**Problem 6.45** (Solution by Dongning Guo, June 20, 2001)

The minimization problem can be solved by using Lagrange multipliers with cost function

\[
\sum_{j=1}^{n} (c^\top y[j])^2 + \lambda (c^\top s_1 - 1) + \nu (\|c\|^2 - 1 - \chi). \tag{1}
\]

Equating the gradient with respect to \( c \) to \( 0 \), we get

\[
2 \sum_{j=1}^{n} y^\top[j]y[j]c + \lambda s_1 + 2\nu c = 0, \tag{2}
\]

or, equivalently

\[
c^* = -\frac{1}{2} \lambda \left[ \sum_{j=1}^{n} y^\top[j]y[j] + \nu I \right]^{-1} s_1 \tag{3}
\]

where \( \lambda \) is such that

\[
s_1^\top c^* = 1. \tag{4}
\]

Let

\[
\hat{R}_1[n] = \sum_{j=1}^{n} y^\top[j]y[j] + \nu I \tag{5}
\]

and

\[
\alpha[n] = \left( s_1^\top \hat{R}_1[n]s_1 \right)^{-1}. \tag{6}
\]

Then the optimal detector at time \( n \) is

\[
c^*[n] = \alpha[n] \hat{R}_1^{-1}[n] s_1. \tag{7}
\]