Asset Pricing under Asymmetric Information
Modeling Information & Solution Concepts

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Books:

Articles:
many - see syllabus

Some parts of these slides rely on Princeton lecture notes by Nöldeke (1993)
Two Interpretations of Asymmetric Information

- different information
- different interpretation of the same information (different background information)
Modeling information I

- State space $\Omega$
  - state $\omega \in \Omega =$ full description of reality
    - fundamentals
    - signals
  - state space is common knowledge and fully agreed among agents
• Partition
  • \((\omega_1, \omega_2, \omega_3), (\omega_4, \omega_5), (\omega_6, \omega_7, \omega_8)\)
  • \(\mathcal{P}_1, \mathcal{P}_2, \mathcal{P}_3\) (partition cells)
  • later more about ‘knowledge operators’ etc.

• Field (Sigma-Algebra) \(\mathcal{F}^i\)

• Probability measure/distribution \(P\)
Modeling information III

• Prior distribution
  • Common prior assumption (CPA) (Harsanyi doctrine)
    • any difference in beliefs is due to differences in info
    • has strong implications
  • Rational Expectations
    • prior\(^i\) = objective distribution \(\forall i\)
    • implies CPA
  • Non-common priors
    • Problem: almost everything goes
    • Way out: Optimal Expectations
      (structure model of endogenous priors)

• Updating/Signal Extraction
Modeling information III

- Updating (general)
  - Bayes’ Rule

\[
P^i (E_n|D) = \frac{P^i (D|E_n) P^i (E_n)}{P^i (D)} ,
\]

- if events \( E_1, E_2, ..., E_N \) are a partition

\[
P^i (E_n|D) = \frac{P^i (D|E_n) P^i (E_n)}{\sum_{n=1}^{N} P^i (D|E_n) P^i (E_n)} ,
\]
Updating - Signal Extraction - general case

- Updating - Signal Extraction
  - $\omega = \{v, S\}$
  - desired property: signal realization $S^H$ is always more favorable than $S^L$
  - formally: $G(v|S^H)$ FOSD $G(v|S^L)$
  - Milgrom (1981) shows that this is equivalent to $f_S(S|v)$ satisfies monotone likelihood ratio property (MLRP)
- $f_S(S|v)/f_S(S|\bar{v})$ is increasing (decreasing) in $S$ if $v > (<)\bar{v}$

$$\frac{f_S(S|v)}{f_S(S|v')} > \frac{f_S(S'|v)}{f_S(S'|v')} \forall v' > v \text{ and } S' > S.$$  

- another property: hazard rate $\frac{f_S(S|v)}{1-F(S|v)}$ is declining in $v$
Updating - Signal Extraction - Normal distributions

- updating normal variable $X$ after receiving signal $S = s$

\[
E[X|S = s] = E[X] + \frac{\text{Cov}[X,S]}{\text{Var}[S]} (s - E[S])
\]
\[
\text{Var}[X|S = s] = \text{Var}[X] - \frac{\text{Cov}[X,S]^2}{\text{Var}[S]}
\]

- $n$ multidimensional random variable \((\vec{X}, \vec{S}) \sim \mathcal{N}(\mu, \Sigma)\)

\[
\mu = \begin{bmatrix} \mu_X \\ \mu_S \end{bmatrix} \\ \Sigma = \begin{bmatrix} \Sigma_{X,X} & \Sigma_{X,S} \\ \Sigma_{S,X} & \Sigma_{S,S} \end{bmatrix}
\]

- Projection Theorem \((X|S = s)\)

\[
\sim \mathcal{N} \left( \mu_X + \Sigma_{X,S} \Sigma_{S,S}^{-1} (s - \mu_S), \Sigma_{X,X} - \Sigma_{X,S} \Sigma_{S,S}^{-1} \Sigma_{S,X} \right)
\]
Special Signal Structures

- \( N \)-Signals of form: \( S_n = X + \varepsilon_n \)
  
  (Let \( X \) be a scalar and \( \tau_y = \frac{1}{\text{Var}[y]} \)),

\[
E[X|s_1, \ldots, s_N] = \mu_X + \frac{1}{\tau_X + \sum_{n=1}^{N} \tau_{\varepsilon_n}} \sum_{n=1}^{N} \tau_{\varepsilon_n} (s_n - \mu_X)
\]

\[
\text{Var}[X|s_1, \ldots, s_N] = \frac{1}{\tau_X + \sum_{n=1}^{N} \tau_{\varepsilon_n}} = \frac{1}{\tau_X|s_1, \ldots, s_N}
\]

- If, in addition, all \( \varepsilon_n \) i.i.d. then

\[
E[X|s_1, \ldots, s_N] = \mu_X + \underbrace{\frac{1}{\tau_X + N\tau_{\varepsilon_n}}}_{\text{Var}[X|s_1, \ldots, s_N]} N\tau_{\varepsilon_n} \left( \sum_{n=1}^{N} \frac{1}{N} s_n - \mu_X \right),
\]

where \( \bar{s} := \sum_{n=1}^{N} \left( \frac{1}{N} \right) s_n \) is a sufficient statistic
Special Signal Structures

- $\mathcal{N}$-Signals of form: $X = S + \varepsilon$
  
  $$E [X|S = s] = s$$
  $$Var [X|S = s] = Var[\varepsilon]$$

- Binary Signal: Updating with binary state space/signal
  - $q =$ precision $= \text{prob}(X = H|S = S^H)$

- “Truncating signals”: $\nu \in [\bar{S}, S]$
  - $\nu$ is Laplace (double exponentially) distributed or uniform
  - posterior is a truncated exponential or uniform

(see e.g. Abreu & Brunnermeier 2002, 2003)
Solution/Equilibrium Concepts

• **Rational Expectations Equilibrium**
  - Competitive environment
  - agents take prices as given (price takers)
  - Rational Expectations (RE) $\Rightarrow$ CPA
  - *General Equilibrium Theory*

• **Bayesian Nash Equilibrium**
  - Strategic environment
  - agents take strategies of others as given
  - CPA (RE) is typically assumed
  - *Game Theory*
  - distinction between normal and extensive form games
    simultaneous move versus sequential move
## The 5 Step Approach

| Step 1   | Specify joint priors  
|          | Conject. price mappings  
|          | \( P : \{S^1, \ldots, S^I, u\} \rightarrow \mathbb{R}^J_+ \)  
| Step 2   | Derive posteriors  
| Step 3   | Derive individual demand  
| Step 4   | Impose market clearing  
| Step 5   | Impose Rationality  
|          | Equate undet. coeff.  

### REE

| Step 1   | Specify joint priors  
|          | Conjecture strategy profiles  
| Step 2   | Derive posteriors  
| Step 3   | Derive best response  
| Step 4   | Impose Rationality  
| Step 5   | No-one deviates  

### BNE (sim. moves)
A little more abstract

- **REE**
  Fixed Point of Mapping: \( \mathcal{M}_P(P(\cdot)) \mapsto P(\cdot) \)

- **BNE** (simultaneous moves)
  Fixed Point of Mapping:
  strategy profiles \( \mapsto \) strategy profiles

- What’s different for sequential move games?
  - late movers react to deviation
  - equilibrium might rely on ‘strange’ out of equilibrium moves
  - refinement: subgame perfection

- Extensive form move games with asymmetric information
  - Sequential equilibrium (agents act sequentially rational)
  - Perfect BNE (for certain games)
    - nature makes a move in the beginning (chooses type)
    - action of agents are observable
A Classification of Market Microstructure Models

- simultaneous submission of demand schedules
  - competitive rational expectation models
  - strategic share auctions
- sequential move models
  - screening models: (uninformed) market maker submits a supply schedule first
    - static
      - uniform price setting
      - limit order book analysis
  - dynamic sequential trade models with multiple trading rounds
- signalling models: informed traders move first, market maker second