Equity and Efficiency in Human Capital Investment: The Local Connection

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First version received December 1994; final version accepted December 1995 (Eds.)

A general model of community formation and human capital accumulation with social spillovers and decentralized school funding is used to analyse the causes of economic segregation and its consequences for equity and efficiency. Significant polarization arises from minor differences in endowments, preferences or access to capital markets. This makes income inequality more persistent across generations, but the same need not be true for wealth. Equilibrium stratification tends to be excessive, resulting in low aggregate surplus. Whether state equalization of school resources can remedy these problems hinges on how purchased, social and family inputs interact in education and in mobility decisions.

INTRODUCTION

The accumulation of human capital underlies the evolution of both income inequality and productivity growth. As demonstrated most vividly by the physical blight and social pathology of inner-city schools, certain essential inputs in this process are of a local nature. They are determined neither at the level of individual families nor that of the whole economy, but at the intermediate level of communities, neighbourhoods, firms or social networks. Not only is this the case with school resources when funding is decentralized, but also with many forms of "social capital": peer effects, role models, job contacts, norms of behaviour, crime, and so on. Through these fiscal and sociological spillovers, the next generation's distribution of skills and incomes is shaped by the manner in which the current one sorts itself into differentiated clusters.

The rise in socioeconomic segregation observed in most Western countries is therefore likely to have a lasting influence on their productive and social structures. A crude indicator of residential polarization in the U.S. is the ratio of central city to suburban mean per capita incomes. For the 85 largest metropolitan areas it fell from 105% to 84% between 1960 and 1989, with fifteen cities ending up below two-thirds.¹ More detailed analyses at the census tract level confirm a general increase in economic segregation during the 1970's and especially the 1980's, even as racial segregation marked a small decline (Jargowsky (1994)). European cities, traditionally much more integrated, are now starting to exhibit a similar pattern where urban poverty is both geographically concentrated ("ghettos") and consistently passed on from one generation to the next (the "underclass").²

1. Two were even below one half: Newark (43%) and Paterson (46%). These numbers are from Ledeber and Barnes (1992), whose data comes from the 1990 U.S. census.
2. See for instance the article on "Europe and the Underclass" in The Economist (1994).
The first objective of this paper is to provide a unified analysis of the causes of socioeconomic stratification and their consequences for income distribution and aggregate productivity. It makes clear the roles played by endowments, preferences, capital markets, neighbourhood effects, and local public goods. The paper’s second concern, almost inseparable from the first, is education finance. Constitutional challenges are compelling increasing numbers of U.S. states to reform the traditional system where towns fund their own schools from property tax revenues. The share of elementary and secondary public education expenditures raised from local sources thus declined from 60% in 1960 to about 45% in recent years. Conversely, some European countries are contemplating a greater decentralization of education finance and control. The paper therefore compares overall surplus, student achievement and equality of opportunity under local and state financing of education. While some of these issues have been studied in the literature reviewed below, the comprehensive framework developed here brings to light new, important effects. For instance, modelling both human capital and financial assets can significantly alter the implications of stratification for equity. Similarly, the effects of school finance reform depend crucially on how purchased inputs combine with social capital in the production of education and in parental mobility decisions. The emphasis on such interactions and their analysis within a simple framework constitute some of the distinctive contributions of this paper.

Section 1 presents the model. Heterogeneous families form communities, choose local public expenditures, and accumulate human capital. Section 2 demonstrates how even minor differences in education technologies, preferences or wealth lead to a high degree of stratification; capital market imperfections are not necessary but will compound other sources. The next two sections show that these small causes can have large effects. Section 3 takes a new look at the standard view that stratification makes inequality more persistent across generations. Consistent with Loury (1977), Borjas (1992), Bénabou (1993, 1996) and Durlauf (1996a, 1996b), I show how income convergence is slowed down and ghettos might endogenously appear. But because the model keeps track of financial bequests as well as human capital, it allows me to go further and demonstrate an important caveat: while stratification exacerbates inequality in education and income, the same need not be true for inequality in total wealth. Families who cluster into more desirable communities can end up, in equilibrium, dissipating more of their savings on housing (relatively to poorer ones) than the value of the human capital advantage gained by their children. The determining factor is thus the cost paid by the rich to separate themselves from the poor, or conversely the extent to which they are able to appropriate the rents generated in the process of segregation. I discuss various practices and institutions which serve this purpose, such as discrimination and zoning laws. I show in particular how de jure racial segregation leads to de facto (economic) segregation once legal barriers to mobility are lifted. Section 4 turns from equity to efficiency considerations. Examining how the productivity of a metropolitan area reflects its organization into local communities, I show that the typical pattern of city-suburb polarization is likely to be very inefficient. Optimal community compositions are determined by a general tradeoff between complementarity (of family and community attributes) and curvature (e.g. decreasing returns to community quality). Because mobility decisions internalize the former—possibly distorted by credit constraints, discrimination or other market imperfections—but not the latter, aggregate surplus tends to be too low. In the Appendix I extend the model to overlapping generations and explain how endogenous sorting can reduce income growth even for better-off families ("self-defeating secession"). Interestingly, U.S. metropolitan areas display a significant negative correlation between economic segregation, measured by the disparity between city and
suburban incomes, and overall growth in per capita incomes and employment. Section 4 also discusses a couple of recent econometric studies which lend more formal support to the idea that residential polarization is detrimental to metropolitan performance.

The second issue studied in the paper is education finance. Section 5 shows that the decentralization of school funding and control constitutes a segregating force, and that it need not improve efficiency. Section 6 then examines the implications of a policy to equalize per student expenditures across communities, such as those being adopted by a growing number of states in response to court mandates. Bénabou (1996) and Fernandez and Rogerson (1994) predict very significant gains from such a reform: in addition to reducing inequality it would raise the economy's long-term output, or even growth rate. In this paper I temper this optimistic scenario by downplaying capital market imperfections and emphasizing instead a new aspect of the problem, namely the interactions between purchased and social inputs in education. Indeed, several pieces of evidence appear to cast doubt on the effectiveness of redistributive funding in raising the performance of poor schools, hence in reducing earnings inequality and augmenting aggregate surplus. Some of the states where central intervention in school funding was the most significant have not seen a commensurate reduction in test score inequality (Downes (1992)), and even seem to have experienced a greater decline in average test scores (Peltzman (1993), Hoxby (1995)). Another case discussed in the paper is that of Kansas City, where a massive redistribution of funds to magnet schools in the inner city since 1986 has yet to deliver significant improvements in student performance. I show how empirically plausible complementarities in the education production function and in parental preferences can help explain some of these puzzles. I determine when equalization works and when it is counterproductive; in the latter case, I discuss alternative policies which may still improve both equity and efficiency. Because these policies work through changes in residential composition, however, they may be constrained by a form of irreversibility which I show to be inherent in the stratification process.

The literature to which the paper belongs has its sources in the classic works of Tiebout (1956) on local public goods and Schelling (1978) on segregation and externalities. More recent prominent influences include the work of Loury (1977, 1987) on racial inequality and of sociologists such as Coleman (1988), Wilson (1987) and Jencks and Mayer (1990) on group interactions. Another important motivation is the current debate over local disparities in school funding, perhaps best publicized by Kozol (1991). In the last few years these ideas have spurred a broad strand of research seeking to integrate local public finance, income distribution and macroeconomics. I build here on the models of De Bartolome (1990) and Bénabou (1993, 1996), but the paper is also closely related to the work of Borjas (1992), Durlauf (1996a, 1996b) and Lundberg and Startz (1994) on

3. Leebur and Barnes' (1992) data for the 85 largest Metropolitan Standard Areas (MSA's) show a correlation of 0.43 between the 1989 city/suburb ratio and metropolitan employment growth from January 1988 to August 1991. In Rusk's much smaller sample of fourteen cities, the 1970 ratio has correlations of 0.60 with per capita income growth during 1969–1989 and 0.45 with employment growth during 1973–1989. In the 209 MSA's studied by Glaeser, Scheinman and Schleifer (1995), the 1970 ratio of city to metropolitan incomes has a correlation of 0.25 with 1970–1990 metropolitan population growth (which varies closely with employment growth); the regression excludes four outliers where the ratio was above 1.4, and yields a t-statistic of 3.6. I am very grateful to Edward Glaeser for providing me with these last statistics.

4. Given a low enough intergenerational discount rate, it could even be Pareto-improving. Several schemes for education finance reform and inter-community transfers were studied by Inman (1978), Fernandez and Rogerson (1996) and Cooper (1992), although not a move to centralized funding. These models also assigned a central role to borrowing constraints, and therefore shared the feature that some redistribution of school budgets was desirable. This effect was also present in Glomm and Ravikumar (1992), but that paper focused mostly on the idea that private funding creates better incentives for investment in human capital than public funding.
the persistence of inequality, and to that of Glomm and Ravikumar (1992), Fernandez
and Rogerson (1994, 1996) and Cooper (1992) on education finance. The central role
played by community composition also links it to the more microeconomic literature on
club theory, notably Brueckner and Lee (1989), Oates and Schwab (1991) and Scotchmer
(1994).

1. THE MODEL

1.1. People and communities

There is a continuum of families, with unit measure. Parents are of two types, A and B
(rich and poor, White and Black, etc.), with human capital endowments \( h_A > h_B \). The
proportions of the two types are \( n \) and \( 1 - n \). The average level of human capital is denoted
\( \bar{h} = nh_A + (1 - n)h_B \). These agents live in a city composed of two towns or communities
\( C', j = 1, 2 \), each holding the same number (1/2) of single-family homes. The inelasticity
of land supply and population density is not essential but simplifies the analysis. All land
belongs to absentee landowners; departures from this neutral allocation are considered
later on. The proportion of high human wealth, type A adults, in each community is
denoted as \( x' \), and community 1 is defined as the richer one: \( x^1 \geq 2n - x^1 = x^2 \). In American
cities the two locations would typically correspond to “the suburbs” and “the urban
centre”; in Europe the labels would often be reversed.⁵

1.2. Preferences and technologies

There are two periods. A parent with type \( h \in \{ h_A, h_B \} \) initially chooses a community
\( C', j = 1, 2 \), so as to maximize the resulting utility \( U^j(h) \):

\[
\begin{align*}
U^j(h) & \equiv \max_{c, c'} U(c, c', h), \text{ subject to} \\
c + \rho' + t^j(h) &= \omega(h) + d \\
c' + P(h, d) &= y(h) \\
h' &= F(h, L', E^j).
\end{align*}
\]

In the first period, he consumes \( c \) and pays rent \( \rho' \) plus taxes \( t^j(h) \) out of his initial
resources \( \omega(h) \), augmented by his chosen level of debt \( d \). The interest rate \( r(h, d) \) may
depend on his type as well as the amount borrowed or saved. In the second period, the
resources \( c' \) available for consumption or bequest—both interpretations are possible—
equal current income \( y(h) \), minus debt repayments \( P(h, d) \equiv d(1 + r(h, d)) \). Finally, the
child’s human capital \( h' \) is determined by that of the parent (through at-home education),
by the quality \( L' \) of social interactions in the chosen community, and by the resources
devoted to its schools, measured by the per student budget \( E^j \). Although in reality “human
capital” consists of multiple attributes with different degrees of appropriability (innate

⁵ A model with two types and two communities provides the minimal framework for studying how agents
associate in the provision of formal and informal education. It can also be viewed as a representative “slice” of
a city with many types and communities, where one focuses on any pair of contiguous towns and their popula-
tions. The paper’s results all extend to a model of that type (e.g. Epple, Fillimon and Romer (1984)) and to the
limiting case of a continuum of types and communities (Wheaton (1993)). When convenient, I shall therefore
treat \( \Delta h/h = (h_A - h_B)/h \) as small and replace differences by derivatives. The one assumption which cannot easily
be relaxed is that type is a one-dimensional variable; with the exception of Epple and Romano (1993), there
are virtually no results on multi-dimensional sorting.
talent, formal education, work ethic, etc.), I follow the standard practice of treating it as a one-dimensional quality.  

1.3. Neighbourhood effects

The local spillover \( L' \) captures the non-fiscal channels through which the young’s acquisition of skills is affected by the social mix of neighbouring families. These include peer effects in education, role models (Wilson (1987), Streufert (1991)), norms of behaviours (concerning for instance single parenthood) and social or professional networks. Conversely, negative influences include high rates of unemployment, welfare dependency or crime in the neighbourhood (Montgomery (1991), Sah (1991)). For tractability I shall represent these many forms of “social capital” (Loury (1977), Coleman (1988)) by a single index which emphasizes their general correlation with the level of education in the community. I thus require \( L' = L(x')|h_A, h_B) \) to be stochastically increasing in the local distribution of human capital: \( L'(x) \gtrless 0 \), and \( L \) rises with \( h_A \) and \( h_B \). Without loss of generality I also assume that \( L \) is homogeneous of degree one in \((h_A, h_B)\), with \( L(x|h_A, h_B) = h \) for all \( x, h \). One can thus think of \( L' \) as an average of local residents’ levels of human capital. Its value for a representative sample of population will be denoted \( \bar{L} \equiv L(n|h_A, h_B) \).

Another key feature of the spillover is its response to a mean-preserving spread in the distribution. Do stronger individuals tend to pull the average up to their level, or do weaker ones tend to drag it down to theirs? Clearly, it is important to allow both cases. Consider for instance the CES index \( L(x) = (x h_A^{1-\varepsilon} / \varepsilon + (1 - x) h_B^{1-\varepsilon} / \varepsilon / (1 - \varepsilon)) \). When \( 1/\varepsilon > 0 \) individual levels of human capital are complements, \( L \) is convex in \( x \), and heterogeneity is a source of loss: \( L < \bar{h} \). When \( 1/\varepsilon < 0 \) individual levels are substitutes, \( L \) is concave in \( x \), and heterogeneity is a source of gains: \( L > \bar{h} \). The Leontief (minimum) and “best-shot” (maximum) cases are obtained in the limit as \( 1/\varepsilon \) goes to \( +\infty \) or \( -\infty \). More generally, the curvature of \( L(x) \) will be an important determinant of the efficiency of equilibrium.

1.4. Local school funding

The other local input into education is decentralized school expenditures. Given the distribution \((x', 1-x')\) of rich and poor families in the community—hence, given social capital \( L' \)—their preferences over school budgets and taxation are aggregated by some local political mechanism into funding decisions \( E' = E(x') \) and tax schedules \( t'(h, \rho) = t(h, \rho, x') \), subject to the budget constraint \( x' t(h_A, \rho', x') + (1-x') t(h_B, \rho', x') = E(x') \). I thus allow taxes to depend on individual income and property values. The political

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6. Note also that \( h' = F(h, L, E) \) should be understood as human wealth net of the child’s optimally chosen studying effort (measured in the numeraire good). Incorporating private purchases of education is also straightforward: \( c \) and \( E \) are replaced in (1) by \( c + e \) and \( E + e \), and parents maximize over both \( e \) and \( d \).


8. Brooks-Gunn et al. (1993) find that it is primarily the upper tail of a neighbourhood’s income distribution which affects the development of children and adolescents raised there. This is consistent with Dynarski, Schwab and Zampelli’s (1989) results that average student performance in a school district rises with dispersion among family incomes, \( ceteris paribus \). On the other hand, Hamilton and Macauley (1991) find that income dispersion raises the unit cost of education.

9. Note that the \( E' \) and \( t'(h, \rho) \) chosen by \( C' \) residents may depend on the equilibrium rent \( \rho' \) (as opposed to an individual’s bid \( \rho \)), through its effects on their wealth and on potential revenues from alternative forms of taxation. To keep the notation simple, this dependence on \( j \) is subsumed in the dependence on \( x' \).
mechanism in question could be majority voting, the outcome of lobbying, or delegation to an efficient local planner. For the moment I shall not specify it explicitly but instead work directly with the reduced-form policy functions $E(\cdot)$ and $t(\cdot)$. This will allow me to identify the general features of political mechanisms which promote or hinder segregation.

1.5. Equilibrium

Let us rewrite the utility of an adult of type $h$, living in a community with percentage $x$ of rich households and paying $\rho$ in land rent, as:

$$V(h, \rho, x) = \max_d \{ U(\omega(h) + d - \rho - t(h, \rho, x), y(h) - P(h, d), F(h, L(x), E(x)) \}.$$  \hspace{1cm} (2)

Equilibrium in the land market will result in stratification if the rich (in human capital) are willing or able to bid more than the poor for land in a high human wealth community. Formally, this means that a rich family's iso-utility curve or bid-rent is steeper than a poor one's:

$$R_s(h, \rho, x) \equiv \frac{dp'}{dx} \bigg|_{v(h, \rho', x) - v(h, \rho, x)} = \frac{V_s(h, \rho, x)}{-V_p(h, \rho, x)}$$

increases with $h$. \hspace{1cm} (3)

When this simple sorting condition in the space of community quality and price holds, the slightest divergence from the symmetric allocation $x^1=x^2=n$ (which is always an equilibrium) sets in motion a cumulative process. As soon as $x^1 > x^2$, $C^1$ becomes more desirable than $C^2$ to all households and must therefore command a positive rent premium $\rho^1 - \rho^2$. Rich families, being most willing to trade a higher $\rho$ for a higher $x$, outbid poor ones for land in $C^1$, thereby further raising $x^1$ and lowering $x^2$. This leads to additional rounds of increase in the rent differential, displacement of the poor by the rich in $C^1$ and vice-versa in $C^2$, until at least one community is completely homogeneous. The three possible configurations are indicated in Figure 1, together with the equilibrium condition on $\rho^1$ and $\rho^2$ which expresses the locational indifference of the type present in both communities.\(^{10}\) Formally, let $R_{hx} \equiv \partial R_s / \partial h$; I show in the Appendix:

**Proposition 1.** 1. If $R_n(h, \rho, x) > 0$ for all $\rho, x$, the unique stable equilibrium is stratified. If $n \leq 1/2$, the rich all live in community 1: $x^1 = 2n, x^2 = 0$. If $n \geq 1/2$, the poor all live in community 2: $x^1 = 1, x^2 = 2n - 1$. The symmetric equilibrium $x^1 = x^2 = n$ is unstable.

2. If $R_n(h, \rho, x) < 0$ for all $\rho, x$, the unique equilibrium is completely integrated (i.e. symmetric), and it is stable.

By virtue of this result, the economic forces which lead to stratification or integration can be brought to light by simply examining the marginal rate of substitution between

\(^{10}\) For simple models of the dynamics of "tipping", see Miyao (1978) or Bénabou (1993). Stratification results based on some special version of the sorting condition (3) are common in the literature on community composition, e.g. Westhoff (1977) or Eppe, Filimon and Romer (1984). As in many of these papers I only determine the rent differential which ensures that no one wants to switch communities (e.g. De Bartolome (1990), Wheaton (1993)). The base level of rents could be pinned down by relaxing one of the assumptions of inelastic land supply and fixed number of agents of each type, but this would needlessly complicate the model. With variable housing size, the stratified equilibrium will obtain as long as the (human) wealth elasticity $\delta \log (V_s / V_p) / \partial \log h$ of agents' valuation of community quality is greater than the elasticity of their demand for housing (Wheaton (1993)). If that condition fails, no equilibrium may exist without zoning restrictions, as the poor will always crowd into small dwellings in rich communities.
first-period consumption and child education. Using the Euler equation for the optimal level of borrowing, we have:

$$R_s(h, \rho, x) = \frac{U_3(c, c', h') - t_s(h, \rho, x)}{U_2(c, c', h')} \cdot \frac{F_L \cdot L(x) + F_E \cdot E'(x)}{P_d(h, d)} \cdot (1 + t_p(h, \rho, x))^{-1}.$$  \hspace{1cm} (4)

This equation incorporates the contributions of preferences, technology, capital markets and local public goods. I shall study each of these components in turn, then explore their joint implications for inequality and efficiency. In doing so I shall continue to focus on the case where $R_{hx}$ is of constant sign, resulting in a unique stable equilibrium where segregation is either maximal or minimal (although never complete, unless $n = 1/2$). This is only for expositional convenience: the paper’s main results on inequality and efficiency remain qualitatively unchanged when $R_{hx}$ changes sign.\(^{11}\)

2. THE DETERMINANTS OF STRATIFICATION

2.1. Local complementarities

The simplest case is that where: (a) the only local input into education is community quality, $F = F(h, L)$, so that no collective decisions are involved;\(^{12}\) (b) capital markets are perfect, $P(h, d) = d(1 + r)$; (c) utility is linear, $U(c, c', h') = c + (c' + h')/(1 + r)$, or more generally education is a pure investment good: $c'$ and $h'$ enter additively, so that $U_3/U_2$ is constant. This last assumption seems especially appropriate when $c'$ is a bequest. The bident slope now simplifies to $R_s(h, d) = F'(L)L'(x)/(1 + r)$; therefore stratification occurs if families with higher human capital are more sensitive to neighbourhood quality than those with lower human wealth.\(^{13}\)

11. I actually consider such a case in footnotes 29 and 30. Becker and Murphy (1994) explicitly allow for multiple stable equilibria in a model closely related to this one. They show that all suffer from the same form of inefficiency as the extremal equilibrium which this paper emphasizes.

12. This case admits two interesting alternative interpretations. One is that agents care about community quality $L$ for reasons other than its impact on their child’s education; crime is a good example, with richer agents caring most about it. The second is that $F = F(h, E)$ and communities finance schools by taxing income at a fixed rate, as is the case when preferences are logarithmic (Glomm and Ravikumar (1992)). Then $E(x) = \tau L(x)$, where $L(x) = x \sigma(h_a) + (1 - x) \sigma(h_b)$ is simply average income in period 1.

13. In the words of Baumol (1967): “The individual’s remedy intensifies the community’s problems and each feeds upon the other. Those who leave the city are usually the very persons who care and can afford to care—the ones who maintain the houses, who do not commit crimes, and who are most capable of providing the taxes needed to arrest the process of urban decay. Their exodus therefore leads to further deterioration in urban conditions and so induces yet another wave of emigration, and so on.”
Proposition 2. A small amount of complementarity between family human capital and community quality $F_{nl} > 0$ is sufficient to cause stratification. The distribution of financial wealth is irrelevant.

In contrast to most of the literature on inequality, this result shows that there might be little role for redistributive policies to affect either the distribution of educational attainment or the efficiency of equilibrium. Transfers or subsidized loans may be poor substitutes for the actual integration of schools, neighbourhoods, and social networks. I shall come back to this issue when discussing school funding.

2.2. Capital market imperfections

It may of course not be easy for a poor family which values education highly to borrow enough to move into a high human wealth community. I now show that differences in "ability" to pay for community quality tend to complement, or even replace, differences in "willingness" to pay in generating economic segregation. I abstract from differences in tastes or returns to education across families, $U_3/U_2 = 1$, $F = F(L)$, and allow instead frictions in credit markets: $R_s(h, d) = F'(L)L'(x)/P(d)$.

Proposition 3. Small imperfections in capital markets, resulting in a higher opportunity cost of funds for poor families $(P_{ad} + d'(h)P_{ad} < 0)$ are sufficient to cause stratification.

This case occurs most simply when families with different levels of human wealth face different interest rates: $P(h, d) = (1 + r(h))$, with $r'(h) < 0$. This could reflect different monitoring technologies or degrees of financial sophistication. A differential can also arise endogenously from wealth constraints. Let everyone face the same interest schedule $P(d) = d(1 + r(d))$, with increasing marginal cost of borrowing (or decreasing marginal return on savings) $P'' > 0$, due to standard asymmetric information problems. Let people work only in the first period of life, earning $\omega(h)$, with $\omega'(h) > 0$; during retirement, $y(h) = 0$. In order to live in the same community as a wealthy family, a poor one needs to borrow more: the optimal $d$ decreases with $h$ for any given $\rho$, hence the result. In the absence of offsetting forces, segregation will occur no matter how small the wealth differences or credit market imperfections.

2.3. Wealth effects

I now focus on the term $(U_3/U_2)(c, c', h')$ in (4), which captures preferences for education relative to old age consumption or financial bequest. To isolate the wealth effect, or consumption value of education, let $F = F(L)$ and capital markets be perfect. Finally, let parental utility be additively separable: $U(c, c', h') = u(c) + v(c') + \omega(h')$, with $u, v, w$

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14. While the most recent study finds $F_{nl} > 0$ (Brooks-Gunn et al. (1993), the large empirical literature surveyed in Jencks and Meyer (1990) contains many contradictory estimates of this effect.

15. The simplest case is that of a wedge between borrowing and lending rates, as in Galor and Zeira (1993). Conditions on endowments which ensure that $A$ agents are always lenders and $B$ agents always borrowers are easily found. A tax subsidy to home ownership relative to renting (interest deductions) has the same effect, when only $A$ types can afford a down-payment.

16. The imperfection could also take the form of a borrowing constraint, $d \leq \bar{d}$. When resources are such that it binds for the $B$'s but not for the $A$'s, the Euler condition is an inequality for the former but an equality for the latter, implying again $R_s(h_A, \rho, x) > R_s(h_B, \rho, x)$. When both groups are constrained, education takes on the features of a consumption good; see footnote 17.
increasing and \( u, v \) strictly concave. Then:

\[
R_s(h, \rho, x) = \frac{w'(F(L))}{\nu' (c')} \cdot \frac{F'(L) L'}{1 + r} \frac{\tilde{u}'(z(h) - \rho)}{\tilde{u}'(z(h) - \rho)} F'(L) L',
\]

where \( \tilde{u}(z) \equiv \max_s \{ u(z - s) + v(s(1 + r)) \} \) and \( z(h) \equiv \omega(h) + y(h)/(1 + r) \) is lifetime wealth.

**Proposition 4.** Small differences in lifetime resources \( (\omega'(h) + y'(h))/(1 + r) > 0 \) are sufficient to cause stratification.

More generally, when \( F = F(h, L), U_1/U_2 = w'(F(h, L))/\tilde{u}'(z(h) - \rho) \); stratification occurs if the reduction in the marginal utility of education due to better parental background is less than the reduction in the marginal utility of consumption which results from higher financial wealth. This is the sense in which education must be a normal good. Note finally the close similarity between the stratifying effects of wealth due to preferences and those which arise from credit market imperfections.  

Wealth effects also underlie models where sorting occurs not through land rents—supply is infinitely elastic—but through different levels or rates of taxation (e.g. Fernandez and Rogerson (1996)). Rich agents vote for high taxes to finance public goods such as education, whereas poor ones prefer lower levels. This leads, through a Tiebout (1956) mechanism, to as much segregation as the number of communities will allow. Instead of rents or taxes, wages can also serve as compensating differentials. Such is the case in Fershtman and Weiss’s (1993) analysis of social status, where people derive utility from being in a profession with a high proportion of well educated members. Wealthier individuals are more willing to accept a lower remuneration of human capital in exchange for higher status. Therefore more of them become highly educated, and the labour market stratifies. The working paper version of this article (Bénabou (1994)) shows how simple variations of the model yield sorting results similar to those in Fernandez and Rogerson (1996) and Fershtman and Weiss (1993).

I have until now assumed that all land rents accrue to absentee landowners. This is a “neutrality” assumption with respect to the allocation of the capital gains and losses created by stratification in \( C_1 \) and \( C_2 \). It also simplifies the problem, by making initial wealth levels exogenous: \( \omega(h) = \tilde{\omega}(h) \). To examine how relaxing this assumption affects the results, let each \( A \) household initially own \( \nu_A \) units of land in \( C_1 \) and \( \eta_A \) units in \( C_2 \) respectively. The corresponding endowments for a \( B \) family are \( \nu_B = (1/2 - n \eta_A)/(1 - n) \) and \( \eta_B = (1/2 - n \eta_A)/(1 - n) \). Total wealth levels are then \( \omega(h_i) = \tilde{\omega}(h_i) + \nu_i \rho^i + \eta_i \rho^i, i = A, B \). If \( \nu_A = \eta_A = 1/2 \), both classes share equally in capital gains and losses; this is another neutral case where the results remain unchanged. The converse, perhaps more realistic case, is \( \nu_A = 1/2n, \eta_A = 0 \). More generally, consider any initial allocation where \( \nu_A > \eta_A \), hence \( \nu_B < \eta_B \). As communities become more homogenous the wealth of the \( A \)'s rises while that of the \( B \)'s declines. Propositions 3 and 4 show that this in itself creates a segregating force, which can sustain stratification even when all others are absent.

17. Compare for instance the above specification with one where \( U(c, c', h') = u(c) + v(c + h') \), so that \( U_1/U_2 = 1 \), but households can neither borrow nor save, for simplicity (\( d = 0 \)). The ratio \( R_s(h, \rho, x) = w'(F(L))F'(L)L' \) is equal to \( \tilde{u}'(z(h) - \rho) \) in the first case and to \( u'(\omega(h) - \rho) \) in the second. With \( \omega'(h) > 0 \) and \( z'(h) > 0 \) these expressions vary with \( h \) in similar ways.
2.4. Local public goods, political mechanisms, and discrimination

I defer until Section 5 specifying a particular mechanism through which communities choose taxes and school expenditures. But inspection of the corresponding terms in (4) already indicates a "natural" tendency toward stratification, à la Tiebout (1956).

**Proposition 5.** Political mechanisms whose outcome is such that expenditures $E(x)$ increase with $x$ when $F_{hE}>0$ (respectively, decrease when $F_{hE}<0$) tend to produce stratification. So do mechanisms such that income tax rates fall with the proportion of rich agents in the community ($t_{px}(h, \rho, x)<0$) or with land values ($t_{lp}(h, \rho, x)<0$).

To the extent that a higher proportion of rich families is reflected in taxation and expenditure decisions which are closer to their preferred choices, stratification will occur as people "vote with their feet". Although not all public choice mechanisms are such that the rich obtain better treatment as they become more numerous (e.g. majority voting with income taxes), this requirement seems fairly realistic. It will be satisfied by the schemes studied later on in the paper.

An alternative interpretation of $t_{hx}<0$ or $t_{lp}<0$ is that the poor incur higher costs, relative to the rich, of joining a rich community. These costs are particularly relevant when the $A$'s and $B$'s correspond to different ethnic groups. They could be pecuniary, due to discrimination by owners or intermediaries in the housing market, or non-pecuniary, such as harassment.\(^{18}\)

3. STRATIFICATION AND INEQUALITY

Does families’ tendency to segregate into homogeneous communities make inequality more persistent? The simple answer is positive, as stratification compounds disparities in educational inputs at the family and community levels. The result is greater income inequality than would have occurred if families had remained integrated. In the simple case where purchased inputs play no role, for instance,

$$
\frac{h_A'}{h_B'} = \frac{F(h_A, L^1)}{F(h_B, L^2)} > \frac{F(h_A, \bar{L})}{F(h_B, \bar{L})}. \tag{5}
$$

When school expenditures matter the inequality is reinforced if $E'(x)>0$, which is the empirically relevant case. In the Appendix I extend the model to overlapping generations and show how segregation slows down income convergence, or even causes divergence. The key mechanism can already be understood from the two-period setup: if $F$ has decreasing returns in $h$ alone but increasing returns in $(h, L)$ or $(h, L, E)$ over some range, inequality will increase from one generation to the next. Such local increasing returns, realized only through segregation, are what underlies the ghetto phenomenon.\(^{19}\)

There is, however, an important caveat to the notion that equilibrium segregation worsens inequality. Because the model keeps track of both human capital and financial assets, it brings to light a question which previous studies have generally overlooked: inequality in what, and at what price? In both the theoretical and empirical literatures,

\(^{18}\) Of course, this component of $t$ does not enter into communities' budget constraints. Yet another interpretation of $t_{px}(h, x, \rho)$ is that agents have pure "tastes" for the ethnic mix of their community, as in Schelling (1978) or Miyao (1978).

\(^{19}\) They are at the core of more complex models such as Bénabou (1993), Durlauf (1996a) or Lundberg and Startz (1994).
the standard approach is to track across families the earnings, education levels or professional status of successive generations, as done in (5). But it is not obvious that this is the appropriate measure of inequality. Better community quality and better schools come at a price, namely a higher rent \( \rho^1 > \rho^2 \) and/or tax bill (if \( t_p > 0 \) or \( t_p < 0 \)). Taking into account not only the payoff but also the costs of educational investments may lead to a very different answer.

**Proposition 6.** Equilibrium stratification accentuates disparities in education and earnings. It need not, however, increase inequalities in children’s total wealth (human capital plus bequest), or more generally between rich and poor families’ utility levels. Specifically, let \( V_{hs} \geq 0 \) and \( V_{hp} \geq 0 \), ensuring that the sorting condition (3) holds:

1. If \( n < 1/2 \) then \( U^1(h_A) - U^2(h_B) > \bar{U}(h_A) - \bar{U}(h_B) \), where the left- and right-hand sides correspond respectively to the stratified and integrated equilibrium. But if \( n > 1/2 \) the inequality is reversed, and if \( n = 1/2 \) the ranking is ambiguous.
2. The same is true for the comparison in children’s total wealth \( c' + h' \), when preferences are linear.

This result is proved in the Appendix. It raises the old question of whether child or family inequality is the relevant concept (Stiglitz (1973)), but also some new and more practical ones. What is the cost to the rich of seceding from the poor? Who owns the assets, property rights or legal rights which allow stratification to occur and earn the rents which it creates? Answering this question means here identifying the recipients of the capital gains and losses which stratification generates on land in communities 1 and 2 respectively. Proposition 6 is derived under the “neutral” assumption that those are distributed evenly: all housing initially belongs to absentee landowners, or alternatively to the city’s residents but with no correlation between a person’s type and the location of his property (\( \nu_i = \eta_i = 1/2 \), \( i = A, B \)). A more realistic assumption might be that \( A \) individuals own all the land: \( \nu_A = \eta_A = 1/2n \), \( \nu_B = \eta_B = 0 \). Yet even then, stratification need not increase inequality in total wealth: capital gains in \( C^1 \) may or may not offset capital losses in \( C^2 \). What is required is that the rich capture a larger share of the rents which their exodus creates, and/or that the poor be left holding a larger share of the corresponding losses: \( \nu_A > \nu_A \), \( \eta_B > \eta_A \).

These results point to the role played in the generation and persistence of inequality by certain collective practices and institutions which allow the rich to capture the benefits of their secession, by raising—even temporarily—the relative cost to the poor by joining or remaining in a community which is appreciating. Prime among these is racial discrimination, whether enforced by law or custom. As long as Blacks are simply not allowed to bid for land in the suburbs to which White families move, the latter can regroup without dissipating too much of the resulting rents on higher land values. When, later on, Blacks are allowed in, those who want to come must pay the full value of the community’s social capital. Although the education gap between their children and those of their white neighbours will close, the gap in total wealth will not. In fact, the inequality in wealth which occurs the moment de jure segregation is lifted and property values adjust can even be sufficient to sustain de facto economic segregation. Consider the specification of Section 2.3, with \( n = 1/2 \). Suppose Whites and Blacks have the same amount \( z \) of total non-land wealth, and each own their houses in \( C^1 \) and \( C^2 \) respectively. Proposition 4 implies that the two groups remain separated even once free mobility is allowed, through a rent differential \( \rho^1 - \rho^2 \) such that \( \bar{u}(z + \rho^1 - \rho^2) < w(F(h_A)) - w(F(h_B)) < \bar{u}(z + \rho^2 - \rho^1) \). In recent years overt discrimination has been replaced, as the main device for keeping out
the poor, by the right given to a town’s residents to impose zoning laws which essentially amount to minimum wealth requirements.20

Proposition 6 also has implications for empirical studies which attempt to measure the role of peer effects, neighbourhood spillovers or school quality in the intergenerational persistence of inequality. These should try to take into account not only earnings but also financial assets: how much of the human wealth afforded to a child by a better educational environment is reflected in a reduction of the bequest he receives? Answering this question could also help determine whether the peer effects found by Borjas (1992) among members of the same ethnic group truly represent “ethnic capital”, or whether they actually operate through local spillovers in ethnically segregated neighbourhoods. While the former are inherited “for free” through birth, the latter are subject to choice and carry a price in the form of rent or tax differentials.

4. STRATIFICATION AND EFFICIENCY: THE BASIC CASE

I now turn from distributional issues to the question of whether it is efficient for different social classes to sort into “separate and unequal” neighbourhoods and school systems. I initially abstract from local school funding and taxes, so as to highlight the role of pure social spillovers. Decentralized education expenditures are incorporated in the next section, making their additional contribution easily identifiable. The efficiency criterion I use is aggregate surplus: are the productivity gains of rich communities greater or smaller than the losses of poor ones?21 The surplus generated by a community with a fraction $x$ of rich households is $S(x)/2,$ where

\[ S(x) = xF(h_A, L(x)) + (1 - x)F(h_B, L(x)). \] (6)

A social planner would allocate the two types of populations so as to maximize the productivity of the whole metropolitan area, $S(x) + S(2n - x),$ over $0 \leq x \leq \min \{2n, 1\}$. If $S$ is convex the optimum is at a corner and coincides with the stratified market equilibrium. On the other hand, if $S$ is concave there are decreasing social returns to the concentration of human capital, and the optimum is interior and symmetric: $S'(x) = S'(2n - x),$ so $x = n.$22

20. See for instance Wheaton (1993). Equivalently, Durlauf (1996b) assumes that a neighbourhood’s land price adjusts to the equilibrium value which keeps out the marginally poorer family, only once all richer families have moved in and bought the land at cost. The opposite of zoning, in a sense, is rent control: imposing $\rho^* = \rho^3$ undermines the sorting mechanism described in Proposition 1. To the extent that this paper identifies adverse effects of stratification on intergenerational mobility and aggregate productivity, some of the arguments for rent control can be rationalized. On the other hand rent control is notoriously difficult to enforce and adversely affects the supply of housing.

21. Surplus-maximization is here equivalent to constrained Pareto-optimality, provided the utility from $h'$ in $U(c', h')$ derives only from the income which education will generate, and the latter can be costlessly transferred across families. No restrictions on the form of $U(\cdot)$ or the features of the credit market are then required. In the overlapping generations version of the model studied in the Appendix I show how economy-wide complementarities can extend the results of Pareto-inefficiency to the case where redistributions are not feasible, and how welfare comparisons might differ between the short and long run.

22. The intuition is apparent when comparing the gains $S(x^*) - S(n)$ from stratification in $C^1$ with the losses $S(n) - S(x^*)$ in $C^2$. Naturally, if $S$ changes curvature partial segregation may be optimal. This leaves the main result unaffected, namely that the decentralized equilibrium is generally characterized by excessive segregation; see footnotes 29 and 30, which analyse a case where both the equilibrium and the optimum may occur at interior solution for $x$. Also, Becker and Murphy (1994) show in a framework closely related to ours that any stable equilibrium typically involves inefficiently high segregation.
To assess the efficiency of equilibrium let us therefore evaluate $S''$:  

$$S' = F(h_A, L) - F(h_B, L) + (xF_L(h_A, L) + (1-x)F_L(h_B, L))L';$$  

$$S'' = 2(F_L(h_A, L) - F_L(h_B, L))L' + (xF_{LL}(h_A, L) + (1-x)F_{LL}(h_B, L))(L')^2$$  

$$(7) + (xF_L(h_A, L) + (1-x)F_L(h_B, L))L''$$

where $L = L(x)$, etc. This expression reveals the key tradeoff between complementarity and curvature which governs efficiency. The first term represents the answer to the question: who benefits most from an increase in community quality? If $F_{hL} > 0$ it is the better-educated family, hence an efficiency gain from stratification. The second term measures whether an increase in $L$ is more valuable to the average resident when starting from a low or from a high level; if $F_{LL} < 0$ the marginal productivity of community quality is decreasing, implying an efficiency loss from stratification. The last term in (7) represents the answer to the question: where does a marginal well-educated family contribute most to raising the quality of its community? If $L'' > 0$ it is much less valuable in the rich community to which it moves than in the poor one which it leaves behind; hence another loss from segregation.  

Differential valuations of community composition constitute the simplest of all sorting mechanisms, by which agents locate solely on the basis of the private gains represented by the first term in (7); see Section 2.1. A social planner must take into account not only $F_{hL}$ but also the “concentration effects” $F_{LL}$ and $L''$, which could be much larger and have the opposite sign. More generally, wealth disparities and capital market imperfections also shape the equilibrium, even though they are unrelated to the productivity of education. For instance if $F_{LL} = L'' = 0$ and $F_{hL} < 0$, integration is optimal as $S'' < 0$; yet differentials in opportunity costs like those studied in Sections 2.2 and 2.3 can still cause $R_s = (U_1/U_2)(c, c', h) \cdot F_L(h, L) \cdot L'/P_d(h, L)$ to increase with $h$, resulting in a stratified city.

**Proposition 7.** Agents segregate or integrate depending on $R_{hs} \geq 0$, no matter how small. The equilibrium is inefficient whenever $S'' \cdot R_{hs} < 0$. The social loss can thus be very large compared to the private benefits shaping residential decisions ($|R_{hs}| \ll |S''|$).

This result is important, because it indicates that the typical pattern of city-suburb polarization is likely to be inefficient: the concentration of human wealth on one side and low skills on the other depresses the growth rate of metropolitan surplus. 24 Empirical work on the aggregate implications of community composition is still in its infancy (in contrast to the large literature on individual outcomes), but the available evidence appears supportive of the view that stratification is a negative-sum game. Rusk (1993) and Ledebur and Barnes (1992) show that U.S. metropolitan areas display a significant inverse correlation between economic segregation, measured by the ratio of suburban to central city mean incomes, and overall growth in per capita income and employment. In a recent econometric study, Cutler and Glaeser (1995) examine how a city’s degree of racial segregation affects the schooling, employment and single-parenthood outcomes of Blacks and Whites. Blacks do significantly worse in more segregated cities, whereas Whites outcomes are essentially

23. Interestingly, the results of Brooks-Gunn et al. (1993) suggest $F_{hL} > 0 \gg F_{LL}, L''$. Note that half of $2(F_L(h_A, L) - F_L(h_B, L))L'$ represents the gains accruing to a pair of $A$ and $B$ agents who trade places. The other half is external: a marginal rise in $L$ is more valuable, ceteris paribus, where there are more L-sensitive agents; see the expression for $S'$.

24. While the case $R_{hs} > 0 > S''$ seems most relevant, Proposition 7 also makes clear how inefficient mixing can occur.
unaffected. Closely related to the idea that stratification is detrimental to metropolitan performance is the view that suburbs cannot durably prosper at the expense of their central cities. Voith (1994) shows that the correlation between city and suburban growth, negative or insignificant during the 1960's, became strongly positive during the 1970's and 1980's. City growth, instrumented to avoid simultaneity, is found to be an important explanatory factor for suburban growth. Improving efficiency with respect to the market allocation requires some implicit or explicit form of price discrimination. One way in which this might happen is through competition among communities for the more desirable A types, through rent subsidies or other targeted benefits reflecting the wedge between their private and social marginal products. As is known from club theory, a competitive equilibrium would be Pareto-efficient but with non-trivial efficient community size it typically fails to exist (e.g. Scotchmer (1995)). Another problem, emphasized by Becker and Murphy (1994), occurs when discriminatory membership prices are not feasible; rich communities might then choose to over-consume certain "exclusive" public goods for which the poor have low willingness to pay. Efficiency thus generally requires city-wide coordination, whether through a unified local government or a monopolistic developer. If individuals' types are observable, this planner can increase aggregate surplus by using direct, differentiated taxes and subsidies to reallocate families across communities. When types are not observable or discrimination is not allowed, the same outcome can be obtained if there is some local public good which A's and B's value differently. As shown in Section 6.3 with education expenditures, the planner can then use it to induce families to self-select into the surplus-maximizing residential pattern. Whether a Pareto-improvement can be achieved, on the other hand, depends on the planner's ability to redistribute the net gains from integration. If he is able to extract from B families enough of the gains to compensate the A's for their losses, everyone can be made better off. But if poorer parents can not be made to pay up ex ante due to credit constraints (which the planner is in no better position than the market to remedy), and if their children cannot be held responsible for the debt (whether due to legal constraints or because, once educated, they move to another city to produce), the market allocation is constrained Pareto-efficient: total surplus can be increased, but only with a net redistribution from poorer to richer families. Section 6.3 briefly discusses some of the political economy implications of such a situation.

5. SCHOOL FINANCE

I now explicitly incorporate local school expenditures and taxes into the analysis. I show how they represent an additional segregating force, then examine the consequences for

25. See Footnote 3 for the correlations which I computed from the data in Rusk (1993) and in Ledebur and Barnes (1992). While the former cover only 14 MSA's the latter cover the 85 largest ones. Cutler and Glaeser study the 209 MSA's which have at least 100,000 people and 10,000 Blacks. In addition to controlling for standard background variables (e.g. parental education), they instrument for the endogeneity of location choice by using a variety of institutional and topographical variables which affect opportunities for segregation.

26. Bénabou (1993, 1996) and Cooper (1992) provide models where production complementarities tie together the economic fate of city and suburban residents, so that stratification can ultimately depress every group's income. Proposition 12 in Appendix A obtains a related "self-defeating secession" result in the much simpler setting developed in this paper. In addition to Voith (1994), whose analysis cover all MSA's, Savitch et al. (1993) also provide evidence of city-suburb interdependence.

27. The basic intuition can be obtained from our two-community model when the efficient allocation is asymmetric. This cannot be supported as a (pure strategy) competitive equilibrium, because the two communities would yield different levels of rents for their landowners or developers. On the related issue of school competition with type-specific student vouchers, see Eden (1992) and Eppe and Romano (1993).
efficiency and inequality. These results will also form the basis for the policy evaluation conducted in the next section.

5.1. The choice of education expenditures

From here on I focus on the most simple case where education is a pure investment good, capital markets are frictionless, and local schools are financed by lump-sum or property taxes (with inelastic lot size, these are equivalent). Extensions of the results to income tax financing or wealth effects due to tastes or credit constraints can be found in the working paper version of this article. With the present assumptions each family’s tax bill equals per-student expenditures \( E \), which are chosen by the town’s residents independently of its current financial resources. In a community with composition \((x, 1-x)\) and spillover quality \( L \), let this level of school funding be determined as:

\[
E^*(x, L) = \text{arg max}_E \{xF(h_A, L, E) + (1-x)F(h_B, L, E) - (1+r)E\}. \tag{8}
\]

This decision rule reflects in a continuous manner the proportions and preferences of local residents (unlike majority voting). The underlying political mechanism can be thought of as unrestricted vote-trading or delegation to a municipal planner. Indeed, (8) yields the efficient level of investment, conditional on community composition. This allows me to highlight the issue of inter- rather than intra-community efficiency.\(^{26}\) Assuming \( F_E < 0 \), we have:

\[
\begin{align*}
\frac{\partial E^*(x, L)}{\partial x} & = \frac{F^A_E - F^B_E}{-xF^A_E - (1-x)F^B_E} - F^A_E \cdot \Delta h \\
\frac{\partial E^*(x, L)}{\partial L} & = \frac{xF^A_E + (1-x)F^B_E}{-xF^A_E - (1-x)F^B_E} - F^B_E
\end{align*}
\]

where \( F^*_E \) stands for \( F^*_E(h_i, L, E^*(L, x)) \), etc. The approximations hold when \( \Delta h/\bar{h} \) is small, with all derivatives evaluated at \((\bar{h}, L(n), E(n))\). Although this condition is not required for any of the results, it simplifies notation and makes intuitions more transparent. The school budget \( E(x) = E^*(x, L(x)) \) varies with community composition as:

\[
E'(x) = \frac{F^A_E - F^B_E + (xF^A_E + (1-x)F^B_E)L'(x)}{-xF^A_E - (1-x)F^B_E} - F^A_E \cdot \Delta h + F^B_E \cdot L'. \tag{9}
\]

When \( E \) and \( L \) are complements, rich communities may thus spend more on their students than poor ones even though, were they placed in the same environment, students from disadvantaged backgrounds would have a higher marginal product of education expenditures than those from richer families.

5.2. Stratification

Having determined \( E(x) \) and \( t(h, \rho, x) = E(x) \), we can now replace them in the bid-rent differential \( R_{h\rho}(h, \rho, x) = (F_{hL'} + F_{hE}E')/(1+r) \), or equivalently in

\[
\text{SEG} = (1+r)(R_x(h_A, \rho, x) - R_x(h_B, \rho, x)) = (F^A_L - F^B_L)L' + (F^A_E - F^B_E)E'. \tag{10}
\]

\(^{26}\) In any case, the most relevant scenario is where the equilibrium involves a high degree of stratification. As stressed by Tiebout (1956) there is then near-unanimity in each community, so that all political mechanisms have the same outcome.
This expression defines the net private gain, in terms of children’s human capital, to a marginal pair of \( A \) and \( B \) families who switch places during the stratification process. Given (9) it takes the form:

\[
SEG = \left( F^A_L - F^B_L + (F^A_E - F^B_E) \frac{x \cdot F^A_{EL} + (1 - x) \cdot F^B_{EL}}{-x \cdot F^A_{EE} - (1 - x) \cdot F^B_{EE}} \right) L' + \frac{(F^A_E - F^B_E)^2}{-x \cdot F^A_{EE} - (1 - x) \cdot F^B_{EE}} h.
\]

\[
\approx \left( \frac{F^A_{hL} + F^B_{hL} \cdot F^A_{EL}}{F^B_{EE}} \right) L' + \frac{(F^B_{EE})^2 \Delta h}{-F^B_{EE}} \Delta h. \tag{11}
\]

The term multiplying \( L' \) represents the \textit{complementarity}, both direct and indirect through \( E \), between parental education and community quality. The other term, always positive, captures everyone’s incentive to move to a jurisdiction where agents with \textit{similar preferences}, being more numerous, have greater weight in the setting of policy. The first implication of (11) is that sorting always occurs in the absence of local externalities (\( F_L = 0 \)). The second one is that local funding of education may \textit{induce} segregation which would otherwise not have occurred (\( F_{hL} < 0 < SEG \)). This is probably an important contributing factor to the greater residential segmentation found in the United States than in other industrialized countries.\(^{29}\)

5.3. \textit{Efficiency}

Recall that \( E(x) \) is chosen optimally, given \( x \). The efficient (surplus-maximizing) residential allocation is thus again determined by the concavity of net community output, now defined as

\[
S(x) \equivxF(h_A, L(x), E(x)) + (1 - x)F(h_B, L(x), E(x)) - (1 + r)E(x). \tag{12}
\]

From (8) and the envelope theorem, \( S' = F^A - F^B + (xF^A_L + (1 - x)F^B_L)L' \). Using (9) then yields:

\[
S'' = 2(F^A_L - F^B_L) L' + (xF^A_L + (1 - x)F^B_B)(L')^2 + (xF^A_L + (1 - x)F^B_L) L''
- (xF^A_E + (1 - x)F^B_E)(E')^2. \tag{13}
\]

The first three terms are due to local spillovers, which affect education both directly and through their interaction with \( E \). Their interpretation is the same as in (7). The last term, always positive, arises from the optimal adjustment of expenditures to community composition \( x \) and quality \( L \). In the absence of local spillovers (and other market imperfections), the tendency to segregate in response to differing preferences over education promotes efficiency (Tiebout (1956)). In general, however, it may have the opposite effect:

\begin{center}
\textbf{Proposition 8.} The decentralization of school funding can trigger stratification which is inefficient: it suffices that \( F_{hL} < 0 < SEG \) and \( S'' < 0 \).
\end{center}

\(^{29}\) De Bartolome (1990) considers the case where \( F_{hL} < 0 = F_{EL} < F_{EE} \) and these two opposing effects result in an asymmetric but incompletely stratified equilibrium. Although his political mechanism and preferences are somewhat different, this can be understood as a solution \( n < x < \min \{ 2n, 1 \} \) to the equation \( \int_0^{2n} SEG(x) dx = F(h_A, L(x)), E(x) - F(h_B, L(x'), E(x')) = 0 \), which defines interior equilibria in our model.
Indeed, we can rewrite:

\[ S'' = SEG + (F^A_L - F^B_L)L' + (xF^A_{LL} + (1-x)F^B_{LL})(L')^2 + (xF^A_L + (1-x)F^B_L)L'' + (xF^A_{LE} + (1-x)F^B_{LE})L'E'. \]  

(14)

The term \( SEG \) represents the net private gains from stratification; the elements which determine its sign were discussed in (11). All others terms constitute wedges between social and private values. The first three of those involve only the local spillover, and were discussed in Section 4. The last one involves the interaction of \( E \) and \( L \). As families congregate into more homogeneous groups, they ignore their own effect on community quality \( L \), including the resulting impact on the productivity of \( E \). Assuming, realistically, that \( E'(x) > 0 \), the effect is favourable if \( L \) and \( E \) are complements: a planner would also like, ceteris paribus, for \( L \) to be high in the community where \( E \) is high. It is detrimental if they are substitutes.\(^{30}\)

6. POLICY IMPLICATIONS

6.1. The debate

In the U.S., court decisions are compelling an increasing number of states to close the wide gaps which exist between the school budgets of different communities. The underlying view is that these disparities, stemming from the lower property tax base of poorer towns, do not give equal starting chances to their young. Equalizing transfers or state control of education funding are then needed to reduce inequality as well as possible inefficiencies.\(^{31}\)

There is also a contrarian view, of which a recent article on Kansas City in The Economist (1993) is representative. Under a 1986 court order to remedy racial segregation in its schools, the city "scraped its existing school system... and replaced it with the best... money could buy," at the cost of an extra $1.3 billion, or $36,111 per student over the normal school budget. Three-quarters of the cost were born by the state, resulting in severe cutbacks in other districts.

"So far, however, all this lavish expenditure has produced few of the desired results... The racial balance in the schools is the same today as it was when the order was issued. The past six years have seen no improvement whatsoever in children’s scores in standardized tests of reading and math... Pupils in elementary schools which have not been turned into magnet schools regularly outperform pupils in generously funded magnet schools... The drop-out rate has risen every year, without fail, since the decree was handed down, and now stands at a disgraceful 60%.”

\(^{30}\) Proposition 8 readily extends to the case where the equilibrium is only partially stratified. As seen in footnote 29, this corresponds to a solution to \( \int_{x_0}^{x_1} SEG(x)dx = 0 \). Thus whenever the integral of the last three terms in (14) is negative, \( S'(x^*) - S'(2n-x^*) = \int_{x_0}^{x_1} S''(x)dx < 0 \), violating optimality. Note that this does not require global concavity of \( S \), so the planner’s solution could still be asymmetric. Such is indeed the case in De Bartolome (1990), where inefficiency follows from the assumptions \( F_{ML} < 0, F_{LL} < 0 \) and \( L' = F_{EL} = 0 \). Finally, Proposition 8 can also be extended to the general model of Section 1. It suffices that the sign of \( R_{nx} \) switch from positive to negative once \( E', t_n, t_p \) are replaced by zero in (4), and that either \( F_{ML}, F_{LL} \) or \( L' \) be sufficiently negative to make the appropriate welfare function concave in \( x \).

\(^{31}\) The most recent example is Michigan, where voters recently approved a proposal to replace local property taxes by an increase in the state sales tax as the primary source of funding for schools. For a review of the legal debate over funding see Henke (1986). For quantitative assessments of the impact of reforms and litigation on the level and allocation of funds see Manwaring and Shefrin (1995) and Murray, Evans and Schwab (1995).
Not surprisingly, Kansas City school officials vigorously dispute this bleak assessment. But other pieces of evidence appear consistent with the view that lack of financial resources is not the main impediment faced by mediocre or failing schools. Expenditures per student are higher in some urban school districts than in neighbouring suburban or rural ones, thanks to a larger industrial tax base or to state subsidies. The latter nonetheless achieve better test scores and lower dropout rates. Reviewing the empirical literature on education production functions, Hanushek (1986) observes that most studies fail to demonstrate a significant link between school expenditures and student achievement. Perhaps most importantly, the increased centralization of education funding experienced by several states (starting with the 1976 California Supreme Court “Serrano” decision) appears to have significantly reduced spending inequalities, but not achievement inequalities, as measured from SAT scores (Downes (1992)). Moreover, it seems to have been accompanied by a concomitant decline in those states’ average test scores (Peltzman (1993), Hoxby (1995)).

Opponents of the “lack of resources” hypothesis often point instead to culprits such as teachers’ unions, the education bureaucracy, or the disincentive effects of the welfare system. I shall advance a very different explanation of the evidence, based on complementarities between purchased and social inputs in education. I will show for instance how equalizing school expenditures can fail to reduce segregation and lead instead to a decline in overall academic performance, hurting the rich more than it helps the poor. Yet policies designed to affect not just the allocation of funds or even students, but that of families, could still improve both equity and efficiency.

6.2. Equalizing school budgets

I continue to model equilibrium prior to state intervention using the simple specification of education expenditures introduced in (8): \( E' = E^*(x', L(x')) \), where \( x' \) is the fraction of high human capital families in community \( j \). Thus I deliberately abstract from financing constraints at the individual or community level. This is not because I consider them unimportant; they played an essential role in my previous analysis of local and centralized school funding (Bénabou (1996)), as well as that of Fernandez and Rogerson (1994). What I seek to do here is to highlight the limits of redistributive policies, and explore whether there remains a role for the state even in the absence of such constraints.32

I first mention some ways in which attempts to equalize school budgets might be defeated at the local level. Clearly, outright redistributions of wealth leave \( E^1 \) and \( E^2 \) unaffected; only consumption adjusts. Mandating that \( E^1 \) be reduced and \( E^2 \) increased may induce creative accounting. A rich town might shift, say, athletic facilities or a library from the school budget to some other line item. A poor one might claim as education-related some expenditures which have mostly consumption value: nicer buildings, teacher and administrator salaries, etc. There is also evidence that rich communities find alternative ways of raising significant amounts of money: voluntary contributions by parents, school fairs, etc. (Henke (1986)). Finally, taxing \( E^1 \) and rebating the proceeds to schools in community 2 as a block grant does reduce spending in \( C^1 \) but changes nothing in \( C^2 \), where residents simply reduce their contributions to education by the amount of the transfer. As a result, total expenditures fall, as observed by Downes (1992) for California following Serrano, and so do test scores.

32. Credit constraints were also at the centre of earlier models where education was privately purchased, such as Loury (1981) or Glomm and Ravikumar (1992).
But let us now take as given that the state is successful in enforcing equalization, either through matching grants (Feldstein (1975)) or simply by centralizing funding. Indeed, the combined State and Federal share in elementary and secondary public school spending rose from 40% to 56% between 1960 and 1991, with a corresponding decline in reliance on local resources.\textsuperscript{33} We start from a stable equilibrium where the population is stratified ($SEG>0$ in (11)) and the richer town spends more on education ($E'(x)>0$ in (9)). After state intervention $E^1$ is reduced and $E^2$ increased, to some common value $\bar{E}$; thus $E'(x)$ is replaced by zero in (10), and mobility decisions are governed by $F_{hl}$.\textsuperscript{34} There are two cases to consider.

**Case 1. Segregation persists**

If $F_{hl}>0$, the composition of each community remains unchanged. More generally this could be due to any of the other stratifying forces discussed earlier, including some group’s preference for racial separation. Conditional on each $x'$ decentralized expenditures were set optimally, so state intervention necessarily reduces surplus. What is interesting is the way in which this occurs, illustrated on Figure 2(a) for the case of perfect sorting ($n=1/2$). In $C^1$, students’ human capital is reduced by $-\Delta h^1 = x'(F(h_A, L^1, E^1) - F(h_A, L^1, \bar{E})) + (1-x')(F(h_B, L^1, E^1) - F(h_B, L^1, \bar{E})) > (1+r)(E^1 - \bar{E})$, due to the concavity of the maximization problem (8). In $C^2$ the corresponding increase is $\Delta h^2 = x^2(F(h_A, L^2, \bar{E}) - F(h_A, L^2, E^2)) + (1-x^2)(F(h_B, L^2, \bar{E}) - F(h_B, L^2, E^2)) < (1+r)(\bar{E} - E^2)$, by the same argument.

**Proposition 9.** When $F_{hl}>0$, equalizing school budgets leaves community composition unchanged but reduces total surplus. Student achievement and subsequent earnings decline more in rich communities than they improve in poor ones (per dollar of cutback or subsidy). If state-wide expenditures are kept constant, $\bar{E} = (E^1 + E^2)/2$, average student performance declines.

So while income inequality is reduced, it is less by lifting up the poor than bringing down the rich. The intuition is that school expenditures are more productive in a human capital-rich community, due to complementarity either between $h$ and $E$, or more interestingly, between $E$ and $L$; see (9).\textsuperscript{35} Top-notch teachers, computers and other educational resources may not do much in a school plagued by discipline problems, the lack of motivation from role models in the community, peer pressure not to study, neighbourhoods, gangs, etc. Redistributing expenditures without simultaneously “redistributing” social capital is ineffective.

**Case 2. Segregation is undone**

If $F_{hl} \leq 0$ (and the other stratifying factors are not too strong), equalizing expenditures removes the incentive to segregate. The stable equilibrium then shifts from the stratified to the integrated configuration. Both purchased and non-purchased educational inputs, $E$

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33. If one combines public and private funding, the state and federal share grew from about 35% in 1960 to 41% in 1991. These data come from various issues of the Digest of Educational Statistics.

34. More generally, wealth differences, credit frictions and discrimination will matter as well. The role of $F_{hl}$ here is thus in large part to represent all the forces which shape residential equilibrium in the absence of local school funding.

35. Downes and Pogue (1993) find that communities with a higher fraction of students from disadvantaged backgrounds have higher per-student costs, for given levels of educational achievement. While one must be careful about identifying average and marginal costs, this suggests that school expenditures in these communities have a lower marginal product.
and $L$, are equalized across students, achieving a greater reduction in inequality than in the previous case. There remains the question of whether academic achievement, income and the surplus from education rise more or less for poor students than they fall for rich ones. The answer is easily obtained in the natural case where $\bar{E}$ is set at the optimal level for an integrated community, $\bar{E} = E^*(n, L(n))$. The net change in surplus is then $S(n) - (S(x^1) + S(2n-x^1))/2$, where $S$ is given by equation (13); hence the following results, proved in the Appendix.
Proposition 10. When $F_{hl} < 0$, equalizing school budgets leads to integration. If state-wide expenditures are chosen optimally, $\hat{E} = E^*(n, L(n))$, total surplus increases provided $S$ is concave. If, moreover, $\Delta h/h$ is small and $L$ is a CES index, surplus increases even when state-wide expenditures remain fixed, $\hat{E} = (E^1 + E^2)/2$. In that case average student achievement also rises.

The conditions under which $S$ is concave, so that integration improves both efficiency and equality, were analysed earlier. Figure 2(b) illustrates the case where equalizing school budgets leads to this desirable outcome. The levelling of $L$ which occurs in response to this policy shifts up the returns to educational expenditure on type $B$ students, much more than it shifts down the returns to expenditure on type $A$ students. These gains, which correspond to the vertical distance between the average of the two solid curves and that of the two dashed ones, are measured (to a second-order approximation) by the sum of the first three terms in (13). They swamp the distortions from constraining expenditures to be the same across communities and individuals; those losses, captured by the last term in (13), are reflected in the slope differential between the two dashed curves on the diagram.

6.3. Is segregation reversible?

Most likely, equalizing school budgets is not sufficient to undo segregation. Indeed many stratifying forces other than the decentralized provision of public goods were identified in Section 1; for convenience I continue to focus on the complementarity $F_{hl} > 0$. As long as $S$ is concave, however, I showed that the private benefits shaping residential decisions could be greatly inferior to the social costs, so that integration would still yield the efficiency gains described in Proposition 10 and illustrated on Figure 2(b). A state government concerned with inefficiency (if $S'' < 0$) or even just with equality of opportunity could, in principle, bring about integration by making poor communities sufficiently attractive to families with high human capital, and vice-versa. This could be done directly, through tax incentives or housing subsidies ($\hat{t}_{hx}(h, x) > 0$), or indirectly, through a contingent allocation of educational resources $\hat{E}(x)$:

\[(1 + r)\hat{R}_{hx} = F_{hl}(h, L(x), \hat{E}(x)) L'(x) + F_{hx}(h, L(x), \hat{E}(x)) \hat{E}'(x) - (1 + r)\hat{t}_{hx}(h, x) \leq 0\]  

for all $x$. The new stable equilibrium will be symmetric, with equal expenditures $\hat{E}(n)$ everywhere; $\hat{E}(n)$ can even be chosen optimally, to equal $E^*(n, L(n))$. But what this differential condition shows is that stratification is likely to be much harder to undo once it has occurred than it is to stop in its tracks early on. Due to the cumulative nature of the process, the amount of transfers required to induce the first few rich families to come back is considerably larger than what it would have taken to make them stay in the first place. For instance, when using only education expenditures, with $F_{hh} > 0$,

\[\hat{E}(2n-x^1) - \hat{E}(x^1) \geq \hat{E}(n-\varepsilon) - \hat{E}(n+\varepsilon),\]  

where $\varepsilon$ is small. Large transfers between the school budgets of rich and poor towns, even if only temporary, may not be feasible. First, they could be precluded by the government’s limited tax-raising and borrowing ability. This is a simple but general point about how a

36. The use of $E$ (which does not require family characteristics to be observable) was first considered by De Bartolome (1990), who showed how it could even deliver a Pareto improvement.
planner facing an upward-sloping opportunity cost of funds schedule can bring the economy back to its efficient equilibrium only if it has not strayed too far from it. Second, the fact that moving is costly creates a time-consistency problem: well-educated families may rightly wonder whether subsidies will still be forthcoming once they have returned to the central city. Third, as discussed in Section 4 it may not be possible to make poor families pay for much of the benefits which their children will reap from integration. This sets a limit on the degree of initial segregation consistent with rich families' being adequately compensated for their losses, a precondition for them to support the policy. Equation (16) then suggests that both the symmetric and the stratified allocations could be stable politico-economic equilibria: the first supported by a wide consensus to correct any small deviations from \( x^1 = x^2 = n \), the second by the opposition of more wealthy households to the significant redistribution required to take the first step towards integration. This idea of history-dependence is formally pursued in Bénabou (1995), where I show how multiple steady-states arise from the constraints which political equilibrium puts on desegregation policy, school finance reform, and surplus-increasing policies more generally.

This kind of irreversibility may explain why magnet schools and similar programmes recently implemented in some U.S. cities are not very successful at fostering racial and economic desegregation of schools. The scope of redistribution required for such policies to be effective—advocates speak of a "Marshall Plan" for the inner cities—seems to be politically unacceptable, economically impractical, or both. In metropolitan areas which are new or have retained a balanced urban–suburban composition, on the other hand, modest policy interventions seem to be effective; see Rusk (1993). The point is perhaps most relevant for European countries where despite a centralized, fairly egalitarian allocation of school resources, economic and racial stratification appears to be on the rise, with ghettos emerging in or at the periphery of major cities. If action is to be taken, it should be before polarization has reached the point where large gains and losses have become locked in.

7. CONCLUSION

The model developed in this paper links together important issues of local public finance, income distribution and productivity growth. Five main conclusions emerge. First, minor differences in education technologies, preferences, or wealth can lead to a high degree of stratification. Imperfect capital markets are not necessary but will compound these other sources. Second, stratification makes inequality in education and income more persistent across generations. Whether or not the same is true of inequality in total wealth depends on the ability of the rich to appropriate the rents created by their secession. Third, the typical pattern of city–suburb polarization is likely to be very inefficient, especially in the long run. Fourth, the effects of school finance reform depend critically on how purchased and social inputs interact in the education production function and in the induced mobility decisions of parents. When a state-wide equalization of expenditures is insufficient to reduce stratification, it may improve educational achievement in poor communities much less than it lowers it in richer ones. Yet policies designed to affect not just the allocation of funds or even students, but that of families, could still improve both equity and efficiency. Fifth, because of the cumulative nature of the stratification process and its effects on the distribution of human and financial wealth, it is likely to be much harder to reverse once it has run its course than to arrest at an early stage.
A. Overlapping generations

I extend here the analysis to a dynamic setting, demonstrating how stratification leads to ghettos and how these can in turn adversely affect overall growth. Let the two periods considered until now represent any two overlapping generations. For tractability the distribution of human wealth must be the only state variable, determining both residential equilibrium and the relevant measure of inequality. The first condition holds if utility is linear ($U_1/U_2=1$ more generally) and capital markets are perfect, or alternatively if there are no financial bequests: $\omega(h), \gamma(h)$ represent labour income in the two periods of life, and all of $\gamma$ is consumed in old age. As discussed in Section 3, the second condition requires that either earnings inequality be the criterion of interest, or that the

\[ h_{t+1} \]

\[ h^* \]

\[ f \]

\[ h_i \]

\[ h_{i+1} = f(h_i, h_i^0) \]

\[ \theta \]

\[ 1 \]

\[ F(h, L_t) \]

\[ F(h, L_1) \]

\[ F(h, L_0) \]

\[ L_t \]

\[ h_{t+1} \]

\[ h^* \]

\[ f \]

\[ h_i \]

\[ h_{t+1} = f(h_i, L_t), L_t = (h_i^0)^{1/2}(h_i^0)^{1/2} \]
first generation of A's who secede capture the whole present value of rents created in C, leaving the B's holding the less valuable land in C. With all housing owner-occupied, human capital inequality translates directly into inequality of total wealth or utility.

For simplicity I abstract from purchased local public goods E, so that $h' = F(h, L)$. Finally, let $n = 1/2$; thus whether integrated or segregated, the two groups always remain homogeneous. Also, in either case, if $h_{A,t} > h_{B,t}$, the same is true in period $t+1$. Therefore if the sorting condition (3) is satisfied at all $(h, p, x)$, rich and poor segregate in every generation. The resulting impact on persistence is most readily apparent when $F(h, L) = \Theta h^\alpha L^\beta$, with $-\alpha + \beta \leq 1 \leq \theta$. In the integrated equilibrium, inequality converges to zero at the rate $1 - \alpha: \chi_{t+1} = \log (h_{A,t+1}/h_{B,t+1}) = \alpha \chi_t$. In the stratified equilibrium, initial conditions vanish more slowly and perhaps even not at all: $\chi_{t+1} = \log (h_{A,t+1}/h_{B,t+1}) = (\alpha + \beta) \chi_t$. Disparities can even be magnified if $F(h, L)$ has decreasing returns in $h$ alone, but increasing returns in $(h, L)$ jointly over some range. Suppose for instance that local spillovers involve a threshold effect:

$$F(h, L) = \Theta h^\alpha \max \{L - f, 0\}^{1-\alpha}$$ (A1)

where $f > 0$ and $L(x) = (h_d)^{\alpha}(h_b)^{1-\alpha}$ is defined as the geometric average. All human capital magnitudes $h, L, F$, etc., are measured here as deviations from some fixed, basic level $h_0 > 0$; thus $h' = 0$ is not to be taken literally. When the two classes are segregated, $h_{A,t+1} = F(h_{A,t}, h_{B,t})$, $t = A, B$. When they remain mixed, $h_{A,t+1} = F(h_{A,t}, L_t)$; given (A1) and the definition of $L$, this implies $L_{t+1} = F(L_t, L_t)$. Figure 3 illustrates these dynamics, which diverge from the fixed point $h^* = F(h^*, h^*) \equiv f/(1 - \theta^{1/(1-\alpha)})$.

**Proposition 11.** Let human capital accumulation be given by $h_{A,t+1} = F(h_{A,t}, L_t)$ as defined in (A1), and let $f < h_0 < h^* < L_0$. Under integration, inequality converges to zero and both $h_{A,t}$ and $h_{B,t}$ grow asymptotically at the rate $\theta - 1 > 0$. Under segregation, the human capital of poor families converges to $h$ in finite time while that of rich families grows asymptotically at the rate $\theta - 1 > 0$.

Equilibrium stratification (due to $F_{h_0} > 0$ or the other factors discussed earlier) confines poorer families to a steady state of low productivity and income, which cohabitation with better-educated dynasties would have allowed them to escape. Two recent papers provide supporting evidence for the kind of non-convexity which underlies the ghettos phenomenon. Crane (1991) finds threshold effects in a way a neighbourhood's professional mix influences local rates of high-school dropout and teenage pregnancy. Using a regression tree analysis to detect non-linearities in income dynamics, Cooper, Durlauf and Johnson (1994) find that low levels of mean county income countervail stratification. Stratification may even hurt the productivity and income of rich dynasties, when in addition to local spillovers in education the different classes interact at the *global*, or economy-wide level. The simplest way in which city and suburban residents can be bound together is as complementary factors in the production of jobs or knowledge. Let for instance the more educated A types be managers, and the less educated B agents workers. A representative firm's output is $Y = (H_{A}^{(\sigma - 1)/\sigma} + H_{B}^{(\sigma - 1)/\sigma})^{\sigma/\sigma - 1}$, where $H_A = n_h A_h$ is managerial human capital and $H_B = (1 - n_h) h_B$ worker human capital. The resulting incomes, $y_d = H_d^{(\sigma - 1)/\sigma} (Y/n)^{1/\sigma}$ and $y_B = H_B^{(\sigma - 1)/\sigma} (Y/(1 - n))^1/\sigma$ are clearly interdependent. Alternatively, individuals could be linked through the funding of regional or national public goods, or through the kinds of economy-wide knowledge spillovers commonly found in endogenous growth models. These ties can all be captured by writing individual productivity as:

$$y(h) = h^\gamma H^{1-\gamma}$$ (A2)

where $H$ is some economy-wide index of human capital. All variables are still measured as deviations from $h$, which represents a basic level of skills and earnings. Like that of the local index $L$, the sensitivity of $H$ to heterogeneity is a key determinant of the costs and benefits of stratification. As shown by the above example

37. Estimating such a log-linear relationship for the descendants of immigrants to the U.S., Borjas (1992) finds that human capital spillovers slow down convergence markedly: $\alpha$ and $\beta$ both equal about 0.25 and are statistically significant.

38. Tamura (1996) and Galor and Tsiddon (1992) present models which combine increasing returns at the individual (country or family) level with an economy-wide technological externality, through which the poor eventually catch up with the rich. The implicit assumption is that the rich do not segregate (i.e. limit interactions to their own group), as they do when given the opportunity in our model.

39. Task assignments are here exogenous, but a model with occupational choice and competitive labour markets would have very similar implications; see Bénapou (1993). Alternatively, Tamura (1991) and Bénapou (1996) present models where increasing returns cause agents to specialize in imperfectly substitutable intermediate inputs.
of a CES production function, this sensitivity reflects the degree of complementarity among agents’ skill levels. Here I shall simply assume that \( H \), like \( L \), is a geometric average: \( H = (h_A)^\gamma (h_B)^{1-\gamma} \).

**Proposition 12.** Assume the same conditions as in Proposition 11, and the production technology (A2). Under integration inequality converges to zero, and all earnings grow asymptotically at the rate \( \theta - 1 \). Under segregation human capital inequality keeps rising, but all earnings converge to \( h \) in finite time.

Productivity growth slows down and eventually peters out, because the secession of “managerial” dynasties prevents “worker” dynasties from acquiring the skills necessary to keep up. The results of Glaeser, Scheinkman and Shleifer (1993) confirm the importance of a well-educated general labour force, in contrast to a high concentration of human capital on a small elite. In explaining a city’s growth they find the proportion of residents with 12 to 15 years of education (high school graduates and some college) to be both more important and statistically more significant than the proportion with higher education. Society may thus face an intertemporal tradeoff. Initially stratification benefits the wealthy more than it hurts the poor, so that overall growth increases (\( S^* > 0 \) in Section 4). But ultimately, the secession of the rich is self-defeating: excessive heterogeneity acts as a drag on the growth of every lineage’s productivity and income.  

**B. Proofs**

**Proof of Proposition 1.** For all \((\rho^1, x^1, \rho^2, x^2)\) let \( \Delta V(h|\rho^1, x^1, \rho^2, x^2) = V(h, \rho^1, x^1) - V(h, \rho^2, x^2) \). Standard reasoning shows that if \( R_{xx} > 0 \) (respectively, \( < 0 \)) everywhere, then \( \Delta V(h|\rho^1, x^1, \rho^2, x^2) = 0 \) implies \( \Delta V(h|\rho^1, x^1, \rho^2, x^2) \) for all \( h^* > h \) (respectively, \( h^* < h \)), with strict inequality if \( x^1 > x^2 \). It is then straightforward to show that an equilibrium is a solution to one of the following three conditions:

(a) \( \Delta V(h_A) = \Delta V(h_B) = 0 \). This requires \( x^1 = x^2 = n \), hence \( \rho^1 = \rho^2 \). Conversely, this symmetric allocation is always an equilibrium.

(b) \( \Delta V(h_A) > \Delta V(h_B) = 0 \). This requires \( x^1 = 0 \), \( x^2 = 2n \), hence \( 2n \leq 1 \). Clearly, one can always find \( \rho^1, \rho^2 \) such that \( \Delta V(h_A) = \rho^1 2n, \rho^2 = 0 \). When \( R_{xx} > 0 \) (respectively, \( < 0 \)), the other condition is automatically (respectively, never) satisfied, hence the allocation is (respectively, cannot be) an equilibrium.

(c) \( \Delta V(h_A) = 0 > \Delta V(h_B) \). This requires \((1 - x^1)/2 = 0, (1 - x^2)/2 = 1 - n \), hence \( 2n \geq 1 \). One can always find \( \rho^1, \rho^2 \) such that \( \Delta V(h_A) = \rho^1 1, \rho^2 2n - 1 = 0 \). The rest of the proof is identical to case (b).

This concludes the derivation of equilibria. I now turn to stability, which is defined very simply. An equilibrium \((\rho^1, x^1, \rho^2, x^2)\) is said to be stable if, for all small \( \epsilon \), moving \( \epsilon \) agents from \( c^2 \) to \( c^1 \) and the same number of \( B \) agents in the reverse direction makes \( B \) agents the highest bidders for land in \( c^1 \), and vice versa. Formally:

\[
\epsilon(\Delta V(h_A) = \rho^1, x^1 + \epsilon, \rho^2, x^2 - \epsilon) = \Delta V(h_B) \rho^1, x^1 + \epsilon, \rho^2, x^2 - \epsilon) < 0, \quad (B1)
\]

for any feasible \( \epsilon > 0 \). Clearly, the symmetric equilibrium is stable if and only if \( R_{xx} < 0 \). The stratified equilibrium, which exists only when \( R_{xx} > 0 \), is then stable since it satisfies \( \Delta V(h_A) - \Delta V(h_B) > 0 \), which is preserved by continuity for small perturbations \( \epsilon < 0 \) (which are the only feasible ones).

**Proof of Proposition 6.** (1) If \( n < 1/2 \), some \( B \) types remain in \( c^1 \) after stratification, so \( U^1(h_A) - U^1(h_B) = U^1(h_A) - U^1(h_B) = (V(h_A, \rho^1, x^1) - V(h_B, \rho^1, x^1) > V(h_A, \rho, x^1) - V(h_B, \rho, x^1) = V(h_A, \rho, x^1) - V(h_B, \rho, x^1) \), since \( x^1 > x^2 \) and \( \rho^1 \geq \rho \). The opposite reasoning applies when \( n > 1/2 \) and it is \( c^1 \) which remains mixed after agents move. Part (2) follows immediately. Note that similar results can be shown using only (3), rather than the stronger conditions \( V_{xx}, V_{xx} > 0 \), in a version of the model with a continuum of types and communities.

**Proof of Proposition 10.** Only the case where \( E = E^1 + E^2/2 \) remains to be proved. With expenditures fixed, the change in average student achievement equals the change in total surplus. This, in turn, equals the change in surplus achieved with the optimal \( E^*(n, L(n)) \), minus the distortion from choosing \( E \) instead. By the envelope theorem, this distortion is of second order in \( \delta = \delta = E^*(n, L(n)) = (E(x^1) + E(x^2))/2 - E((x^1 + x^2)/2) \). But \( \delta \) is itself of second order in \( \Delta h \), as can be seen from (9): when \( L \) is a CES with elasticity \( \epsilon \), \( L = L' \approx \Delta h \) and \( L' \approx (\Delta h)^2/2 \epsilon \); hence \( E'(x) \) is of order \( \Delta h \), and, differentiating (9), \( E''(x) \) is of order \( (\Delta h)^2 \). The distortion \( \delta^2 \) is

40. Making workers’ skills essential to production (by defining \( H \) as the geometric average, or \( Y \) as a Cobb–Douglas) is only a convenient shortcut. When education uses real resources (schools, R&D), as in the general model \( h = F(h, L, E) \), reductions in productivity feed back into each family’s human capital accumulation. Thus even when the elasticity of substitution \( \sigma \) between \( h_A \) and \( h_B \) in \( Y \) is greater than one, and even in the absence of threshold effects, stratification can lower the steady-state path of human capital and income to which rich families converge. See Bénabou (1996).
therefore negligible compared to the gains \( S(x) - \frac{1}{2}S(x^1) - \frac{1}{2}S(x^2) \) which are of second order in \( \Delta \), as shown by the expression (13) giving \( S^* \).

Acknowledgements. I am grateful for the helpful comments of Olivier Blanchard, Patrick Bolton, Peter Diamond, Dick Eckaus, Jonathan Gruber, Michael Kremer, Glenn Louy, Julio Rotemberg, Richard Romano, Richard Startz and Jean Tirole. I also thank Ian Jewitt and two referees for useful suggestions. Financial support from the NSF (SES92-09267) is gratefully acknowledged.

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