Lecture for Sept 18

Dornbusch, Fischer, Samuelson (1977): 160 years of international economics in one paper

One factor, labor. 2 countries.

Continuum of goods, ranked in order of Home comparative advantage; relative productivity $A(z)$:

Given relative wage $w$, all goods with relative productivity $>z_{bar}$ produced in Home, $<z_{bar}$ in Foreign
Cobb-Douglas demand: each good receives share \( b(z) \) of spending. Let

\[
\Gamma(z_{bar}) = \int_{0}^{z_{bar}} b(z)dz
\]

That’s the share of spending on Home goods.

Market clearing:

\[
wL = \Gamma(z_{bar})[wL + L^*]
\]

Hence

\[
w = \frac{L^* \Gamma(z_{bar})}{L (1 - \Gamma(z_{bar}))}
\]
Now do transfer problem. Suppose Home gets transfer $D$

$$wL = \Gamma(zbar)(wL + D + L^* - D)$$

Hmm. That’s not going anywhere (Ohlin position)

But now suppose that Home and Foreign both spend a share $v$ of income on nontraded goods; then

$$wL = [v + \Gamma(zbar)](wL + D) + \Gamma(zbar)(L^* - D)$$

So now we’re getting somewhere:

$$w = \frac{L^*}{L} \frac{\Gamma}{1 - v - \Gamma} + \frac{vD}{1 - v - \Gamma}$$

An inward transfer (a trade deficit) shifts $B(z)$ up, so that equilibrium $w$ rises.
But how big is the effect? Eaton-Kortum

They think of $A()$ as coming from a random process of allocating technologies to countries, with a specific distribution (Frechet) that happens to work. $T$ is an index of a country’s overall technology level. It turns out that

$$A(z) = \left( \frac{T}{T^*} \right)^{1/\theta} \left( \frac{1 - z}{z} \right)^{1/\theta}$$

in their formulation, while all goods are symmetric in demand, so that $\Gamma(z) = (1-v)z$ where $v$ is the share of nontraded goods in spending.

So you have two equations:

$w = A(z)$ as above, and

$$w = \frac{L^*}{L} \frac{(1 - v)}{1 - v - (1 - v)z} + \frac{vD}{1 - v - (1 - v)z}$$

These can be solved for $w$ and $z$ given deficit $D$

But how can you estimate $\theta$?

Geography!
Figure 1.—Trade and geography.

Figure 2.—Trade and prices.
They use retail prices – kind of funny.

But basic point is that strong distance-trade relationship suggests fairly high elasticity of substitution – in effect, high Armington elasticity

Their estimates:

Preferred:
US relative wage down 6.8%, real wage down 0.5%

Low elasticity case:
Relative wage down 13.5%, real wage down 1.1%

Why are Eaton-Kortum so different from Obstfeld-Rogoff (and conventional wisdom)?

1. High Armington elasticity
2. But also, different assumption on intersectoral mobility!
   OR say fixed production of N, EK have labor perfectly mobile among sectors

Bottom line of all this: relative prices play key role in trade adjustment. How close are we to a one-good world? Not very, except possibly in the long run.
So let’s assume a one-good world!

The basic monetary model:

Purchasing power parity: \( P = S P^* \) (\( S \) for spot exchange rate)

Money demand: \( M/P = Y^{\alpha} e^{\beta i} \) where \( Y \) is real GDP

Full employment: real GDP exogenous, \( P \) does the adjusting

So \( P = M Y^{\alpha} e^{\beta i} \) (why the exponential? So that it comes out log-linear)

Everything symmetric in Foreign

Use lower case for logs:

\[ p = m - \alpha y + \beta i \]

\[ p^* = m^* - \alpha y^* + \beta i^* \]

\[ s = p - p^* = m - m^* - \alpha (y - y^*) + \beta (i - i^*) \]

Other things equal, exchange rate reflects relative money supplies

What about \( i - i^* \)? Expectations

Invest $1 at home => $1+i after 1 year

Invest abroad: get €(1/S) => €(1/S)(1+i*) after one year, which can be re-exchanged as $(S^e/S)(1+i^*)$, where \( S^e \) is the exchange rate expected to prevail
Ignore risk. Then

$$1 + i = (1 + i^*)(Se / S)$$

$$\frac{1 + i}{1 + i^*} = \frac{Se}{S}$$

Or, approximately,

$$1 + i - i^* = 1 + (Se - S)/S$$

Or

$$i - i^* = E[\Delta s]$$

So the exchange rate equation becomes (suppressing constant terms)

$$s = m - m^* + \beta E(\Delta s)$$

This breaks nicely into a “fundamentals” component and a “speculative” component

Even better if we assume rational expectations

Use subscript for time period; change notation so that m is log of relative money supply, formerly m-m^* (or hold m^* constant)
Then

\[ s_t = m_t + \beta(E[s_{t+1}] - s_t) \]

\[ \Rightarrow s_t = \frac{1}{1+\beta} m_t + \frac{\beta}{1+\beta} E[s_{t+1}] \]

\[ \Rightarrow s_t = \frac{1}{1+\beta} m_t + \frac{\beta}{1+\beta} \frac{1}{1+\beta} E[m_{t+1}] + \left[ \frac{\beta}{1+\beta} \right]^2 E[s_{t+2}] \]

\[ \Rightarrow s_t = \frac{1}{1+\beta} m_t + \frac{\beta}{1+\beta} \frac{1}{1+\beta} E[m_{t+1}] + \left[ \frac{\beta}{1+\beta} \right]^2 \frac{1}{1+\beta} E[m_{t+2}] + \ldots \]

So the current exchange rate is an exponentially declining sum of current and all future relative money supplies -- basically like a stock price.

Empirically, this is a total bust except in high-inflation situations. But useful in thinking about expectations and their role.
Target zones:

Central bank sets upper and lower limits to allowed exchange rate,

$-\bar{s} \leq s \leq \bar{s}$

The question: clearly the existence of the band will affect exchange rate inside the band, because it changes what can happen in the future. Summing over paths of m is a complete mess. But there’s another approach:

Intuition on TZ:

What has to happen:
Trying to derive this

Monetary exchange rate equation, but in continuous time

\[ s = m + v + \beta E[ds]/dt \]

Let \( m=0 \) within the band

Velocity follows a random walk with variance \( \sigma^2 \) per unit time:

\[ dv = \sigma dz \]
Let’s just guess that in the solution, XR is just a function of v,

\[ s = G(v) \]

Now we can use a bit of stochastic calculus. All you need to know is that when v follows Brownian motion, a second-order Taylor series is right for expected rate of change:

\[
\frac{E[ds]}{dt} = \frac{1}{2} \sigma^2 G''(v)
\]

So, we guess at a functional form for G()

\[ s = v + A[e^{\rho v} - e^{-\rho v}] \]

(where’s that from? I did it in discrete time, found that it had roots \( r, 1/r \), and analogized)

Meanwhile, this must also be true:

\[ s = v + \beta E[ds]/dt \]

But

\[
\frac{E[ds]}{dt} = \frac{1}{2} \sigma^2 G''(v) = \frac{1}{2} \sigma^2 \rho^2 A[e^{\rho v} - e^{-\rho v}]
\]

So find rho such that

\[
\frac{\beta}{2} \sigma^2 \rho^2 = 1
\]
Yes, we can! And if $A<0$, we get the “$S$” shape. But which $S$?

“Smooth pasting” – must be tangent.

This is probably the fanciest application of the monetary model ...