Theories of Heterogeneous Firms and Trade

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Abstract

This paper reviews the recent theoretical literature on heterogeneous firms and trade, which emphasizes firm selection into international markets and reallocations of resources across firms. We discuss the empirical challenges that motivated this research and its relationship to traditional trade theories. We examine the implications of firm heterogeneity for comparative advantage, market size, aggregate trade, the welfare gains from trade, and the relationship between trade and income distribution. While a number of studies examine the endogenous response of firm productivity to trade liberalization, modeling internal firm organization and the origins of firm heterogeneity remain interesting areas of ongoing research.

Keywords: Heterogeneous firms, international trade, within-industry reallocation, selection into exporting

JEL: F12, F16, L22

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1 Introduction

Theoretical research in international trade increasingly emphasizes the decisions of individual plants and firms in understanding the causes and consequences of aggregate trade. This new theoretical emphasis is a response to empirical studies using micro data, which revealed a number of features of producer behavior that were not well explained by pre-existing theories of international trade. There is substantial heterogeneity in productivity, size and other economic characteristics even within narrowly-defined industries. Participation in international trade is relatively rare and is associated with superior values of productivity and other measures of economic performance. Trade liberalization is accompanied by reallocations of resources within industries, which raise average industry productivity, as low productivity suppliers exit and high productivity suppliers expand to enter international markets. Trade liberalization is also accompanied by endogenous changes in firm productivity, which in turn influence within-industry resource allocation.

This paper reviews the recent theoretical literature on heterogeneous firms and trade inspired by these empirical findings. We begin in Section 2 by briefly summarizing the main empirical features of micro data on plants and firms that have influenced the development of theoretical research. Section 3 introduces the Melitz (2003) model, which accounts for these main features of the micro data, and has become a key benchmark framework in international trade for analyzing a whole host of issues.¹ The Melitz model can be embedded within the integrated equilibrium framework of traditional trade theory, as shown in Section 4. The resulting framework explains why countries export more in some industries rather than others (“inter-industry trade’’), why nonetheless there is two-way trade within industries (“intra-industry trade’’), and why in industries engaged in the two forms of trade some firms participate in international markets while many others do not.

Modeling firm heterogeneity leads to a number of new insights concerning the determinants and effects of international trade, including the role of market size (Section 5) and the well-known gravity equation relationship for international trade (Section 6). These theories have also been quantitatively successful in explaining patterns of aggregate and disaggregate trade, as reviewed in Section 7.

While theories of firm heterogeneity and trade stress reallocations of resources across firms, the benchmark model of firm heterogeneity features a frictionless labor market. As a result, all workers are employed for a common wage and are affected symmetrically by the opening of trade. More recent research has emphasized labor market imperfections, such as search frictions, and the way in which these can influence the income distributional effects of trade liberalization across workers, as discussed in Section 8.

¹While we focus on the Melitz model and other related approaches in the light of its theoretical influence and empirical success, alternative approaches to modeling firm heterogeneity and trade include Bernard et al. (2003) and Yeaple (2005).
Although models of firm heterogeneity and trade often treat firm productivity as fixed and focus on selection and reallocation across firms, a number of studies have modeled endogenous firm productivity and organization, as discussed in Section 9. This line of research has considered several dimensions of firm decisions, including the vertical integration decision, the choice of product mix, investments in new technologies, and adjustments to the skill composition of the workforce. While the theoretical literature on heterogeneous firms and trade has advanced rapidly in breadth and depth, there remain areas that are relatively unexplored, and Section 11 offers some concluding comments about possible areas for further research.

2 Empirical Challenges

As micro data on plants and firms became increasingly available from the late 1980s and early 1990s onwards, a number of empirical challenges for existing theories of international trade began to emerge. One of the most striking features of these micro data is the tremendous heterogeneity in productivity, size and other economic characteristics, even within narrowly-defined industries. In influential work, Bernard & Jensen (1995, 1999) showed that this heterogeneity is systematically related to trade participation. Within an industry, some firms export while many others do not and, even among exporters, the fraction of shipments exported is often small. Exporters are larger, more productive, and pay higher wages than other firms within the same industry. Additionally, exporters are observed in both net exporting and net importing industries, although the fraction of exporting firms and the fraction of exported output vary across industries with correlates of comparative advantage.

None of these features of the data are well explained by existing theories of international trade, with their assumption of a representative firm within industries. In comparative-advantage-based models, such as the Heckscher-Ohlin model, the emphasis is on net trade across industries (“inter-industry trade”), and the assumptions of perfect competition and constant returns to scale often imply that firm size is indeterminate. In variety-based models, such as Krugman (1980), firms specialize in distinct horizontally-differentiated varieties and there is two-way trade within industries (“intra-industry trade”), but consumer love of variety typically implies that all firms export.

Further evidence relating firm heterogeneity and trade participation comes from empirical studies of trade

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2For a more detailed discussion of this empirical literature, see Bernard, Jensen, Redding & Schott (2007). For complementary theoretical reviews of the literatures on firms, organizations and trade, see Antrás & Rossi-Hansberg (2009) and Helpman (2006).

3The finding that exporters are more productive than non-exporters within industries raises the question of the direction of causality between exporting and productivity. While there is substantial evidence of selection into exporting, there is less evidence of “learning by exporting.” This literature is reviewed in Lopez (2005) and, more recently, a number of papers have provided evidence that exporting enhances incentives for technology adoption, as discussed in Section 9.
liberalization episodes. While traditional trade theories emphasize reallocation across industries, much of the observed reallocation in the aftermath of trade liberalization is found to occur across firms within narrowly-defined industries (e.g. Levinsohn 1999). Trade liberalizations are accompanied by the exit of low productivity plants and firms and increases in aggregate industry productivity through the resulting changes in industry composition. While much of the evidence of these intra-industry relocations comes from studies of large-scale trade liberalizations in developing countries (e.g. Tybout & Westbrook 1991, Pavcnik 2003), similar results hold for developed countries (e.g. Bernard, Jensen & Schott 2006, Trefler 2004).

Another related feature of micro data is ongoing reallocation and selection even in the absence of trade liberalization or other large-scale changes. In U.S. Census Data, around one third of plants enter or exit every five years, and there are systematic differences in productivity, size and other economic characteristics between exitors and survivors (see in particular Dunne et al. 1989). Similarly, gross job creation and destruction at the plant level are large relative to net changes in industry employment, and these rates of job creation and destruction are positively correlated across industries (see in particular Davis & Haltiwanger 1992). None of these features are well explained by existing theories of international trade, with their abstraction from firm and industry dynamics.

3 The Melitz Model

The Melitz (2003) model addresses the above empirical challenges by combining a model of industry equilibrium featuring heterogeneous firm productivity, as in Jovanovic (1982) and Hopenhayn (1990), with a model of trade based on love of variety preferences and increasing returns to scale, as in Krugman (1980).

3.1 Preferences and Endowments

The world consists of many countries, such that each country trades with $n \geq 1$ foreign countries. We begin by considering the case of symmetric countries, before discussing in a later section country asymmetries. Labor is the sole factor of production in inelastic supply $L$ for each country and is immobile across countries.

Consumer preferences are defined over consumption of a continuum of horizontally-differentiated varieties within an industry. Preferences take the Constant Elasticity of Substitution (CES) or Dixit & Stiglitz (1977) form:

$$ C = \left[ \int_{\omega \in \Omega} q(\omega)^\rho d\omega \right]^{\frac{1}{\rho}}, \quad 0 < \rho < 1, \quad (1) $$

Derivations of expressions in this and subsequent sections are contained in a web appendix.
where $\omega$ indexes varieties, $\Omega$ is the (endogenous) set of varieties, and the price index dual to (1) is:

$$P = \left[ \int_{\omega \in \Omega} p(\omega)^{1-\sigma} d\omega \right]^{\frac{1}{1-\sigma}}, \quad \sigma = \frac{1}{1-\rho} > 1,$$

where $\sigma$ corresponds to the elasticity of substitution between varieties.

The model can be interpreted as capturing an industry within an economy. The assumption of CES preferences implies a strong “love of variety”: there is diminishing marginal utility from the consumption of any given variety; utility is increasing in the measure of varieties consumed; and the marginal utility from consumption of any given variety approaches infinity as consumption approaches zero. Given these preferences, the revenue for a variety supplied to the domestic market is:

$$r_d(\omega) = R \left( \frac{p_d(\omega)}{P} \right)^{1-\sigma}, \quad (2)$$

where $p_d(\omega)$ is the price of variety $\omega$ in the domestic market; $R$ denotes aggregate revenue, which equals aggregate income, which equals aggregate expenditure; the price index $P$ summarizes the prices of competing varieties.

### 3.2 Production Technology

There is a competitive fringe of potential entrants that can enter by paying a sunk entry cost of $f_e$ units of labor. Potential entrants face uncertainty about their productivity in the industry. Once the sunk entry cost is paid, a firm draws its productivity $\varphi$ from a fixed distribution, $g(\varphi)$. As firms with the same productivity behave symmetrically, we index firms from now on by $\varphi$ alone. Productivity remains fixed after entry, but firms face a constant exogenous probability of death $\delta$, which induces steady-state entry and exit of firms in the model. The assumption that the probability of firm death is uncorrelated with firm productivity is strong. However, the model still captures empirical findings that exiting firms are on average of lower productivity than surviving firms, because, among the cohort of entering firms each period, those that draw low productivity exit immediately as discussed below.

The market structure is monopolistic competition. Production of each variety involves a fixed production cost of $f_d$ units of labor and a constant variable cost that depends on firm productivity. The total labor required to produce $q(\varphi)$ units of a variety is therefore:

$$l(\varphi) = f_d + \frac{q(\varphi)}{\varphi}. $$
With CES preferences, the fixed production cost is central to matching empirical findings that exiting firms are on average of lower productivity than surviving firms, because firms that draw a sufficiently low productivity cannot generate enough variable profits to cover the fixed production cost. If firms decide to export, they face a fixed exporting cost of $f_x$ units of labor and iceberg variable costs of trade, such that $\tau > 1$ units of each variety must be exported in order for one unit to arrive in a foreign country. With CES preferences, the fixed exporting cost is key to matching empirical findings that only the most productive firms export, because only firms that draw a sufficiently high productivity can generate enough variable profits to cover the fixed exporting cost. Otherwise, in the presence of only variable trade costs, all firms would export, since CES preferences imply that the marginal utility of consuming any given variety approaches infinity as consumption of that variety approaches zero.

### 3.3 Production and Exporting Decisions

As each firm supplies one of a continuum of varieties, it is of measure zero relative to the industry as a whole, and hence takes the aggregate price index as given. The first-order condition for profit maximization yields the standard result that equilibrium prices are a mark-up over marginal cost that depends on the elasticity of demand. Given the same constant elasticity of demand in the domestic and export markets, equilibrium prices in the export market are a constant multiple of those in the domestic market due to the variable costs of trade:

$$p_x(\phi) = \tau p_d(\phi) = \tau \left( \frac{\sigma}{\sigma - 1} \right) \frac{w_d}{\phi} = \frac{\tau}{\rho \phi},$$  \hspace{1cm} (3)

where we choose the wage in one country as the numeraire and use country symmetry. Together these imply $w = 1$ for all countries.$^5$

Substituting the pricing rule into firm revenue (2), we obtain the following expression for equilibrium firm revenue in the export and domestic markets:

$$r_x(\phi) = \tau^{1-\sigma} r_d(\phi) = \tau^{1-\sigma} (\rho \phi)^{\sigma-1} R P^{\sigma-1}. \hspace{1cm} (4)$$

In deriving this expression for equilibrium revenue, firm varieties were assumed to enter utility symmetrically

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$^5$Horizontal product differentiation and the pricing rule (3) have implications for empirical measures of productivity. The representation of consumer preferences (1), in which varieties enter utility symmetrically, implicitly imposes a choice of units in which to measure the quantity of each variety. There is no necessary relationship between this normalization and the units in which physical quantities of output are measured for each firm in the data. As a result, data on physical quantities of output cannot be directly compared across firms, and revenue rather than quantity-based empirical measures of productivity are appropriate. Additionally, the constant mark-up implied by CES preferences in (3) ensures that firm prices are inversely proportional to firm productivity. Hence revenue-based measures of productivity, $p(\phi) q(\phi) / l(\phi)$, only vary across firms because of the fixed costs. More productive firms have higher variable labor input and higher revenue, with the result that the fixed labor input is spread over more units of revenue.
in (1). However, it is straightforward to allow for different weights for each firm variety to capture, for example, differences in product quality. With CES preferences and monopolistic competition, product quality enters firm revenue in the same way as firm productivity.\(^6\)

From equilibrium revenue (4), the relative revenue of any two firms within the same market depends solely on their relative productivities:

\[
\frac{r_d(\varphi'')}{r_d(\varphi')} = \frac{r_x(\varphi'')}{r_x(\varphi')} = \left( \frac{\varphi''}{\varphi'} \right)^{\sigma-1},
\]

Additionally, from equilibrium revenue (4), the relative revenue of a firm with a given productivity in the domestic and export markets depends solely on variable trade costs. Together these two features of relative revenues greatly simplify the characterization of industry equilibrium.

Consumer love of variety and a fixed production cost imply that no firm would ever export without also serving the domestic market. Therefore we can apportion the entire fixed production cost to the domestic market and the fixed exporting cost to the foreign market.\(^7\) Adopting this convenient accounting device, the pricing rule (3) implies that variable profits in each market are proportional to revenue, while firm profits in each market equal variable profits minus the relevant fixed cost:

\[
\pi_x(\varphi) = \frac{r_x(\varphi)}{\sigma} - f_x, \quad \pi_d(\varphi) = \frac{r_d(\varphi)}{\sigma} - f_d.
\]

The fixed production cost implies that there is a zero-profit cutoff productivity \((\varphi^*_d)\) below which firms would make negative profits if they entered, and hence they exit immediately:

\[
r_d(\varphi^*_d) = (\rho\varphi^*_d)^{\sigma-1}RP^{\sigma-1} = \sigma f_d.
\]

Similarly, the fixed exporting cost implies that there is an exporting cutoff productivity \((\varphi^*_x)\) below which surviving firms would make negative profits if they exported, and hence they serve only the domestic market:

\[
r_x(\varphi^*_x) = \tau^{1-\sigma}(\rho\varphi^*_x)^{\sigma-1}RP^{\sigma-1} = \sigma f_x.
\]

\(^6\)An empirical literature has sought to use variation in unit values (revenue divided by physical quantity) to distinguish between models of horizontal product differentiation featuring productivity and quality differences across firms, including Baldwin & Harrigan (2007) and Johnson (2009). As discussed in footnote 5, there is no necessary relationship between the normalization imposed by the symmetric demand representation (1) and the units in which physical quantities of output are measured for each firm in the data. As a result, data on physical quantities of output cannot be directly compared across firms in the presence of horizontal product differentiation, which complicates the interpretation of variation in unit values across firms.

\(^7\)This is merely a convenient accounting device. Instead of analyzing the decision to export by comparing export profits to the fixed exporting cost, we could instead equivalently compare the sum of domestic and export profit profits to the sum of the fixed production and exporting costs.
Combining the zero-profit and exporting cutoff conditions, (7) and (8), with the relationship between variety revenues within the same market (5), we obtain the following relationship between the two productivity cutoffs:

\[ \varphi^*_x = \Lambda \varphi^*_d, \]  
\[ \Lambda \equiv \tau \left( \frac{f_x}{f_d} \right)^{\frac{1}{\sigma-1}}. \]  

(9)

Therefore, for sufficiently high values of fixed and variable trade costs, the model features selection into export markets: \( \Lambda > 1 \). Only the most productive firms export, while intermediate productivity firms serve only the domestic market, and low productivity firms exit.\\(^9\)

### 3.4 Steady-state Industry Equilibrium

The steady-state industry equilibrium is characterized by constant masses of firms entering, producing and exporting, as well as stationary \textit{ex post} distributions of productivity among producing and exporting firms. With firm productivity fixed at entry and a constant independent probability of firm death, these stationary \textit{ex post} distributions for productivity take a particularly tractable form. The \textit{ex post} productivity distributions in the domestic and export markets, \( \mu_d(\varphi) \) and \( \mu_x(\varphi) \) respectively, are truncations of the \textit{ex ante} productivity distribution, \( g(\varphi) \), at the zero-profit and exporting cutoff productivities respectively:

\[ \mu_d(\varphi) = \begin{cases} \frac{g(\varphi)}{1-G(\varphi_d)} & \text{if } \varphi \geq \varphi^*_d \\ 0 & \text{otherwise} \end{cases}, \quad \mu_x(\varphi) = \begin{cases} \frac{g(\varphi)}{1-G(\varphi^*_x)} & \text{if } \varphi \geq \varphi^*_x \\ 0 & \text{otherwise} \end{cases}. \]  

(10)

In an equilibrium with positive firm entry, we require that the free entry condition holds, which equates the expected value of entry to the sunk entry cost. The expected value of entry depends on the value of a firm with each productivity \( \varphi \), which equals the maximum of zero (if the firm exits) and the net present value of the stream of future profits (if the firm enters). Assuming, for simplicity, no time discounting, the value of a firm with productivity \( \varphi \) is:

\[ v(\varphi) = \max \left\{ 0, \frac{\pi(\varphi)}{\delta} \right\}. \]

The free entry condition therefore takes the following form:

\[ v_e = [1 - G(\varphi^*_d)] \frac{\pi}{\delta} = [1 - G(\varphi^*_x)] \frac{[\bar{\pi}_d + \chi n \bar{\pi}_x]}{\delta} = f_e, \]  

(11)

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\(^8\)Empirical estimates of the fixed costs of exporting are typically large: see for example Das et al. (2007) and Roberts & Tybout (1997). Arkolakis (2008) argues that these large values for fixed costs are hard to reconcile with small export shipments and proposes instead a model of endogenous market entry costs, which is discussed further below.

\(^9\)While the discussion here concentrates on exporting, Helpman et al. (2004) extend the analysis to consider exporting and horizontal foreign direct investment (FDI) as alternative modes of serving foreign markets.
where the expected value of entry equals the probability of successful entry, $[1 - G(\varphi^*_d)]$, times expected profits conditional on successful entry, $\bar{\pi}$. Expected profits conditional on successful entry equal expected profits in the domestic market conditional on serving that market, $\bar{\pi}_d$, plus the probability of exporting, $\chi = [1 - G(\varphi^*_x)]/[1 - G(\varphi^*_d)]$, times the number of export markets, $n$, times expected profits in each export market conditional on serving that market, $\bar{\pi}_x$. Expected profits in the domestic and export markets, $\bar{\pi}_d$ and $\bar{\pi}_x$, themselves depend on the ex post productivity distributions (10).

In a steady-state equilibrium with a constant mass of firms producing, we also require that the mass of successful entrants that draw a productivity above the zero-profit cutoff equals the mass of firms that die, which yields the following steady-state stability condition:

$$[1 - G(\varphi^*_d)] M_e = \delta M_d,$$  

(12)

where $M_e$ denotes the mass of entrants and $M_d$ denotes the mass of producing firms.

While aggregate variables, such as the price index, depend on integrals over values for firm productivity, the assumption of CES preferences again greatly simplifies the analysis, because all aggregate variables can be written in terms of a weighted average of firm productivity. As a result, aggregate variables in the Melitz model take the same value as in a model in which all firms have a common productivity equal to weighted-average productivity, but weighted-average productivity is itself endogenously determined by firm decisions.

Using the equilibrium pricing rule (3), the zero-profit and exporting cutoff productivities, (7) and (8) respectively, the ex post productivity distributions (10), and country symmetry, we obtain the following expression for the price index:

$$P = \left[ M_d p_d(\bar{\varphi}_d)^{1-\sigma} + \chi n M_d \bar{\pi}^{1-\sigma} p_d(\bar{\varphi}_x)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}$$

$$= M_t^{\frac{1}{1-\sigma}} p(\bar{\varphi}_t)$$

where $M_t = (1 + \chi n) M_d$; weighted-average productivities in the domestic and export markets, $\bar{\varphi}_d$ and $\bar{\varphi}_x$, are defined in the web appendix and depend solely on the zero-profit and exporting cutoff productivities, $\varphi^*_d$ and $\varphi^*_x$; overall weighted average productivity, $\bar{\varphi}_t$, is itself a weighted average of $\bar{\varphi}_d$ and $\bar{\varphi}_x$, as also defined in the web appendix. These weighted-average productivities correspond to theoretically-consistent measures of aggregate productivity derived from the CES ideal price index.\(^{10}\)

\(^{10}\)From (4), $q_d(\varphi)/q(\bar{\varphi}) = (\varphi/\bar{\varphi}_d)^\sigma$, which implies that $\bar{\varphi}_d$ can be written as $\bar{\varphi}_d = \int_0^\infty \varphi^{-1} [q(\varphi)/q(\bar{\varphi})] [g(\varphi)/(1 - G(\varphi^*_d))] d\varphi$. Therefore, $\bar{\varphi}_d$ is the weighted harmonic mean of the $\varphi$’s, where the weights $(q_d(\varphi)/q(\bar{\varphi}))$ are firms’ relative output shares. The interpretation of $\bar{\varphi}_x$ is directly analogous.
With symmetric countries, the steady-state industry equilibrium can be referenced by a quadruple \( \{ \varphi_d^*, \varphi_x^*, R, P \} \), in terms of which all other endogenous variables of the model can be written. As the model has a recursive structure, it is straightforward to determine this steady-state equilibrium for a general continuous productivity distribution, as shown in the web appendix.

### 3.5 Trade Liberalization and Intra-industry Reallocation

To examine the comparative statics of opening the closed economy to trade, we compare the steady-state equilibrium in the closed and open economy.\(^{11}\) Using the relationship between variety revenues and the zero-profit and exporting cutoff conditions, the free entry condition (11) can be written as:

\[
v_e = \frac{f_d}{\delta} \int_{\varphi_d^*}^\infty \left( \frac{\varphi}{\varphi_d^*} \right)^{\sigma - 1} g(\varphi) d\varphi + \frac{n f_x}{\delta} \int_{\varphi_x^*}^\infty \left( \frac{\varphi}{\varphi_x^*} \right)^{\sigma - 1} g(\varphi) d\varphi = f_e. \tag{13}
\]

Substituting for \( \varphi_x^* = \Lambda \varphi_d^* \) from the relationship between the productivity cutoffs (9), a unique equilibrium value of the zero-profit cutoff productivity, \( \varphi_d^* \), can be determined independently of the other endogenous variables of the model. Having determined \( \varphi_d^* \), the exporting cutoff productivity, \( \varphi_x^* \), follows immediately from the relationship between the productivity cutoffs (9).

The free entry condition (13) defines a downward-sloping relationship between \( \varphi_x^* \) and \( \varphi_d^* \), where the closed economy free entry condition corresponds to the limiting case of infinitely large trade costs, where \( \varphi_x^* \to \infty \). As the closed economy opens to trade and the exporting cutoff productivity, \( \varphi_x^* \), falls to a finite value, the zero-profit cutoff productivity, \( \varphi_d^* \), must rise in order for the expected value of entry in (13) to remain equal to the unchanged sunk entry cost. Intuitively, the opening of trade increases expected profitability in the industry, \( \bar{\pi} \), because of the positive probability of drawing a productivity sufficiently high to export, which implies that the probability of successful entry, \( [1 - G (\varphi_d^*)] \), must fall to restore equality between the expected value of entry and the sunk entry cost. While we have derived these results for the opening of the closed economy to trade, similar results hold for reductions in variable trade costs in the open economy equilibrium.

The rise in the zero-profit cutoff productivity, \( \varphi_d^* \), induced by trade liberalization leads to within-industry reallocations of resources across firms. There is exit by low-productivity firms with productivities above the old but below the new zero-profit cutoff. Additionally, intermediate-productivity firms that serve only the domestic market experience a contraction in revenue as a result of the rise in the zero-profit cutoff, since \( r_d (\varphi) = (\varphi/\varphi_d^*)^{\sigma - 1} \sigma f_d \). While high-productivity exporting firms also experience a contraction in revenue in

\(^{11}\)Following the opening of trade, there are transition dynamics from the closed to the open economy steady-state equilibrium (see for example Chaney 2005). In a later section, we follow much of the literature in considering a static version of the Melitz model, with no exogenous firm death and no steady-state entry and exit of firms.
the domestic market, they enjoy an expansion in revenue in the export market. As shown in the web appendix, the expansion in export market revenue dominates, so that the total revenue of high-productivity exporting firms rises. Each of these firm responses induces reallocations of resources towards more productive firms, which raises aggregate productivity.\(^\text{12}\)

The mechanism behind these within-industry reallocations of resources is the differential impact of trade liberalization on exporters and non-exporters. In the special case where fixed exporting costs are equal to zero \((f_x = 0)\), all firms export and the opening of trade has no effect on the zero-profit cutoff productivity, because the open economy free entry condition (13) takes exactly the same form as in the closed economy. Although the opening of trade leads to entry by foreign varieties into the domestic market, which reduces the domestic price index, this reduction in the price index has a symmetric effect on the domestic revenue of all firms in (4). As a result, there is a change in the mass of firms at each productivity, \(M_{dg}(\varphi)/[1 - G(\varphi^*_d)]\), but no change in the zero-profit cutoff productivity below which firms exit.\(^\text{13}\)

### 3.6 Welfare

While theories of international trade have typically emphasized comparative advantage and product variety as sources of welfare gains from trade, theories of heterogeneous firms and trade point to within-industry reallocations of resources as a new channel through which trade can affect welfare. Indirect utility in both the closed and open economy can be expressed solely in terms of the zero-profit productivity cutoff below which firms exit:

\[
V = \frac{1}{P} = \rho \left( \frac{L}{\sigma f} \right)^{1/(\sigma - 1)} \varphi^*_d.
\]

As the opening of trade raises the zero-profit productivity cutoff below which firms exit, \(\varphi^*_d\), there are necessarily welfare gains from trade in the Melitz model.

### 3.7 Pareto Distribution

While the above analysis was undertaken for any continuous productivity distribution, the case of a Pareto productivity distribution has received particular attention in the literature and will be considered in a later

\(^{12}\)One caveat is that \(\tilde{\varphi}_d\) takes into account output lost in transit (from \(\tau\)), which implies that it is possible for \(\tilde{\varphi}_d\) to be lower than the closed economy value of \(\tilde{\varphi}_d\) when \(\tau\) is high and \(f_x\) is low. However, any productivity average based on output “at the factory gate” is necessarily higher in the open economy equilibrium with selection into export markets than in the closed economy equilibrium, as shown in Melitz (2003) and the web appendix.

\(^{13}\)From the relationship between the productivity cutoffs (9), there is a range of parameter values with positive fixed exporting costs for which all firms export. For this range of parameter values, the opening of trade does affect the zero-profit cutoff productivity in the free entry condition (13), but only because it increases fixed costs for all firms. As a result, some previously viable low productivity firms can no longer generate enough revenue to cover fixed costs and exit.
section. In this case:

\[ g(\varphi) = k \varphi^k \varphi^{-(k+1)}, \quad G(\varphi) = 1 - \left( \varphi_{\min} \varphi \right)^k, \]  

(14)

where \( \varphi_{\min} > 0 \) is the lower bound of the support of the productivity distribution; lower values of the shape parameter \( k \) correspond to greater dispersion in productivity; and we require \( k > (\sigma - 1) \) for the distribution of firm revenue to have a finite mean.

The Pareto distribution has two key attractive features for the Melitz model. First a random variable with this distribution remains Pareto distributed when it is truncated from below. Second the CES demand system implies that firm variables, such as revenue, are power functions of productivity, and a power function of a Pareto-distributed random variable is itself Pareto distributed. For these reasons, the model becomes particularly tractable for the case of a Pareto distribution, which has been found to provide a reasonable approximation to the observed distribution of firm size (e.g. Axtell 2001).

3.8 Summary

Taken together, the Melitz model addresses each of the empirical challenges discussed above. The model features producer heterogeneity, steady-state entry and exit of firms, and the accompanying steady-state job creation and destruction. With positive fixed exporting costs, and for sufficiently large values of fixed and variable trade costs, only some firms export. These exporting firms are larger and more productive than firms that only serve the domestic market. And for values of trade costs for which there is selection into export markets, trade liberalization induces reallocations of resources across firms within industries, as low-productivity firms exit and high-productivity firms expand to enter export markets.

4 Integrated Equilibrium

While the Melitz model emphasizes reallocations of resources within industries, traditional theories of international trade, such as the Heckscher-Ohlin model, stress comparative advantage and reallocation across industries. In this section, we review Bernard, Redding & Schott (2007), which embeds the Melitz model within the standard framework of general equilibrium trade theory using the concept of integrated equilibrium, as used in Dixit & Norman (1980) and Helpman & Krugman (1985).

Consider a world of two countries (home and foreign), two industries (good one and good two) and two factors of production (skilled and unskilled labor). Home is assumed to be skill-abundant relative to foreign. Consumer preferences are identical and homothetic and for simplicity the upper-tier of utility across the two
sectors is assumed to take the Cobb-Douglas form. Within each sector, preferences are defined over consumption of a continuum of horizontally differentiated varieties as in the previous section:

\[
U = \left(\int_{\omega \in \Omega_1} q_1(\omega)^\rho d\omega\right)^{\frac{\alpha}{\rho}} \left(\int_{\omega \in \Omega_2} q_2(\omega)^\rho d\omega\right)^{\frac{1-\alpha}{\rho}}, \quad 0 < \alpha < 1.
\]

As in the standard Heckscher-Ohlin model, production technologies are the same across countries but differ in factor intensity across industries. Good one is assumed to be skill-intensive relative to good 2. In each sector, there is a fixed production cost and a constant variable cost that depends on firm productivity. The production technology is assumed to be homothetic, so that the fixed and variable cost use the two factors of production with the same intensity. Therefore the total cost of producing \(q(\varphi)\) units of a variety in sector \(i\) is:

\[
\Gamma_i = \left[f_i + \frac{q_i(\varphi)}{\varphi}\right] (w_S)^{\beta_i} (w_L)^{1-\beta_i}, \quad 1 > \beta_1 > \beta_2 > 0
\]

where \(w_S\) is the skilled wage and \(w_L\) is the unskilled wage.

When fixed and variable trade costs are equal to zero, the concept of integrated equilibrium can be used to determine the set of factor allocations to the two countries for which trade in goods alone can equalize factor prices. Within this factor price equalization set, the four theorems of the Heckscher-Ohlin model – the Factor Price Equalization, Stolper-Samuelson, Rybczynski and Heckscher-Ohlin Theorems – continue to hold in the presence of firm heterogeneity.

When fixed and variable trade costs are not equal to zero, factor price equalization breaks down and, for parameter values for which there is selection into export markets, the opening of trade results in intra-industry reallocations across firms. As these intra-industry reallocations are driven by the differential impact of the opening of trade on exporters and non-exporters, they are stronger in the comparative advantage industry, where export opportunities are relatively more attractive. Although there is a decline in the relative mass of firms in the comparative disadvantage industry, as factors of production are reallocated in accordance with comparative advantage, exit by low productivity firms is strongest in the comparative advantage industry. As a result, the comparative advantage industry has a lower exporting cutoff, a higher zero-profit cutoff and a higher average productivity than the comparative disadvantage industry in the open economy equilibrium. This differential impact of the opening of trade on the productivity cutoffs depending on Heckscher-Ohlin-based comparative advantage influences both the welfare gains from trade and the impact of trade on income distribution.
5 Trade and Market Size

While the Melitz model captures reallocations of resources across firms within industries, the property of constant mark-ups imposed by CES preferences stands at odds with empirical studies of trade liberalization episodes, which typically find pro-competitive effects (see for example Tybout 2003). Relatedly, the CES demand structure implies that the within-industry productivity distribution is invariant to market size, which only affects the mass of firms. In contrast to these predictions, Campbell & Hopenhayn (2005) present empirical evidence that retail establishments in larger markets have higher sales and employment. Similarly, Syverson (2004) examines the concrete industry as an example of a good with high transport costs, and finds that larger markets have both higher average plant size and higher average productivity.

In this section, we review the Melitz & Ottaviano (2008) model, which combines firm heterogeneity with quasi-linear preferences to generate endogenous mark-ups that vary with firm productivity, market size and trade integration. Labor is the sole factor of production and each country \( i \) is endowed with \( L^i \) workers. The representative consumer’s preferences in each country are defined over consumption of a continuum of differentiated varieties, \( q^c_\omega \), and consumption of a homogeneous good, \( q^c_0 \):

\[
U = q^c_0 + \alpha \int_{\omega \in \Omega} q^c_\omega d\omega - \frac{1}{2} \gamma \int_{\omega \in \Omega} (q^c_\omega)^2 d\omega - \frac{1}{2} \eta \left( \int_{\omega \in \Omega} q^c_\omega d\omega \right)^2,
\]

where higher \( \alpha \) and lower \( \eta \) increase demand for differentiated varieties relative to the numeraire, while higher \( \gamma \) implies greater love of variety, with \( \gamma = 0 \) corresponding to the special case of perfect substitutes. Each country’s labor endowment is assumed to be sufficiently large that it both consumes and produces the homogeneous good, which is chosen as the numeraire, so that \( p^c_0 = 1 \). Using the first-order conditions for utility maximization, the inverse demand curve for a differentiated variety is:

\[
p_\omega = \alpha - \gamma q^c_\omega - \eta Q^c, \quad Q^c = \int_{\omega \in \Omega} q^c_\omega d\omega,
\]

where demand for a variety is positive if \( p_\omega \leq \alpha - \eta Q^c \), which defines a “choke price” above which demand for a variety is zero.

The homogeneous good is produced under conditions of perfect competition and constant returns to scale with a unit labor requirement. Differentiated varieties are produced under conditions of monopolistic competition and constant returns to scale. To enter the differentiated sector, a firm must incur a sunk entry cost of \( f_e \)

---

14Other possible frameworks for modeling firm heterogeneity with endogenous mark-ups include Bertrand competition in Bernard et al. (2003), translog preferences and monopolistic competition following Feenstra (2003), and Constant Absolute Risk Aversion preferences and monopolistic competition following Behrens & Murata (2006).
units of labor, after which its unit labor requirement or cost, \( c \), is drawn from a cumulative distribution function \( G(c) \) with support on \([0, c_M]\). As firms with the same cost, \( c \), behave symmetrically, firms are indexed from now on by \( c \) alone. If a firm decides to export, it faces iceberg variable costs of trade, such that \( \tau^j > 1 \) units of a variety must be exported to country \( j \) in order for one unit to arrive.

With quasi-linear preferences, the marginal utility of consumption is finite at zero consumption. Therefore fixed production costs and fixed exporting costs are not needed to generate firm exit and selection into export markets respectively. Firms drawing a marginal cost above the choke price in the domestic market exit in equilibrium, because they cannot generate positive profits from production. Additionally, a firm’s marginal costs may lie below the choke price in the domestic market, but may be above the choke price in the foreign market once variable trade costs have been taken into account. Thus sufficiently high variable trade costs are enough to induce selection into export markets.

The consideration of equilibria where each country both consumes and produces the homogeneous good greatly simplifies the analysis. As long as the homogeneous good is produced, productivity in this sector pins down the wage in each country as equal to one. Additionally, as long as the homogeneous good is consumed, quasi-linear preferences imply that the demand for differentiated varieties can be determined independently of income. A key advantage of these features is the resulting tractability of the model, which means that it can be used to consider multiple countries with arbitrary differences in country size and physical geography (variable trade costs). A disadvantage is that changes in income are accommodated solely through changes in consumption of the homogeneous good and have no general equilibrium effects in the differentiated sector.

As markets are assumed to be segmented and the production technology exhibits constant returns to scale, the supplier of each differentiated variety maximizes independently the profits earned from domestic and export sales.\(^{15}\) Using the first-order conditions for utility and profit maximization, firm variables such as the price in the domestic and export market can be determined solely as a function of a firm’s own cost and the cost cutoff above which it is not profitable to serve a market:

\[
p^d_i (c) = \frac{1}{2} (c^d_i + c), \quad p^j_x (c) = \frac{\tau^j}{2} (c^j_x + c),
\]

where countries are indexed by \( i \) and \( j \); \( c^d_i \) is the cost cutoff in the domestic market for country \( i \); \( c^j_x \) is the cost cutoff for firms exporting from country \( i \) to \( j \); and \( c^j_x = c^j_d / \tau^j \).

From the above equilibrium pricing rule, more productive firms again charge lower prices, but linear de-

\( ^{15} \)Using the equilibrium pricing rules derived below, it can be shown that equilibrium variety prices are such that there exist no profitable arbitrage opportunities across markets.
mand implies that firms charging lower prices face more inelastic demand, and hence they choose optimally to set a higher mark-up of price over marginal cost. As a result, the lower costs of more productive firms are not fully passed on to consumers in the form of lower prices.\textsuperscript{16}

To characterize the model’s comparative statics, we follow Melitz & Ottaviano (2008) in assuming that firm productivity $1/c$ follows a Pareto distribution with lower productivity bound $1/c_M$ and shape parameter $k > 1$. In this case, the cumulative distribution function of firm costs is given by $G(c) = (c/c_M)^k$ for $c \in [0, c_M]$.

We begin by considering the closed economy, which again corresponds to the limiting case of infinitely large trade costs, where $c_x \to 0$. With quasi-linear preferences, market size (the measure of consumers $L$) affects the closed economy cost cutoff. Larger markets have lower cost cutoffs ($c_D$), lower average costs ($\bar{c}$), and lower average prices ($\bar{p}$). Comparing a large to a small market, some low productivity firms have marginal costs above the choke price in the larger market and exit, while surviving firms price on a more elastic segment of their demand curve, and hence charge lower mark-ups than in the smaller market. Therefore consumers in larger markets face lower average prices and enjoy higher welfare, because of both higher average productivity and lower average mark-ups.

Opening the closed economy to trade has similar effects on the domestic market cost cutoff as an increase in closed economy market size. Consumers experience welfare gains from trade through lower average prices, because of both higher average productivity and lower average mark-ups. Once the economy is open to trade, unilateral, multilateral and preferential trade liberalization have distinct effects on welfare, because they have different effects on the entry and production decisions of differentiated-sector firms.

6 Gravity

One of the most successful empirical relationships in economics is the gravity equation, which relates the value of trade between countries to their size and the economic distance between them. Models of firm heterogeneity and trade have yielded new insights for the gravity equation by highlighting a distinction between the extensive margin (the measure of exporting firms) and the intensive margin (average exports conditional on exporting). To highlight this distinction, Chaney (2008) considers a version of the Melitz model with a Pareto productivity distribution and a fixed measure of potential firms rather than free entry. Subsequently, Arkolakis et al. (2008) have shown that the same results hold with free entry, and hence we follow their exposition.

16 As firm prices now fall less than proportionately with firm productivity, revenue-based measures of productivity, $p(\varphi)q(\varphi)/l(\varphi)$, vary across firms even in the absence of fixed costs.
model is considered with a single differentiated sector; the world consists of many (potentially asymmetric) countries;\textsuperscript{17} productivity follows the Pareto distribution (14); and fixed exporting costs are incurred in the consuming rather than the producing country.\textsuperscript{18} Total exports from country \(i\) to destination market \(j\) can be written in terms of the extensive and intensive margins of exports:

\[
X_{ij} = \left( \frac{1 - G(\varphi^*_{ij})}{1 - G(\varphi^*_i)} \right) M_i \left\{ \int_{\varphi_{ij}}^{\infty} (\varphi^*)^{-1} (\tau_{ij} \varphi^*_{ij})^{-1} \frac{g(\varphi)}{1 - G(\varphi^*)} d\varphi \right\}
\]

where \(M_i\) is the mass of producing firms in country \(i\) and \(w_j L_j\) is total income, which equals total expenditure on differentiated varieties, in destination market \(j\); \(\varphi^*_{ij}\) is the productivity cutoff for firms located in country \(i\) and serving destination market \(j\); \(\tau_{ii} = 1\) and \(\tau_{ij} > 1\) for \(j \neq i\). Using the Pareto productivity distribution, we obtain:

\[
X_{ij} = \left( \frac{\varphi^*_{ii}}{\varphi^*_{ij}} \right)^k M_i \left\{ \int_{\varphi_{ij}}^{\infty} (\varphi^*)^{-1} (\tau_{ij} \varphi^*_{ij})^{-1} \frac{g(\varphi)}{1 - G(\varphi^*)} d\varphi \right\} \]

Perhaps surprisingly, the intensive margin of trade is independent of variable trade costs with a Pareto productivity distribution. On the one hand, higher variable trade costs reduce exports of a given firm to a given country, which reduces average exports per firm. On the other hand, higher variable trade costs induce low productivity firms to exit the export market, which raises average exports per firm. With a Pareto productivity distribution these two effects exactly offset one another, so as to leave the intensive margin independent of variable trade costs.

Using total exports (15) and the equilibrium mass of producing firms in country \(i\), the share of country \(j\)'s expenditure on goods produced in country \(i\), \(\vartheta_{ij}\), can be evaluated as:

\[
\vartheta_{ij} = \frac{L_i \varphi^k_{i \min} (\tau_{ij} \varphi^*_{ij})^{-k} \int_{\varphi_{ij}}^{\infty} (\varphi^*)^{-1} (\tau_{ij} \varphi^*_{ij})^{-1} \frac{g(\varphi)}{1 - G(\varphi^*)} d\varphi}{\sum_s L_s \varphi^k_{s \min} (\tau_{sj} \varphi^*_{sj})^{-k} \int_{\varphi_{sj}}^{\infty} (\varphi^*)^{-1} (\tau_{sj} \varphi^*_{sj})^{-1} \frac{g(\varphi)}{1 - G(\varphi^*)} d\varphi}
\]

This expression for the trade share takes a familiar gravity equation form, where trade between countries \(i\) and \(j\) depends on both bilateral resistance (trade costs between \(i\) and \(j\)) and multilateral resistance (trade costs between all countries \(s\) and \(j\)). The elasticity of the trade share with respect to variable trade costs depends not on the elasticity of substitution, \(\sigma\), but on the parameter determining the dispersion of productivity, \(k\).

\textsuperscript{17}For an analysis of asymmetries between a home and foreign country in a model with a differentiated and homogeneous sector, see Demidova (2008).

\textsuperscript{18}While we follow the specification in Arkolakis et al. (2008), whether fixed exporting costs are incurred in the consuming or producing country makes little difference.
This feature is closely related to the property of the Pareto productivity distribution noted above, namely that changes in variable trade costs only affect trade flows through the extensive margin, where the extensive margin response depends on the dispersion of productivity. Notably, the trade share (16) takes a directly analogous form to that in Eaton & Kortum (2002), despite the stark differences between the two models: monopolistic competition and increasing returns to scale versus perfect competition and constant returns to scale. This similarity of the trade shares reflects the fact that, under the distributional assumptions of both models, variable trade costs only influence trade flows through the extensive margin.

As shown in Arkolakis et al. (2008), further insight into the welfare gains from trade can be garnered by using the cutoff productivity conditions and the trade share (16) to write indirect utility as follows:

\[
V_j = \frac{w_j}{P_j} = \vartheta_{jj}^{-1/k} L_j^{1/(\sigma-1)} \left( \frac{\varphi_{jmin} f_{jj}^{1-k/(\sigma-1)} \sigma - 1}{f_e \left( \frac{\sigma}{\sigma-1} \right)^k \sigma k/(\sigma-1) k - \sigma + 1} \right)^{1/k}.
\]

Since in the closed economy \( \vartheta_{jj} = 1 \), while in the open economy \( 0 < \vartheta_{jj} < 1 \), we again obtain the result that there are necessarily welfare gains from trade. Furthermore, a country’s trade share with itself, \( \vartheta_{jj} \), is a sufficient statistic for welfare in the sense that estimating the welfare gains from trade relative to autarky requires only a measure of a country’s trade share with itself, \( \vartheta_{jj} \), and an estimate of the elasticity of trade flows with respect to variable trade costs, \( k \), which can be obtained from bilateral trade data. Notably, the property that the trade share is a sufficient statistic for welfare is shared with Eaton & Kortum (2002) and a wider class of models.\(^{19}\)

One of the striking features of the Melitz model is that welfare gains from trade still occur even if the mass of varieties available for domestic consumption falls. As shown in Arkolakis et al. (2008), total exports (15) and the definition of the trade share, \( \vartheta_{ij} = X_{ij} / w_j L_j \), can be used to express the total mass of varieties available for consumption in country \( j \) as:

\[
\sum_s M_{sj} = \frac{L_j}{f_{jj} \frac{\sigma_k}{k-\sigma+1}} + \frac{L_j}{k-\sigma+1} \sum_{s \neq j} \vartheta_{sj} \left( \frac{1}{f_{sj}} - \frac{1}{f_{jj}} \right).
\]

A sufficient condition for the mass of varieties available for domestic consumption to fall following the opening of trade is that the fixed cost of exporting to each foreign country, \( f_{sj} \) for \( s \neq j \), exceeds the fixed cost of

\(^{19}\)Arkolakis et al. (2009) characterize the class of models for which the trade share is such a sufficient statistic for welfare. Given the same trade share and the same elasticity of trade with respect to trade costs, this class of models implies the same welfare gains from trade. Of course, the trade share is an endogenous variable, which varies with the parameters of each model and in general can differ across models within this class. Though Atkeson & Burstein (2010) show that the free entry condition constrains the overall magnitude of the welfare gains from trade in a model of product variety with CES preferences and monopolistic competition, as discussed in a later section.

19
supplying the domestic market, $f_{jj}$, which generalizes a result for two countries reported in Baldwin & Forslid (2004). The intuition for why the mass of varieties available for domestic consumption can fall following the opening of trade is as follows. Since the opening of the closed economy to trade raises the domestic productivity cutoff, average firm size rises, which with a given labor endowment implies a decline in the mass of domestically-produced varieties.\footnote{In Krugman (1979), there is a similar rise in firm size as a result of the opening of trade, which reduces the mass of domestically-produced varieties, but firms are homogeneous and the rise in firm size occurs as a result of a variable elasticity of substitution.} For the parameter values discussed above, this reduction in the mass of domestically-produced varieties dominates the access to new varieties from foreign markets, so that the overall mass of varieties available for domestic consumption falls.

7 Quantitative Analysis

While the discussion above highlighted the ability of models of heterogeneous firms to account for the qualitative features of micro data, there is also evidence that they can account quantitatively for the observed pattern of trade across firms and countries.

Eaton et al. (2008) use simulated method of moments to estimate an extended version of the Melitz model using moments based on French trade data by firm and destination market. These data exhibit a number of striking empirical regularities to be explained. First, the number of French firms selling to a market (relative to French market share) increases with market size according to an approximately log linear relationship. This pattern of firm export market participation exhibits an imperfect hierarchy, where firms selling to less popular markets are more likely to sell to more popular markets, but do not always do so. Second, export sales distributions are similar across markets of very different size and extent of French participation. While the upper tail of these distributions is approximately Pareto distributed, there are departures from a Pareto distribution in the lower tail where small export shipments are observed. Third, average sales in France are higher for firms selling to less popular foreign markets and to more foreign markets.

In their econometric analysis, Eaton et al (2008) consider a static version of the Melitz model, with a Pareto productivity distribution, and a fixed measure of potential firms, as in Chaney (2008). But to account for the empirical regularities noted above, the model is extended in a number of directions. In order to explain variation in export participation with market size in a model with CES demand, fixed market entry costs are required. But to generate the departures from a Pareto distribution in the lower tail where small export shipments are observed, these fixed market entry costs are allowed to vary endogenously with a firm’s choice of the fraction of consumers within a market to serve, $f$, as in Arkolakis (2008). Additionally, to allow for imperfect hierarchies
of markets, fixed market entry costs are assumed to be subject to an idiosyncratic shock for each firm $\omega$ and destination market $j$, $\varepsilon_{j\omega}$, as well a common shock for each source country $i$ and destination market $j$, $E_{ij}$. Market entry costs are therefore:

$$E_{ij\omega} = \varepsilon_{j\omega} E_{ij} M (f),$$

where the function $M (f)$ determines how market entry costs vary with the fraction of consumers served and takes the following form, which is derived from microfoundations in Arkolakis (2008):

$$M (f) = \frac{1 - (1 - f)^{1-1/\lambda}}{1 - 1/\lambda},$$

where $\lambda > 0$ captures the increasing cost of reaching a larger fraction of consumers. Any given consumer is served with probability $f$, so that each consumer receives the same measure of varieties, but the particular varieties in question can vary across consumers.\textsuperscript{21}

Finally, to allow for idiosyncratic variation in sales conditional on entering a given export market for firms with a given productivity, demand is also subject to an idiosyncratic shock for each firm $\omega$ and destination market $j$, $\alpha_{j\omega}$:

$$X_{ij\omega} = \alpha_{j\omega} f_{ij\omega} X_j \left( \frac{\tau_{ij} P_{i\omega}}{P_j} \right)^{1-\sigma},$$

where $X_j$ is total expenditure in market $j$ and the presence of $f_{ij\omega}$ reflects the fact that only a fraction of consumers in each market are served. A firm’s decision to enter a market depends on the composite shock, $\eta_{j\omega} = \alpha_{j\omega} / \varepsilon_{j\omega}$, but a firm with a given productivity can enter a market because of a low entry shock, $\varepsilon_{j\omega}$, and yet still have low sales in that market because of a low demand shock, $\alpha_{j\omega}$.

The properties of the model depend on five key parameters: a composite parameter that includes both the elasticity of substitution and the Pareto shape parameter, the convexity of marketing costs, the variance of demand shocks, the variance of entry shocks, and the correlation between demand and entry shocks. As shown in Eaton et al. (2008), these parameters can be precisely estimated from moments of the French data and the resulting parameterized model provides a good fit to the observed data. For the estimated parameter values, firm productivity accounts for around half of the observed variation across firms in export market participation, but explains substantially less of the variation in exports conditional on entering a market.

\textsuperscript{21}To generate the observed departures from a Pareto distribution in the lower tail of the export sales distribution, one requires $0 < \lambda < 1$, which implies an increasing marginal cost of reaching additional consumers. An alternative potential explanation for the departures from a Pareto distribution in the lower tail is a variable elasticity of substitution, though such an explanation also needs to be consistent with an upper tail of the export sales distribution that is well approximated by a Pareto distribution. Both endogenous market entry costs and a variable elasticity of substitution provide potential explanations for empirical findings that most of the growth in trade following trade liberalization is in goods previously traded in small amounts, as in Kehoe & Ruhl (2009).
Given the estimated parameter values, the model can be used to examine counterfactuals, such as a 10 percent reduction in bilateral trade barriers for all French firms. In this counterfactual, total sales by French firms rise by around $16 million, with most of this increase accounted for by a rise in sales of the top decile of firms of around $23 million. In contrast, every other decile of firms experiences a decline in sales, with around half of the firms in the bottom decile exiting. Taken together, these results suggest that the intra-industry reallocations emphasized by theories of heterogeneous firms and trade can be quantitatively large, even for empirically reasonable changes in trade frictions.\footnote{For a quantitative analysis of European integration using a model of firm heterogeneity and trade, see Del Gatto et al. (2006). For a quantitative analysis of the effect of China’s productivity growth on world welfare, see Hsien & Ossa (2010). For a quantitative analysis of the apparel sector in Bangladesh, see Cherkashin et al. (2010).}

Modeling the extensive margin of export participation also enables theories of heterogeneous firms and trade to account for another important feature of international trade data, which is the prevalence of zero bilateral flows between countries. Helpman et al. (2008) develop a multi-country version of the Melitz model, in which the productivity distribution is a truncated Pareto. In this case, no firm exports from country \(i\) to destination market \(j\) if the productivity cutoff, \(\varphi_{ij}^*\), lies above the upper limit of country \(i\)’s productivity distribution. Estimating a structural gravity equation, they show that controlling for both the non-random selection of positive trade flows and the extensive margin of exporting firms has quantitatively important implications for estimates of the effects of standard trade frictions on trade flows.

8 Labor Markets

Although trade liberalization has uneven effects across firms in the Melitz model, all workers are affected in the same way, because there is a frictionless labor market in which all workers are employed for a common wage. More recently, theories of heterogeneous firms and trade have begin to explore the implications of labor market frictions for the impact of trade liberalization on the distribution of income across workers. One line of research has considered models of fair or efficiency wages, including Davis & Harrigan (2007), Egger & Kreickemeier (2009), and Amiti & Davis (2008). A second line of work has examined the implications of search frictions in the labor market, including Felbermayr et al. (2008), Helpman & Itskhoki (2010), and Helpman et al. (2010).\footnote{The key early paper introducing search frictions into a general equilibrium model of trade is Davidson et al. (1988). For a synthesis of the broader literature on search frictions and trade, see Davidson & Matusz (2009).}

In the presence of labor market frictions, equilibrium unemployment can exist and provides a channel through which trade can affect the distribution of income. While Helpman & Itskhoki (2010) emphasize reallocations across sectors with different degrees of search frictions, Felbermayr et al. (2008) stress the role of productivity gains from trade liberalization in reducing effective search costs. Helpman & Itskhoki (2010)
consider a two-country, two-sector model of international trade, in which one sector produces a homogeneous product, while the other sector produces differentiated products, and both are subject to search frictions. Differences in labor market institutions across countries and industries provide a source of comparative advantage and shape the impact of trade liberalization on aggregate unemployment. Reductions in a country’s labor market frictions in the differentiated sector raise its own welfare, by expanding the size of its differentiated sector and reducing its differentiated-sector price index. This expansion in the differentiated sector in one country intensifies competition in the export market faced by firms in the other country’s differentiated sector. As a result, the other country’s differentiated sector contracts, which reduces its welfare. In contrast, proportional reductions in labor market frictions in the differentiated sector in both countries raise welfare in each country, by expanding the size of the differentiated sector in each country. Variation in search frictions across sectors generates differences in sectoral unemployment rates, which implies that the opening of trade affects aggregate unemployment by changing the relative shares of the labor force searching for employment in each sector.

As well as introducing unemployment, labor market frictions can also give rise to equilibrium differences in wages across firms. A large empirical literature finds that larger firms pay higher wages than smaller firms, and many empirical studies have found that exporters pay higher wages than non-exporters. Models of heterogeneous firms and trade naturally generate these two features of the data if firm wages are systematically related to firm revenue. In Egger & Kreickemeier (2009) and Amiti & Davis (2008), firm wages are assumed to be related to productivity, revenue or profits, because of fair wage concerns. In contrast, in Helpman et al. (2010), the relationship between firm wages and revenue is derived from search frictions and costly screening of workers to obtain information about worker ability.

When wages vary with revenue across firms, within-industry reallocations of resources across firms provide a new channel through which trade can affect income distribution. As shown in Helpman et al. (2010), the opening of the closed economy to trade raises wage inequality within industries in a class of models satisfying the following three conditions: wages and employment are power functions of productivity, only some firms export and exporting raises the wage paid by a firm with a given productivity, and productivity is Pareto distributed. When these three conditions are satisfied, the distribution of wages across workers within an industry,

\[ \text{Another setting in which cross-country differences in labor market institutions can provide a source of comparative advantage is where volatility varies across sectors, as in Cufat & Melitz (2010).} \]

\[ \text{See for example the survey by Oi & Idson (1999).} \]

\[ \text{See in particular Bernard & Jensen (1995, 1997).} \]
\( G_w(w) \), can be expressed as:

\[
G_w(w) = \begin{cases} 
S_{t,d} G_{w,d}(w) & \text{for } w_d \leq w \leq w_d \left( \frac{\varphi^*_x}{\varphi^*_d} \right)^{\zeta_w}, \\
S_{t,d} & \text{for } w_d \left( \frac{\varphi^*_x}{\varphi^*_d} \right)^{\zeta_w} \leq w \leq w_x, \\
S_{t,d} + (1 - S_{t,d}) G_{w,x}(w) & \text{for } w \geq w_x,
\end{cases}
\]

where \( \zeta_w \) is elasticity of firm wages with respect to firm productivity; \( w_d \) is the lowest wage paid by domestic firms; \( w_d \left( \frac{\varphi^*_x}{\varphi^*_d} \right)^{\zeta_w} \) is the highest wage paid by a domestic firm; \( w_x = w_d \Upsilon_{x}^{\psi w} \left( \frac{\varphi^*_x}{\varphi^*_d} \right)^{\zeta_w} \) is the lowest wage paid by an exporting firm, where \( \Upsilon_{x}^{\psi w} \) is the increase in wages at the productivity threshold for entry into export markets; \( S_{t,d} \) is the employment share of domestic firms; \( G_{w,d}(w) \) is a truncated Pareto distribution of wages across workers employed by domestic firms; \( G_{w,x}(w) \) is an untruncated Pareto distribution of wages across workers employed by exporters. Both \( G_{w,d}(w) \) and \( G_{w,x}(w) \) have the same shape parameter, which is determined by the elasticity of wages and employment with respect to productivity and the shape parameter of the Pareto productivity distribution, as shown in the web appendix.

When the three conditions discussed above are satisfied, Helpman et al. (2010) show that there is strictly greater wage inequality in the open economy when only some firms export than in the closed economy, and there is the same level of wage inequality in the open economy when all firms export as in the closed economy. It follows that wage inequality is at first increasing in trade openness and later decreasing in trade openness. The intuition for these results stems from the increase in firm wages that occurs at the productivity threshold above which firms export, which is only present when some but not all firms export. When no firm exports, a small reduction in trade costs increases wage inequality, because it induces some firms to start exporting and raises the wages paid by these exporting firms relative to domestic firms. When all firms export, a small rise in trade costs increases wage inequality, because it induces some firms to stop exporting and reduces the wages paid by these domestic firms relative to exporting firms.

\section{Endogenous Firm Productivity}

While treating productivity as a fixed parameter of the firm highlights the role of selection and reallocation across firms, firm productivity is in general endogenous, and its response to trade liberalization influences the within-industry allocation of resources.
9.1 Multi-Product Firms

One line of research has explored within-firm reallocation as a source of endogenous firm productivity growth. Multi-product firms exhibit substantial heterogeneity across products with firms and dominate production (Bernard, Redding & Schott 2010) and exports (Bernard, Redding & Schott 2006). Motivated by these empirical findings, a growing theoretical literature in international trade has modeled multi-product firms. As shown in Bernard et al. (2006), the Melitz model in Section 3 can be extended to allow heterogeneous firms to optimally choose the range of products to produce and export.\(^{27}\) Suppose that the representative consumer derives utility from the consumption of a continuum of symmetric products \(h\) defined on the interval \([0, 1]\):

\[
U = \left[ \int_0^1 C_h \nu dh \right]^\frac{1}{\nu}, \quad 0 < \nu < 1.
\]

Within each product, a continuum of firms supply differentiated varieties of the product. Incurring the sunk entry cost \(f_e\) creates a firm brand and a blueprint for one horizontally differentiated variety of each product that can be supplied using this brand. After incurring the sunk entry cost, a firm observes two components of productivity: “ability” \(\varphi \in (0, \infty)\), which is common to all products and drawn from a distribution \(g(\varphi)\), and “expertise” \(\lambda_h \in (0, \infty)\), which is specific to each product \(h\) and drawn from a distribution \(z(\lambda)\). The firm faces fixed production and exporting costs for each product, \(f_d\) and \(f_x\), in the domestic and export market. Similarly, there is a fixed cost to becoming a firm and to becoming an exporter, \(F_d\) and \(F_x\), which are independent of the number of products supplied.

Industry equilibrium can be determined using a similar approach to that used in Section 3. There are cutoff values for expertise for a firm of each ability \(\varphi\) at which the firm supplies a product to the domestic or export market, \(\lambda^*_d(\varphi)\) and \(\lambda^*_x(\varphi)\):

\[
r_d(\varphi, \lambda^*_d(\varphi)) = \sigma f_d, \quad r_x(\varphi, \lambda^*_x(\varphi)) = \sigma f_x.
\]

Using the fact that the relative revenues of any two firms and products depend solely on relative values of ability and expertise, the product cutoffs for expertise for each firm ability can be expressed as:

\[
\lambda^*_d(\varphi) = \left( \frac{\varphi^*_d(\varphi)}{\varphi} \right) \lambda^*_d(\varphi^*_d), \quad \lambda^*_x(\varphi) = \left( \frac{\varphi^*_x(\varphi)}{\varphi} \right) \lambda^*_x(\varphi^*_x).
\]

\(^{27}\)Other recent models of multi-product firms and trade include Arkolakis & Muendler (2010), Eckel & Neary (2010), Feenstra & Ma (2008), Mayer et al. (2010), and Nocke & Yeaple (2006).
where $\lambda^*_d(\varphi^*_d)$ is the expertise cutoff for the lowest-ability firm and $\lambda^*_x(\varphi^*_x)$ is the expertise cutoff for the lowest-ability exporter.

With a unit continuum of symmetric products and independent and identical distributions of product expertise, the law of large numbers implies that the fraction of products supplied to a market by a firm with a given ability $\varphi$ equals the probability of drawing an expertise above the relevant cutoff, $[1 - Z(\lambda^*_d(\varphi))]$ and $[1 - Z(\lambda^*_x(\varphi))]$ for the domestic and export market respectively. Firms with higher abilities have lower expertise cutoffs in the domestic and export market, because they can generate sufficient variable profits to cover product fixed costs, $f_d$ and $f_x$, at lower values for ability. As a result, firms with higher abilities produce and export a wider range of products.

While the decision whether to supply a product to a market is determined by the product expertise cutoffs, $\lambda^*_d(\varphi)$ and $\lambda^*_x(\varphi)$, there are also firm ability cutoffs in the domestic and export markets, $\varphi^*_d$ and $\varphi^*_x$, which determine the firm’s decision whether to produce and export. A firm drawing an ability below these ability cutoffs cannot generate sufficient variable profits across the small range of profitable products to cover the fixed costs of operating in the domestic and export markets, $F_d$ and $F_x$.

In this setting with multi-product firms, the opening of the closed economy to trade leads to reallocations of resources both across and within firms. For parameter values for which there is selection into export markets, the opening of trade raises the zero-profit cutoffs for firm ability, $\varphi^*_d$, and for product expertise for a given firm ability, $\lambda^*_d(\varphi)$. Low-ability firms exit and surviving firms drop marginal products and focus on more successful higher-expertise products. As a result, even though firm ability and expertise are fixed parameters, there is a reallocation of resources within firms, which raises firm productivity.

While the discussion here interprets expertise, $\lambda$, as a component of productivity, it can also be interpreted as a component of demand, which could be specific to countries as well as products. As shown in Bernard et al. (2009), the resulting framework provides a natural explanation for a number of key features of the distribution of exports across firms, products and countries. While higher trade costs reduce exports of a given firm and product, they also lead to changes in export composition towards higher export value firms and products. Consistent with this, distance is negatively and statistically significantly related to exports of a given firm and product, the number of exporters and the number of exported products, but has a positive and statistically insignificant effect on average exports per firm and product.\(^{28}\)

Finally, there is evidence of ongoing reallocation and selection within firms, even in the absence of trade

\(^{28}\)Modeling multi-product firms expands the number extensive margins of trade to include firms, products and destinations. As shown in Bernard, Jensen, Redding & Schott (2009), these extensive margins together account for most of the cross-country variation in bilateral trade.
liberalization or other large-scale changes. As reported in Bernard, Redding & Schott (2010), around one half of surviving U.S. firms add and/or drop products from their existing range every five years, and the contribution of these added and dropped products to aggregate output is of around the same magnitude as the contribution of firm entry and exit. Consistent with a natural generalization of models of industry dynamics to incorporate selection with firms, the probability that a product is dropped within a surviving firm exhibits age and scale dependence, as observed for the probability that a firm exits. Taken together, these findings suggest that reallocation may be even more important than hitherto thought, in so far as it occurs across products within firms as well as across firms.

9.2 Technology and Skills

The theoretical literature on heterogeneous firms and trade has also considered a number of other sources of endogenous firm productivity growth. One key insight from this literature is that there is a complementarity between entry into export markets and other revenue-enhancing investments that involve fixed costs. Exporting expands the size of the market over which other fixed costs can be amortized, while at the same time other revenue-enhancing investments raise the return to incurring the fixed costs of exporting.

One such revenue-enhancing investment that typically involves fixed costs is technology adoption. As shown in Constantini & Melitz (2008), the complementarity between exporting and technology adoption complicates the interpretation of causality in the relationship between productivity and exporting. Anticipation of trade liberalization can induce firms to bring forward the decision to adopt new technologies in advance of export market entry. Empirical evidence of this complementarity between exporting and technology adoption is found by Bustos (2010), using data on Argentinian firms and Mercosur, and by Lileeva & Trefler (2010), using data on Canadian plants and the Canada-U.S. Free Trade Agreement.

Technological progress can take the form of product or process innovation, as emphasized in the endogenous growth literature. Atkeson & Burstein (2010) consider a dynamic heterogeneous firm model, which features both product innovation, in the form of the introduction of new firm varieties, and process innovation, in the form of stochastic reductions in firm costs. The central finding of their paper is that trade liberalization can lead to substantial changes in firms’ exit, export and process innovation decisions, but the impact of these changes on welfare is largely offset by the response in product innovation. The intuition for this result stems from the free entry condition, which requires the increase in profits from product innovation induced by trade liberalization to be offset by an increase in the real wage. To a first-order approximation, the increase in ag-

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aggregate productivity required to sustain this increase in the real wage is independent of the channel through which the increase in aggregate productivity is achieved. As a result, changes in firms’ exit, export and process innovation decisions must be offset by adjustments in product innovation, so as to ensure that the response in aggregate productivity is consistent with equilibrium. While these theoretical results are derived analytically for symmetric countries and depend on the assumptions of CES preferences and monopolistic competition, the paper highlights that introducing more disaggregate margins of firm adjustment does not necessarily imply greater aggregate welfare gains from trade liberalization.

Other research has emphasized complementarities between technology and skills. In Yeaple (2005), firms are *ex ante* homogeneous but *ex post* heterogeneous, because of endogenous choices about technology and workforce skill composition. In Verhoogen (2008), more-productive plants produce higher-quality goods than less-productive plants, and they pay higher wages to maintain a higher-quality workforce. Only the most productive plants enter the export market, and Southern exporters produce higher-quality goods for export than for the domestic market, to appeal to richer Northern consumers. An expansion of export opportunities induces more-productive Southern plants to increase exports, upgrade quality, and raise wages relative to less-productive plants within the same industry, increasing within-industry wage dispersion. In Bustos (2007), trade liberalization induces more firms to enter the export market and to adopt skill-biased new technologies.

### 9.3 International Production Networks

Although theories of heterogeneous firms have largely focused on firm export behavior, recently available transactions-level trade data have revealed that firm import behavior exhibits many of the same features. To account for these empirical findings, one line of theoretical research has incorporated firm importing into theories of heterogeneous firms in an analogous way to firm exporting, as in Amiti & Davis (2008) and Kasahara & Lapham (2008). More generally, the presence of both importing and exporting within firms suggests the relevance of theories of the international fragmentation of production, including Dixit & Grossman (1982), Yi (2003) and Grossman & Rossi-Hansberg (2008).

While the international fragmentation of production relates to the decision whether to onshore or offshore stages of production, another separate decision is whether to insource or outsource these stages of production: i.e. whether to organize them within or outside the boundaries of the firm. Antràs & Helpman (2004, 2008) develop a model of North-South trade with firm heterogeneity and incomplete contracts to jointly analyze the

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30See, for example, Bernard, Jensen, Redding & Schott (2007).
31Recent empirical evidence suggests that imported intermediate inputs may play a prominent role in shaping the overall impact of trade liberalization, as in Amiti & Konings (2007) and Goldberg et al. (2010), and in influencing measured firm productivity, as in Halpern et al. (2009) and Kugler & Verhoogen (2008).
onshoring/offshoring and insourcing/outsourcing decisions. Firms sort endogenously based on their productivity into integrated companies that produce inputs in the North (do not engage in foreign trade in inputs), integrated companies that produce inputs in the South (engage in foreign direct investment (FDI) and intra-firm trade), disintegrated companies that outsource in the North (do not engage in foreign trade in inputs), and disintegrated companies that outsource in the South (import inputs at arm’s length).

Another strand of the theoretical literature on organizations and trade emphasizes the international allocation of talent in assignment models, including Antrás, Garicano & Rossi-Hansberg (2006), Burstein & Monge-Naranjo (2009), Dasgupta (2010), Ohnsorge & Trefler (2007), and Yeaple (2005). While these assignment models address a variety of issues, and often do not take a stand on whether production activities occur within or beyond the boundaries of the firm, this line of research provides another promising route for deriving firm heterogeneity as the endogenous result of an underlying microeconomic mechanism.

10 Firm and Aggregate Dynamics

While the Melitz model focuses on the production and exporting decisions of firms of a given productivity, a recent theoretical literature has begun to explore firm dynamics in models of international trade.

Using Colombian transactions-level trade data, Eaton et al. (2008) find that around one half of Colombian exporters in any given year were not exporters in the previous year. Among these new exporters, most do not survive into the next year. But conditional on survival, these new exporters grow rapidly and account for a substantial proportion of export growth over longer time horizons such as a decade.

Theoretical models seeking to account for these export market dynamics have emphasized learning about foreign markets and trade relationships, including Albornoz et al. (2010) and Eaton et al. (2010), and stochastic shocks to firm productivity, including Arkolakis (2009) and Ruhl & Willis (2008). Another somewhat separate line of research has examined the relationship between disaggregate firm heterogeneity and aggregate dynamics, including Alessandria & Choi (2007), Bilbiie et al. (2007), and Ghironi & Melitz (2005).

11 Conclusion

The literature on heterogeneous firms and trade has reconciled trade theory with empirical features of micro data on plants and firms. In so doing, it has highlighted a new dimension of reallocation across firms within industries and a new channel through which trade liberalization can affect income distribution.

32 For empirical evidence on intra-firm versus arms-length trade, see for example Bernard, Jensen, Redding & Schott (2010) and Nunn & Trefler (2008).
While the theoretical literature on heterogeneous firms and trade has advanced in great breadth and depth, there remain a number of open questions. One area for further research is gaining a deeper understanding of the origins of firm heterogeneity and the role of internal firm organization. Another area for further work is the microeconomic modeling of the trade costs that induce firm selection into export markets, including the role of wholesale and retail distribution networks.

Finally, as noted in Neary (2009), most existing research has been undertaken within the framework of monopolistic competition. But empirical findings that trade is concentrated in the hands of a relatively small number of firms suggest that theories of strategic interaction and dynamic games could prove a promising avenue for further inquiry.
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Literature Cited


