Problem Set 3

1. Consider a two-person exchange economy with endowments \( \omega_1 = (1, 3); \omega_2 = (3, 1) \) and the preferences of both consumers are given by the vector ordering preference relation: i.e. for any two commodity bundles \((x, y)\) and \((x', y')\) we have: 
   \( (x, y) \succ (x', y') \) if and only if \( x > x', y > y' \).
   
a. Illustrate in an Edgeworth box the Pareto efficient allocations.
   b. What is the set of Walrasian equilibria for this economy?

2. The city of Beverly Hills considers displaying a statue on Rodeo Drive. The statue has already been commissioned and therefore has to be paid no matter whether it will be displayed or not. Let \( v_i, i \in \{1, \ldots, I\} \) describe the value that the \( I \) residents assign to the display of the statue. Inhabitants of Beverly Hills have no problem assigning monetary values to art and hence \( v_i \) should be interpreted as the amount of money that \( i \) would be willing to pay if the statue were displayed. (Note that \( v_i \) could be positive or negative!) Thus, the utility of \( i \) when she has \( m_i \) units of money is
   \[
   v_i + \begin{cases} 
   v_i & \text{if the statue is displayed} \\
   m_i & \text{otherwise}.
   \end{cases}
   \]
   (As usual, \( m_i \) can be any real number.)
   
a. Characterize Pareto efficient outcomes. What decision rule should the city use when deciding whether or not to display the statue?
   b. Extend the commodity space so that this situation can be described within the competitive model. (The city of Beverly Hills should be modeled as a firm; define the appropriate consumption sets, budget sets, etc.)
   c. Find the competitive equilibria.

3. Show that \( \bar{p}, \bar{y} \) (as defined in class) are non-empty, convex valued and uhc correspondences (under the assumptions made in class).

4. Consider an economy with two goods and one consumer and one firm. Assume that the consumer has a utility function that is given by \( u(x_1, x_2) = x_1 + ax_2 \) and has strictly positive endowments of both goods. The firm can produce either nothing (choose \((0, 0)\)) or use the following technology: \( Y = \{(y_1, y_2) \mid y_1 + b(y_2 + c) \leq 0, y_2 \leq -c\} \). Assume \( a, b \) and \( c \) are strictly positive. For what values of \( a, b, \) and \( c \) is there a Walrasian equilibrium for this economy? For those cases where an equilibrium exists, determine equilibrium prices and quantities.

5. Consider an exchange economy with strictly convex and strictly monotone preferences. Suppose that the initial endowments are a Pareto optimal allocation. Show that the equilibrium allocation is unique.
6. Suppose that $L = 2$ and there is a continuum of consumers. All consumers have the same initial endowments; they are not rational, however. Given a budget set, each consumer takes a random draw from her budget line (according to a uniform distribution on the non-negative consumptions). Let $z(p)$ denote the average excess demand ($=\text{aggregate demand minus endowment}$). Show that $z$ is the same as the excess demand generated from a continuum of identical utility maximizing consumers with Cobb-Douglas utility.