_t$ times

$$\beta^* = \frac{1 - e^{-\lambda \eta \kappa}}{\lambda} (g - r)$$
Endogenous crash

⇒ Bubble bursts at $t_0 + \eta \kappa + \tau^*$

$h = \frac{\lambda}{1 - e^{-\lambda(t_i + \eta \kappa + \tau^* - t)}}$

bubble appreciation
bubble size

lower bound: $\frac{g-\tau}{\beta} > \frac{\lambda}{1-e^{\lambda \eta \kappa}}$

$t_i - \eta \quad t_i - \eta \kappa \quad t_i$

$t_i - \eta + \eta \kappa + \tau^* \quad t_i + \tau^* \quad t_i + \eta \kappa + \tau^*$

conjectured attack
optimal
Exogenous crash for low $\bar{\tau}$ (i.e. low $\bar{\beta}$)

- Proposition 2: Suppose $\frac{\lambda}{1-e^{-\lambda \eta \kappa}} \leq \frac{g-r}{\bar{\beta}}$.
  - Unique trading equilibrium
  - Traders begin attacking after a delay of $\tau^1 < \bar{\tau}$ periods.
  - Bubble does not burst due to endogenous selling pressure prior to $t_0 + \bar{\tau}$. 
Delayed attack by $\tau'$

⇒ Bubble bursts at $t_0 + \min\{\eta \kappa + \tau', \bar{\tau}\}$

$h = \frac{\lambda}{1 - e^{-\lambda(t_i + \eta \kappa + \tau_i - t)}}$

lower bound: $\frac{g-r}{\beta} > \frac{\lambda}{1-e^{\lambda \eta \kappa}}$

bubble appreciation
bubble size
Delayed attack by $\tau'$

$\Rightarrow$ Bubble bursts at $t_0 + \min\{\eta \kappa + \tau', \bar{\tau}\}$

hazard rate of $t_0 + \eta \kappa + \tau'$

$$h = \frac{\lambda}{1 - e^{-\lambda(t_i+\eta \kappa+\bar{\tau}-t)}}$$

bubble appreciation
bubble size

lower bound: $\frac{g-r}{\beta} > \frac{\lambda}{1-e^{\lambda \eta \kappa}}$
Delayed attack by $\tau'$

$\Rightarrow$ Bubble bursts at $t_0 + \min\{\eta \kappa + \tau', \bar{\tau}\}$

hazard rate of $t_0 + \eta \kappa + \tau'$

$$h = \frac{\lambda}{1 - e^{-\lambda (t_i + \eta \kappa + \bar{\tau} - t)}}$$

Bubble appreciation
Bubble size

lower bound: $\frac{g - r}{\beta} > \frac{\lambda}{1 - e^{\lambda \eta \kappa}}$

$\Rightarrow$ Bubble bursts for exogenous reasons at $\bar{\tau}$
Lack of common knowledge

⇒ standard backwards induction can’t be applied

\[ t_0 + \eta \kappa + \tau^* \]

\( \kappa \) traders know of the bubble

everybody knows that everybody knows that everybody knows of the bubble

\((\text{same reasoning applies for } \kappa \text{ traders})\)
Role of synchronizing events

- News may have an impact disproportionate to any intrinsic informational (fundamental) content
  - News can serve as a synchronization device
- Fads & fashion in information
  - Which news should traders coordinate on?
- When “synchronized attach” fails, then the bubble is temporarily strengthened
Setting with synchronizing events

- Focus on news with no info content (sunspots)
- Synchronizing events occur with Poisson arrival rate
  - Note that pre-emption argument does not apply since event occurs with zero probability
- Arbitrageurs who are aware of the bubble become increasingly worried about it over time.
  - Only traders who became aware of the bubble more than $\tau_e$ periods ago observe (look out for) this synchronizing event.
Synchronizing events – market rebounds

- Proposition 5: In ‘responsive equilibrium’
  Sell out  a) always at the time of the public event \( t_e \),
  b) after \( t_i + \tau^{**} \) (where \( \tau^{**} < \tau^* \))
except after a failed attack at , re-enter the market
for \( t \in (t_e, t_e - \tau_e + \tau^{**}) \).

- Intuition for re-entering the market
  - For \( t_e < t_0 + \eta \kappa + \tau_e \) attack fails, agents learn \( t_0 > t_e - \tau_e - \eta \kappa \)
  - Without public event, they would have learnt this only at \( t_e + \tau_e - \tau^{**} \)
  - Density that bubble burst for endogenous reasons is zero
Price cascades and rebounds

- Price drops as a synchronizing event
  - Through psychological resistance line
  - By more than, say 5%

- Exogenous price drop
  - After a price drop
    - If bubble is rip
      - ⇒ bubble bursts and price drops further
    - If bubble is not ripe yet
      - ⇒ price bounces back and he bubble is strengthened for some time
Price cascades and rebounds

- Proposition 6: In ‘responsive equilibrium’
  Sell out  
  a) after a price drop if $\tau_i \leq \tau_p(H_p)$
  b) after $t_i + \tau^{***}$ (where $\tau^{***} < \tau^*$)
  except after a rebound at $t_p$, re-enter the market for $t \in (t_p, t_p - \tau_p + \tau^{**})$.

- Intuition
  - Attack is costly, since price might jump back
    $\Rightarrow$ only arbitrageurs who became aware of the bubble more than $\tau_p$ periods ago attack bubble.
  - After a rebound, an endogenous crash can be temporarily ruled out and hence, arbitrageurs re-enter the market.
  - Even sell out after another price drop is less likely.
Conclusion of Bubbles and Crashes

- **Bubbles**
  - Dispersion of opinion among arbs causes a synchronization problem which makes coordinated price correction difficult.
  - Arbitrageurs time the market and ride the bubble
  - ⇒ Bubbles persist

- **Crashes**
  - Can be triggered by unanticipated news without any fundamental content, since
  - It might serve as synchronization device.

- **Rebound**
  - Can occur after a failed attack which temporarily strengthens the bubble
Hedge Funds & the Technology Bubble

Markus Brunnermeier and Stefan Nagel

Princeton University and Stanford University
Quarterly 13F filings to SEC
Mandatory for all institutional investors
  - With holdings in U.S. stocks of more than $100 million
  - Domestic and foreign
  - At manager level

Caveat: No short positions

53 managers with CDA/Spectrum data
- Excludes 18 managers b/c mutual business dominates
- Incl. Soros, Tiger, Tudor, D.E. Shaw etc.

Hedge fund performance data
- HFR hedge fund style indexes
Fig. 2: Weight of NASDAQ technology stocks (high P/S) in aggregate hedge fund portfolio versus weight in market portfolio.
Did Soros ride the bubble?

Fig. 4a: Weight of technology stocks in hedge fund portfolios versus weight in market portfolio

Proportion invested in NASDAQ high P/S stocks

| Horse         | Mar-98 | Jun-98 | Sep-98 | Dec-98 | Mar-99 | Jun-99 | Sep-99 | Dec-99 | Mar-00 | Jun-00 | Sep-00 | Dec-00 |
|---------------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| Zwieg-DiMenna | ☑      | ☑      | ☑      | ☑      | ☑      | ☑      | ☑      | ☑      | ☑      | ☑      | ☑      | ☑      |
| Soros         | ☑      | ☑      | ☑      | ☑      | ☑      | ☑      | ☑      | ☑      | ☑      | ☑      | ☑      | ☑      |
| Husic         | ☐      | ☐      | ☑      | ☑      | ☐      | ☐      | ☑      | ☐      | ☐      | ☑      | ☐      | ☐      |
| Market Portfolio | ☑ | ☑ | ☑ | ☑ | ☑ | ☑ | ☑ | ☑ | ☑ | ☑ | ☑ | ☑ |
| Tiger         | ☐      | ☐      | ☐      | ☐      | ☐      | ☐      | ☐      | ☐      | ☐      | ☐      | ☐      | ☐      |
| Omega         | ☐      | ☐      | ☐      | ☐      | ☐      | ☐      | ☐      | ☐      | ☐      | ☐      | ☐      | ☐      |
Fund in- and outflows

Fund flows as proportion of assets under management

Quantum Fund (Soros)
Jaguar Fund (Tiger)
Figure 5. Average share of outstanding equity held by hedge funds around price peaks of individual stocks
Did hedge funds’ timing pay off?

Figure 6: Performance of a copycat fund that replicates hedge fund holdings in the NASDAQ high P/S segment
Hedge funds were riding the bubble

- Short sale constrains and “arbitrage” risk are not sufficient to explain this behavior.

Timing best of hedge funds were well placed. Outperformance!

- Rues out unawareness of bubble
- Suggests predictable investor sentiment. Riding the bubble for a while may have been a rational strategy

⇒ Supports ‘bubble-timing’ models