Lecture 13: Review Session, and New Keynesian Model

ECO 503: Macroeconomic Theory I

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Review of Material

- Course had two components
  1. tools
  2. substance

- Let’s go over these, emphasizing what I want you to take away from this course
What I want you to take away: **Tools**

- how to solve deterministic dynamic optimization problems in **discrete time**...
- ... and **continuous time**...
- how to **analyze dynamics** of resulting trajectories (find steady states, analyze their stability)
- how to **numerically solve** dynamic optimization problems (shooting algorithm, linearization)
- how to **calibrate** a model
- how to set up a **Ramsey taxation problem**
- how to construct a **balanced growth path** (BGP)
- how to solve rep. agent models with **externalities**
- how to introduce overlapping generations (**perpetual youth**)  
- how to think about a **stationary equilibrium** in a heterogeneous agent model
What I want you to take away: 

**Substance**

- Basic tradeoffs in growth model
- Income vs. substitution effects
- Effects of response to unexpected productivity increase in growth model (building block of business cycle model)
- Quick convergence of growth model to steady state (or BGP)
- Kaldor facts (and caveats)
- Growth model = decent model of growth experience of U.S. (and other developed countries)...
- ... but bad model of “development dynamics”
What I want you to take away:

Substance

• Ramsey taxation of capital:
  • robust prediction: if possible, want to tax more today than tomorrow
  • not robust: ⇒ zero capital taxes in long-run

• Endogenous growth: all theories make some linearity assumption
  • if it doesn’t jump out at you, it’s hidden somewhere

• Upper tails of income and wealth distribution follow Pareto distributions

• Q: where do Pareto distributions come from?
  A: simplest mechanism: combination of
    1. exponential growth
    2. random death (exponentially distributed)
New Keynesian Model

- New Keynesian model = RBC/growth model with sticky prices

- References:
  - Gali (2008): most accessible intro
  - Woodford (2003): New Keynesian bible
  - Clarida, Gali and Gertler (1999): most influential article
  - Gali and Monacelli (2005): small open economy version
Why Should You Care?

- Simple framework to think about relationship between monetary policy, inflation and the business cycle.
- RBC model: cannot even think about these issues! Real variables are completely separate from nominal variables ("monetary neutrality", "classical dichotomy").
- Corollary: monetary policy has **no effect** on any real variables.
- Sticky prices break "monetary neutrality"
- Workhorse model at central banks (see Fed presentation /DB_EC0521_2012_2013/LectureNotes/MacroModelsAtTheFed.pdf)
- Makes some sense of newspaper statements like: “a boom leads the economy to overheat and creates inflationary pressure”
- Some reason to believe that “demand shocks” (e.g. consumer confidence, animal spirits) may drive business cycle. Sticky prices = one way to get this story off the ground.
Outline

(1) Model with flexible prices

(2) Model with sticky prices
Setup: Flexible Prices

- Households maximize
  \[ \int_0^\infty e^{-\rho t} \left\{ \log C(t) - \frac{N(t)^{1+\varphi}}{1 + \varphi} \right\} dt \]

  subject to

  \[ PC + \dot{B} = iB + WN \]

- C: consumption
- N: labor
- P: price level
- B: bonds
- i: nominal interest rate
- W: nominal wage
- Note: no capital
Households

- Hamiltonian

\[ H(B, C, N, \lambda) = \log C - \frac{N^{1+\phi}}{1 + \phi} + \lambda[iB + WN - PC] \]

- Conditions for optimum

\[ \dot{\lambda} = \rho \lambda - \lambda i \]

\[ \frac{1}{C} = \lambda P \quad \Rightarrow \quad \frac{\dot{C}}{C} = -\frac{\dot{\lambda}}{\lambda} - \frac{\dot{P}}{P} \]

\[ N^\phi = \lambda W \]

- Defining the inflation rate \( \pi = \dot{P}/P \)

\[ \frac{\dot{C}}{C} = i - \pi - \rho \]

\[ CN^\phi = \frac{W}{P} \]
Firms – Final Goods Producer

• A competitive final goods producer aggregates a continuum of intermediate inputs

\[ Y = \left( \int_{0}^{1} y_j^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}} \]

• Cost minimization \( \Rightarrow \) demand for intermediate good \( j \)

\[ y_j(p_j) = \left( \frac{p_j}{P} \right)^{-\varepsilon} Y \]

where

\[ P = \left( \int_{0}^{1} p_j^{1-\varepsilon} dj \right)^{\frac{1}{1-\varepsilon}} \]

Firms – Intermediate Goods Producers

• Continuum of monopolistically competitive intermediate goods producers $j \in [0, 1]$.

• Production uses labor only

$$y_j(t) = A(t)n_j(t).$$

• Solve (drop $j$ subscripts for simplicity)

$$\max_p \ p \left( \frac{p}{P(t)} \right)^{-\varepsilon} Y(t) \ - \ \frac{W(t)}{A(t)} \left( \frac{p}{P(t)} \right)^{-\varepsilon} Y(t)$$

• Solution

$$p(t) = P(t) = \frac{\varepsilon}{\varepsilon - 1} \frac{W(t)}{A(t)}$$

where $P = p_j$ follows because all producers are identical.
Equilibrium with Flexible Prices

• Market clearing:

\[ C = AN \]

• Combining with household FOC \( CN^\varphi = W/P \) and

\[ P = \frac{\varepsilon}{\varepsilon - 1} W/A \]

\[ C = Y = A \left( \frac{\varepsilon}{\varepsilon - 1} \right)^{-\frac{1}{1+\varphi}} \]

• Note: distortion from monopolistic competition

• Back out real interest rate from

\[ r = i - \pi = \rho - \frac{\dot{C}}{C} = \rho + \frac{\dot{A}}{A} = \rho + g \]
Some Notable Features

- Like an RBC model, this model features “monetary neutrality”
  [http://lmgtfy.com/?q=monetary+neutrality](http://lmgtfy.com/?q=monetary+neutrality)
- Equivalently: there is a “classical dichotomy”
  [http://lmgtfy.com/?q=classical+dichotom](http://lmgtfy.com/?q=classical+dichotom)
- Real variables \((C(t), Y(t), N(t), W(t)/P(t), r(t))\) are determined completely separately from nominal variables \((P(t), W(t), \pi(t), i(t))\).
- In fact, \(P(t)\) and \(\pi(t)\) are not even determined in the absence of a description of a determination of the economy’s money stock (e.g. through monetary policy). But this doesn’t matter for real variables.
- As a corollary, monetary policy has **no effect** on real variables.
Sticky Prices

- Everything same except intermediate goods producers.
- Per period profits are still
  \[ \Pi_t(p) = p \left( \frac{p}{P(t)} \right)^{-\varepsilon} Y(t) - \frac{W(t)}{A(t)} \left( \frac{p}{P(t)} \right)^{-\varepsilon} Y(t) \]
- But now have to pay quadratic price adjustment cost
  \[ \Theta_t \left( \frac{\dot{p}}{p} \right) = \frac{\theta}{2} \left( \frac{\dot{p}}{p} \right)^2 P(t) Y(t) \]
- Optimal control problem:
  \[ V_0(p_0) = \max_{p(t), t \geq 0} \int_0^\infty e^{-\int_0^t i(s) ds} \left\{ \Pi_t(p(t)) - \Theta_t \left( \frac{\dot{p}(t)}{p(t)} \right) \right\} dt \]
- \( \theta \): degree of price stickiness
Comparison to Literature

- Note: my formulation uses quadratic price adjustment costs as in Rotemberg (1982).
- Different from standard Calvo (1983) pricing formulation: allowed to change price at Poisson rate $\alpha$
- I like Rotemberg better because pricing is state dependent as opposed time dependent (“Calvo fairy”).
- Closer to “menu cost” models.
- Schmitt-Grohe and Uribe (2004), Fernandez-Villaverde et al. (2011) also use Rotemberg
- I also assume that adjustment costs are paid as a transfer to consumers, $T = \Theta_t(\pi) = (\theta/2)\pi^2PY$. Just a trick to eliminate real resource costs of inflation ($\Theta_t(\pi) \approx 0$ anyway).
Optimal Price Setting

- Hamiltonian (state: $p$, control: $\dot{p}$, co-state: $\eta$):
  \[ H(p, \dot{p}, \eta) = p \left( \frac{p}{P} \right)^{-\varepsilon} Y - \frac{W}{A} \left( \frac{p}{P} \right)^{-\varepsilon} Y - \frac{\theta}{2} \left( \frac{\dot{p}}{p} \right)^2 PY + \eta \dot{p} \]

- Conditions for optimum
  \[ \theta \frac{\dot{p} P}{p p} Y = \eta \]

  \[ \dot{\eta} = i \eta - \left[ (1 - \varepsilon) \left( \frac{p}{P} \right)^{-\varepsilon} Y + \varepsilon \frac{W}{p} \frac{1}{A} \left( \frac{p}{P} \right)^{-\varepsilon} Y + \theta \left( \frac{\dot{p}}{p} \right)^2 \frac{P}{p} Y \right]. \]

- Symmetric equilibrium: $p = P$
  \[ \theta \pi Y = \eta \]

  \[ \dot{\eta} = i \eta - \left[ (1 - \varepsilon) Y + \varepsilon \frac{W}{P} \frac{1}{A} Y + \theta \pi^2 Y \right]. \]
Optimal Price Setting

- Recall the FOC: $\theta \pi Y = \eta$. Differentiate with respect to time

$$\theta \dot{\pi} Y + \theta \pi \dot{Y} = \dot{\eta}$$

- Substitute into equation for co-state and rearrange

Lemma

The price setting of firms implies that the inflation rate $\pi = \dot{P}/P$ is determined by

$$\left( i - \pi - \frac{\dot{Y}}{Y} \right) \pi = \frac{\varepsilon - 1}{\theta} \left( \frac{\varepsilon}{\varepsilon - 1} \frac{W}{P} \frac{1}{A} - 1 \right) + \dot{\pi}.$$
Optimal Price Setting in Equilibrium

• In equilibrium $C = Y$ and Euler equation

$$\frac{\dot{Y}}{Y} = \frac{\dot{C}}{C} = i - \pi - \rho$$

• Substitute into expression on previous slide ⇒ Inflation determined by

$$\rho \pi = \frac{\varepsilon - 1}{\theta} \left( \frac{\varepsilon}{\varepsilon - 1} \frac{W}{P} \frac{1}{A} - 1 \right) + \dot{\pi}. \quad (*)$$

• In integral form (check that differentiating gives back above)

$$\pi(t) = \frac{\varepsilon - 1}{\theta} \int_t^\infty e^{-\rho(s-t)} \left( \frac{\varepsilon}{\varepsilon - 1} \frac{W(s)}{P(s)} \frac{1}{A(s)} - 1 \right) ds$$

• Compare with equation (16) in Chapter 3.3. of Gali’s book and expression just below.
Optimal Price Setting in Equilibrium

• Inflation determined by

\[ \pi(t) = \frac{\varepsilon - 1}{\theta} \int_t^\infty e^{-\rho(s-t)} \left( \frac{\varepsilon}{\varepsilon - 1} \frac{W(s)}{P(s)} \frac{1}{A(s)} - 1 \right) ds \]

• Intuition: term in brackets = marginal payoff to a firm from increasing its price

\[ \Pi_t'(P(t)) = (\varepsilon - 1) Y(t) \left( \frac{\varepsilon}{\varepsilon - 1} \frac{W(t)}{P(t)} \frac{1}{A(t)} - 1 \right). \]

• Positive whenever \( P \) less than optimal markup \( \frac{\varepsilon}{\varepsilon - 1} \) over marginal cost \( W/A \).

• With flexible prices, \( \theta = 0 \): \( \Pi_t'(P(t)) = 0 \) for all \( t \), \( P = \frac{\varepsilon}{\varepsilon - 1} \frac{W}{A} \).

• With sticky prices, \( \theta > 0 \): \( \pi = \text{PDV of all future } \Pi_t'(P(t)) \).

• Adjustment cost is convex. So if expect reason to adjust in the future – e.g. \( W(t)/A(t) \uparrow \) – already adjust now.
IS Curve and Phillips Curve

- Call outcomes under flexible prices, $\theta = 0$, “natural” output $Y^n$ and “natural” real interest rate. Recall

$$Y^n = A \left( \frac{\varepsilon}{\varepsilon - 1} \right)^{\frac{-1}{1+\varphi}}, \quad \frac{\dot{Y}^n}{Y^n} = r - \rho, \quad r = \rho + \frac{\dot{A}}{A}$$

- Define output gap: $X = Y/Y^n$. Recall Euler equation under sticky prices

$$\frac{\dot{Y}}{Y} = i - \pi - \rho$$

- Euler equation in terms of output gap $\dot{X}/X = \dot{Y}/Y - \dot{Y}^n/Y^n$

$$\frac{\dot{X}}{X} = i - \pi - r$$

- This is basically an IS curve.
IS Curve and Phillips Curve

- Can obtain “Phillips Curve” in similar way. Recall

\[ P^n = \frac{\varepsilon}{\varepsilon - 1} \frac{W^n}{A} \Rightarrow \frac{W}{P} \frac{1}{A} = \frac{W/P}{W^n/P^n} \]

- Equation for inflation (*) becomes

\[ \rho \pi = \frac{\varepsilon - 1}{\theta} \frac{W/P - W^n/P^n}{W^n/P^n} + \dot{\pi}. \]

- From FOC \( CN^\varphi = \frac{W}{P} \), and mkt clearing \( C = Y, N = Y/A \)

\[ \frac{W/P}{W^n/P^n} = \left( \frac{Y}{Y^n} \right)^{1+\varphi} = X^{1+\varphi}. \]
IS Curve and Phillips Curve

- Relation between inflation and output gap: “New Keynesian Phillips Curve”

\[ \rho \pi = \frac{\varepsilon - 1}{\theta} (X^{1+\varphi} - 1) + \pi. \]

- In integral form

\[ \pi(t) = \frac{\varepsilon - 1}{\theta} \int_t^\infty e^{-\rho(s-t)} (X(s)^{1+\varphi} - 1) \, ds. \]

- Inflation high when future output gaps are high, i.e. when economy “overheats”
Three Equation Model

• Recall: IS curve and Phillips curve

\[
\frac{\dot{X}}{X} = i - \pi - r \quad \text{(IS)}
\]

\[
\rho \pi = \frac{\varepsilon - 1}{\theta} (X^{1+\varphi} - 1) + \dot{\pi} \quad \text{(PC)}
\]

• To close model: Taylor rule

\[
i = i^* + \phi \pi + \phi_x \log X \quad \text{(TR)}
\]

• “Three equation model,” see modern undergraduate textbooks (e.g. Carlin and Soskice)

• Substitute (TR) into (IS) ⇒ system of two ODEs in \((\pi, X)\), analyze with phase diagram.
Three Equation Model in Literature

- Literature uses log-linearization all over the place.
- Obtain exact analogues by defining
  \[ x \equiv \log X = \log Y - \log Y^n \]
  
- Using that for small \( x \) (Taylor-series)
  \[ X^{1+\varphi} - 1 = e^{(1+\varphi)x} - 1 \approx (1 + \varphi)x \]

- and defining \( \kappa \equiv (\varepsilon - 1)(1 + \varphi)/\theta \)
  \[ \dot{x} = i - \pi - r \quad \text{(IS')} \]
  \[ \rho\pi = \kappa x + \dot{\pi} \quad \text{(PC')} \]
  \[ i = i^* + \phi\pi + \phi_x x \quad \text{(TR')} \]

- Exact continuous time analogues of (21), (22), (25) in Chapter 3 of Gali’s book, same as in Werning (2012)
Phase Diagrams

- For simplicity, assume $\phi_x = 0$. Makes some math easier.
- Also ignore ZLB, $i \geq 0$ (next time).
- Substitute (TR') into (IS')

$$
\begin{align*}
\dot{x} &= i^* - r + (\phi - 1)\pi \\
\dot{\pi} &= \rho\pi - \kappa x \\
\end{align*}
$$

(ODE)

- See phase diagrams on next slide
- Important: both $\pi$ and $x$ are jump-variables. No state variables.
- Two cases:
  - $\phi > 1$: unique equilibrium. “Taylor principle”: $i$ increases more than one-for-one with $\pi$ so that also real rates increase.
  - $\phi < 1$: equilibrium indeterminacy
- From now assume $\phi > 1$
Phase Diagram with $\phi > 1$

Dynamics:

$x > 0 \iff \Pi > \frac{r - i}{\phi - 1}$

$\Pi > 0 \iff x < \frac{p}{k \Pi}$

Equilibrium
Monetary Policy

- Can achieve $\pi = 0$ and $x = 0$ by setting $i^* = r$ (and $\phi > 1$).

- Scenario 1: suppose economy is in $(x, \pi) = (0, 0)$ equilibrium. But at $t = T$, $r$ increases, e.g. because TFP growth increases (recall $r = \rho + \dot{A}/A$).

- Scenario 2: suppose economy is in $(x, \pi) = (0, 0)$ equilibrium. But at $t = T$, someone at the Fed goes crazy and increases $i^*$ (e.g. because mistakenly think that TFP growth goes up).

- Draw time paths for $(x(t), \pi(t))$ for both scenarios.
Optimal Monetary Policy

- Huge literature

Zero Lower Bound

• So far: New Keynesian 3 equation model, derived from microfoundations

• Ignored ZLB (or “liquidity trap”), $i(t) \geq 0$.

• Some references
  
  • Eggertsson and Woodford (2003), “The zero interest-rate bound and optimal monetary policy”
  
  • Christiano, Eichenbaum, Rebelo (2011) “When is the Government Spending Multiplier Large?”
  