Let $\chi$ be a Dirichlet character with conductor $Q$. Though the general case presents no additional difficulties, for ease of exposition, I will assume that $\chi(-1) = 1$. Let $L(s, \chi)$ be the Dirichlet $L$-function associated to $\chi$. What can we say about the $\mathbb{Q}$-dimension $\delta_\chi(a)$ of the space generated by the set 
\{\chi(1), \ldots, \chi(Q-1), L(3, \chi), \ldots, L(a, \chi)\} where $a$ runs through odd values? This question is the focus of my current research. In fact, I prove the following:

**Theorem.** For each $\epsilon > 0$ there is an $A(\epsilon)$ such that for $a > A(\epsilon)$

$$\delta_\chi(a) \geq \frac{1 - \epsilon}{Q + \log(2)} \log \left( \frac{a}{Q} \right).$$

In my talk I will show how one can arrive at such results using a criterion for linear independence and a suitably chosen auxiliary function.