Developing Flow of Gas-Particle Mixtures in Vertical Ducts

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It is well-known that during high-velocity flow of gas-particle mixtures through a vertical duct, the particles are distributed nonuniformly over the cross section. We had shown in an earlier paper that such a nonuniform state is sustained under fully-developed flow conditions by the large scale fluctuations accompanying such flows. In the present study, we have carried out a simple analysis of the developing flow problem to demonstrate that the evolution of the nonuniform distribution is indeed driven by turbophoresis.

1. Introduction

Experimental studies on high-velocity gas-particle flows in vertical pipes have revealed that particles are usually distributed over the cross section of the pipe in a nonuniform fashion (e.g., see Weinstein et al., 1986; Bader et al., 1988) and that such flows are inherently unsteady with large fluctuations in suspension density (e.g., see Schnitzlein and Weinstein, 1988). The presence of clusters and streamers of particles in such flows have also been reported (e.g., see Grace and Tuot, 1979; Gidaspow, 1994). It is generally believed that all these features persist even under fully-developed flow conditions, so many researchers have sought to explain the nonuniform distribution of particles over the cross section through analysis of equations of motion for fully-developed flow (Sinclair and Jackson, 1989; Louge et al., 1991; Dasgupta et al., 1994; Cao and Ahmadi, 1995; Hrenya and Sinclair, 1996). In an earlier paper (Dasgupta et al., 1994), we derived time-averaged equations of motion for fully-developed flow of gas-particle suspensions in a vertical pipe and showed that the occurrence of nonuniform distribution of particles is a direct consequence of the large scale fluctuations accompanying such flows. The present study is concerned with an analysis of the manner in which the flow patterns develop.

The axial development of cocurrent flow of gas-particle mixtures in vertical ducts is characterized by a number of different zones. The particles are initially accelerated by the high-velocity gas stream in the “acceleration zone” where large changes in the particle concentration, pressure gradient, etc., occur. Following the acceleration, the time-averaged particle concentration and velocity fields evolve slowly to their fully-developed states over a longer distance (“transition zone”). This is followed by a fully-developed zone, after which is an “end zone” where the flow patterns change again to conform to the exit geometry. If the duct is not “sufficiently tall”, the fully-developed zone may be absent and the exit and entrance zones will interact strongly. In the present study, we will focus our attention on ducts that are tall enough to prevent such interaction and examine the process of flow development.

The acceleration zone in the entrance region of a vertical riser has been analyzed previously by several researchers using a one-dimensional model (e.g., see Gidaspow, 1994). Such a model does indeed capture nicely the rapid decrease in the cross-sectional average particle concentration resulting from particle acceleration (Gidaspow, 1994). In the present study, we are concerned with the manner in which lateral nonuniformity evolves as flow patterns develop. One-dimensional models, by their nature, are unsuitable for this purpose.

In the present study, we derive the time-averaged equations of motion for developing flow, simplify them through a scaling argument, and employ a K-ε model to close the final system of equations. One can extract the laminar limit of this turbulent flow model by simply setting K to zero. This laminar model predicts that under fully-developed flow conditions the particles will be distributed uniformly over the cross section of the duct, which is known to be incorrect for high-velocity gas-particle flow through risers. Nevertheless, it is useful to analyze the laminar flow model, as a comparison of the predictions of the laminar and turbulent flow models can reveal the stage of flow development at which the turbulent fluctuations begin to play an important role.

Gravity plays an important role in determining the structure of fully-developed flow and the manner in which the flow develops. Therefore, it is useful to analyze the process of flow development in different situations, such as cocurrent downflow and cocurrent upflow. We also consider the case of flow in the absence of gravity to expose the role of gravity more clearly.

In both the laminar and turbulent flow models, the simplified system of equations describing steady developing flow contains only the first derivatives of dependent variables in the axial direction and the first and second derivatives in the transverse direction. This form allows us to analyze the acceleration and transition zones in a developing flow in tall units as an initial value problem. Such an approach can be used for most cocurrent downflow problems and for the hypothetical problems of flow development under gravity-free conditions. It will be seen that internal recirculation arises frequently during flow development in risers, so solving the developing flow as an initial value problem has limited use. Nevertheless, it is useful to examine the flow development problem as an initial value problem because of the tremendous computational advantage it offers over the corresponding boundary value problem. With this in mind, we have performed such calculations for cocurrent upflow, cocurrent downflow, and cocurrent flow in the absence of gravity in order to bring forth some of the features of the flow development process, such as segregation driven by kinematics and turbophoresis.
2. Model Equations

In the present study, we focus attention on vertical gas-particle flow in the gap between two vertical plates. The y-axis points in a direction perpendicular to the plates, and the plates are located at $y = \pm B$. The z-axis is vertical, but points in the direction of mean flow, so that it points up in cocurrent upflow, while for cocurrent downflow it points down.

As in our earlier paper (Dasgupta et al., 1994), we begin with the volume-averaged equations of motion proposed by Anderson and Jackson (1967):

$$\frac{\partial}{\partial t}(\phi_g) + \nabla \cdot (\phi_g \mathbf{u}_g) = 0$$

(1)

$$\frac{\partial}{\partial t}(\phi_s) + \nabla \cdot (\phi_s \mathbf{u}_s) = 0$$

(2)

$$\rho_s \left[ \frac{\partial}{\partial t}(\phi_s \mathbf{u}_s) + \nabla \cdot (\phi_s \mathbf{u}_s \mathbf{u}_s) \right] = \nabla \cdot \mathbf{E}_s + \phi_s \nabla \cdot \mathbf{E}_g + \rho_s \phi_s \mathbf{g} + \beta_1 (\mathbf{u}_g - \mathbf{u}_s)$$

(3)

$$\rho_g \left[ \frac{\partial}{\partial t}(\phi_g \mathbf{u}_g) + \nabla \cdot (\phi_g \mathbf{u}_g \mathbf{u}_g) \right] = \phi_g \nabla \cdot \mathbf{E}_g + \rho_g \phi_g \mathbf{g} - \beta_1 (\mathbf{u}_g - \mathbf{u}_s)$$

(4)

Here $\phi_s$, $\phi_g$, and $\mathbf{u}_s$, $\mathbf{u}_g$ are the volume fractions and velocities of the two phases, respectively, $\rho_s$ and $\rho_g$ are the densities of the solid and the gas, $\mathbf{E}_s$ and $\mathbf{E}_g$ are the stress tensors associated with the two phases, $\beta_1$ is the drag coefficient, and $\mathbf{g}$ is the specific gravity force.

We restrict our attention to heavily particle-laden flows, $\rho_s \phi_g \gg \rho_g \phi_s$, in which case, the gas momentum balance, eq 4, can be simplified by dropping the inertial terms:

$$\nabla \cdot \mathbf{E}_g + \rho_g \mathbf{g} - \beta (\mathbf{u}_g - \mathbf{u}_s) = 0$$

(5)

where $\beta = \beta_1 \phi_g$. Introducing this into (3), we get

$$\rho_s \left[ \frac{\partial}{\partial t}(\phi_s \mathbf{u}_s) + \phi_s \nabla \cdot (\mathbf{u}_s \mathbf{u}_s) \right] = \nabla \cdot \mathbf{E}_s + \nabla \cdot \mathbf{E}_g + \rho g$$

(6)

where $\rho = \rho_s \phi_g + \rho_g \phi_s$.

Adopting the same averaging procedure as in Dasgupta et al. (1994) and specializing the results to steady developing flow, we get

$$\nabla \cdot \mathbf{E}_s = \nabla \cdot \mathbf{E}_g = \rho g$$

In eq 9, the second term on the lower-hand side is the familiar Reynolds stress term, the third represents transport of momentum by turbulent dispersion, and the fourth is simply the contribution due to third-order correlation. It has been common practice to adopt the Newtonian fluid form of closure for the tensors $\mathbf{E}_s$ and $\mathbf{E}_g$ in eqs 3 and 4, with the effective viscosity and pressure terms in the particle phase stress specified explicitly as functions of particle concentration. Time-averaging then generates more complicated expressions for the time-smoothed tensors, $\mathbf{E}_s$ and $\mathbf{E}_g$, because of correlations involving particle concentration. As the Newtonian fluid form for the tensors $\mathbf{E}_s$ and $\mathbf{E}_g$ is, itself, without a firm basis and the key mechanical features such as occurrence of lateral segregation of particles are driven by the velocity correlations in eq 7 (and not the stress tensors $\mathbf{E}_s$ and $\mathbf{E}_g$), in our previous work on fully-developed flow, we simply assumed Newtonian closures directly for the time-smoothed tensors (Dasgupta et al., 1994):

$$\mathbf{E}_s = - \overline{\rho_s (\phi_s) \mathbf{I}} + \mu_{es}(\overline{\mathbf{u}_s}) [\nabla \mathbf{u}_s + (\nabla \mathbf{u}_s)^T - 2/3 (\nabla \cdot \mathbf{u}_s) \mathbf{I}]$$

(11)

where $\mu_{es}$ and $\overline{\rho_s}$ are the effective viscosity and pressure for the solid phase, respectively. The explicit functional form chosen for them are as follows:

$$\overline{\rho_s} = A_p (1 + 4 \phi_s g_0)$$

(12)

$$\mu_{es} = A_v \left[ (1 + 1.6 \phi_s g_0)^2 / g_0 + 9.77 \phi_s^2 g_0^2 \right]$$

(13)

where $g_0 = 1/(1 - (\phi_s \phi_m)^{1/3})$ and $\phi_m$ is the value of $\phi_s$ at random close packing (taken to be equal to 0.65). These are consistent in form with predictions of the granular kinetic theory (Lun et al., 1984), but the kinetic theory also relates the factors $A_v$ and $A_p$ to granular temperature. The effective pressure coefficient, $A_p$, according to the granular kinetic theory, is proportional to $\rho_s$ and the granular temperature, $T$, while $A_v$ is proportional to $\rho_s d_T^{1/2}$. Taking $v_T^{2/3}$ as a measure of $T$ for the present two-phase flow problem, we write the following:

$$A_p = C_p \rho_s d_T^{2/3}$$

(14)

and

$$A_v = C_v \rho_s d_T \left[ \frac{v_T}{\sqrt{3}} \right]$$

where $C_p$ is assigned a value of unity, unless mentioned otherwise, and $C_v$ is assigned a value of $3(3)^{1/2}$ so as to get a laminar particle phase viscosity of the order of 1 poise.

In heavily particle-laden flows, the viscous stresses in the gas phase play a minor role, so we simply write the following:

$$\overline{\mathbf{E}_g} = - \overline{\rho_g \mathbf{I}}$$

As discussed in Dasgupta et al. (1994), we approximate the time-averaged drag force term appearing in (10) as

$$\beta (\mathbf{u}_g - \mathbf{u}_s) \approx \beta (\phi_s \overline{\mathbf{u}_s} - \overline{\mathbf{u}_s})$$

(15)
and we adopt for \( \beta \) the form described by Gidaspaw (1994):

\[
\beta(\phi_g) = \frac{3}{4} C_D \frac{\bar{u}_g \bar{u}_s - \bar{u}_s^2}{d_p} f(\phi_g)
\]

\[
C_D = \begin{cases} 
24 & \text{Re}_g < 1000 \\
0.44 & \text{Re}_g \geq 1000 
\end{cases}
\]

where

\[
\text{Re}_g = \frac{\phi_g \rho_g d_p |\bar{u}_g - \bar{u}_s|}{\mu_g}
\]

and \( f(\phi_g) = (1 - \phi_g)^{-2.65} \)

The model equations presented above can be further simplified by examining the relative importance of the various terms. We are interested in studying flow development in applications where the flow is primarily axial. It is reasonable to expect that the axial velocities of the two phases will be proportional to the mean superficial velocity of the mixture, \( U \). Let \( L \) denote the typical length over which the flow pattern develops. It is well-known that in single-phase turbulent flow \( L \gg B \). We will see below that this is true for two-phase flow as well. It follows from continuity equations that the time-averaged transverse velocities of both phases will be of order \( U (B/L) \) and are therefore much smaller than the axial velocities. We also expect that the fluctuating components of the axial and transverse components of both phases will be proportional to \( U \) itself. Upon inserting these scales for the various velocities into the time-averaged equations above, it is found that the governing equations for steady developing flow in two dimensions can be further simplified to obtain the following:

\[
\frac{\partial}{\partial z} (\phi_g \bar{u}_{gz}) + \frac{\partial}{\partial y} (\phi_g \bar{u}_{gy}) + \frac{\partial}{\partial y} (\phi_g \bar{u}_{sy}) = 0
\]

\[
\frac{\partial}{\partial z} (\phi_g \bar{u}_{gz}) + \frac{\partial}{\partial y} (\phi_g \bar{u}_{gy}) + \frac{\partial}{\partial y} (\phi_g \bar{u}_{sy}) = 0
\]

\[
\rho_s \left[ \frac{\partial}{\partial z} (\phi_s \bar{u}_{sz} + \bar{u}_{sz} \bar{u}_s) + \frac{\partial}{\partial y} (\phi_s \bar{u}_{sz} \bar{u}_s) \right] =
\]

\[
- \frac{\partial p_g}{\partial z} - \frac{\partial p_s}{\partial z} + \gamma \left( \frac{\partial}{\partial y} (\phi_s \bar{u}_{sz}) + d \beta \bar{g} \right)
\]

\[
\rho_s \left( \frac{\partial}{\partial y} (\phi_s \bar{u}_{sy} \bar{u}_y) \right) = - \frac{\partial p_g}{\partial y} - \frac{\partial p_s}{\partial y}
\]

\[
- \frac{\partial p_g}{\partial z} + d \beta \bar{g} - \bar{g} (\bar{u}_y - \bar{u}_{sy}) = 0
\]

\[
- \frac{\partial p_g}{\partial y} - \bar{g} (\bar{u}_y - \bar{u}_{sy}) = 0
\]

The direction of flow is indicated by \( d_i \); \( d_i = 1 \) for cocurrent downflow and \( d_i = -1 \) for cocurrent upflow. Assigning a value of zero for \( d_i \) corresponds to flow in
the absence of gravity. Before we can proceed any further, we must postulate closure relations for the correlations appearing in eqs 17–20, and empiricism is inevitable at this stage. Although direct integration of the time-dependent volume-averaged equations of motion (eqs 1–4) would eliminate the need for this empiricism, such an integration would be extremely time-consuming. For most engineering applications, it is unnecessary, and a simplified model which is more easily solved and, at the same time, can capture the time-averaged behavior is adequate. Developing such a model is, inevitably, an iterative procedure, where one postulates closure relations for the correlations in a time-smoothed form of the equations of motion, calculates the outcome of the model, and refines the closure relations as needed. As in our previous work on fully-developed flow (Dasgupta et al., 1994, 1995), we consider a $K-\epsilon$ closure and write the following:

$$
\phi'_s \mathbf{u}'_s = -\nu_T \nabla \phi'_s \quad \text{and} \quad \phi'_g \mathbf{u}'_g = \nu_T \nabla \phi'_g
$$

$$
\mathbf{u}'_{sx} \mathbf{u}'_{sx} = \mathbf{u}'_{sy} \mathbf{u}'_{sy} = \mathbf{u}'_{sz} \mathbf{u}'_{sz} = 2/3 K \quad \text{and} \quad \mathbf{u}'_{sy} \mathbf{u}'_{sz} = -\nu_T \frac{\partial \mathbf{u}'_{sz}}{\partial y}
$$

where $K$ is the (turbulent) kinetic energy per unit mass associated with the largescale fluctuating motion of the particle clusters and $\nu_T$ is the kinematic viscosity associated with this cluster motion. As in the case of turbulent, single-phase flow, we write $\nu_T = C_{\mu} f_0 (K/\epsilon)$, where $\epsilon$ denotes the rate of dissipation of $K$ and the

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**Figure 2.** Lateral variation of (a) $\phi_s$, (b) $u'_s$, and (c) $u'_g$ in the acceleration zone for cocurrent flow in the absence of gravity, in the laminar limit. $\phi_{\theta_0} = 0.15$. See Tables 2 and 3 for particle and gas properties and the relationship between $v_s, \phi_{\theta_0}$, and $Re z^* = 0.0 (- -); z^* = 0.000 019 (- - -); z^* = 0.000 11 (- - -); z^* = 0.001 93 (- - -).

**Figure 3.** Lateral variation of (a) $\phi_s$, (b) $u'_s$, and (c) $u'_g$ in the transition zone for cocurrent flow in the absence of gravity, in the laminar limit. $\phi_{\theta_0} = 0.15$. See Tables 2 and 3 for particle and gas properties and the relationship between $v_s, \phi_{\theta_0}$, and $Re z^* = 0.0019 (- -); z^* = 0.0108 (- - -); z^* = 0.0487 (- - -); z^* = 0.1437 (- - -); z^* = 1.0 (- - -).
quantities $C_n$ and $f_n$ are defined in Table 1. Applying the K–ε closure scheme developed for single-phase flow to approximate the consequences of particle clusters on heavily particle-laden flows is speculative. It was found in our earlier studies (Dasgupta et al., 1994, 1995) that the K–ε model for the present gas-particle flow problem captured the features of fully-developed flow surprisingly well, which suggests that this closure scheme could serve an interim approximation for the velocity correlations appearing in the model equations until better closure models emerge. It is not yet clear whether this closure scheme will work satisfactorily for developing flow problems, and this is one reason for the present study of developing flows. Introducing these approximations into (17)–(20),

\[
\frac{\partial}{\partial z}(\phi_s U_{sz}) + \frac{\partial}{\partial y}(\phi_s U_{sy}) - \frac{\partial}{\partial y}(\nu \frac{\partial \phi_s}{\partial y}) = 0 \quad (23)
\]

\[
\frac{\partial}{\partial z}(\phi_s U_{sz}) + \frac{\partial}{\partial y}(\phi_s U_{sy}) + \frac{\partial}{\partial y}(\nu \frac{\partial \phi_s}{\partial y}) = 0 \quad (24)
\]

\[
\rho_s \left\{ \frac{\partial}{\partial z} \left[ \phi_s (U_{sz}^2 + 2 \nu K) \right] \right\} =
- \frac{\partial \rho}{\partial z} - \frac{\partial \rho_o}{\partial z} + \frac{\partial}{\partial y}(\mu_o \rho_o \nu_l \frac{\partial u_{sz}}{\partial y}) + d_g \frac{\partial \phi_s}{\partial y} \quad (25)
\]

\[
2 \nu \frac{\partial}{\partial y} \left\{ \rho_s \frac{\partial \bar{\phi}_5}{\partial y} \right\} = - \frac{\partial \rho}{\partial y} - \frac{\partial \rho_o}{\partial y} \quad (26)
\]

One can derive in an analogous manner equations for $K$ and $\epsilon$ and simplify them to obtain eq 27:

\[
\frac{\partial}{\partial z}(\rho_s \dot{\phi}_5 U_{sz} K) = \frac{\partial}{\partial y} \left[ \left( \mu_o + \frac{\mu_s \nu}{\sigma_k} \right) \frac{\partial K}{\partial y} \right] - \rho_s \dot{\phi}_5 \dot{\epsilon} + \frac{\dot{\phi}_5 \nu}{\sigma_l} \left( \frac{\partial u_{sz}}{\partial y} \right)^2 \quad (27)
\]

and

\[
\frac{\partial}{\partial z}(\rho_s \dot{\phi}_5 U_{sz} \epsilon) = \frac{\partial}{\partial y} \left[ \left( \mu_o + \frac{\mu_s \nu}{\sigma_\epsilon} \right) \frac{\partial \epsilon}{\partial y} \right] - C_{1,2} \rho_s \dot{\phi}_5 \epsilon^2 +
C_{1,5} \rho_s \dot{\phi}_5 \nu \left( \frac{\partial u_{sz}}{\partial y} \right)^2 \quad (28)
\]

Equations 21–28 then form the basis for analyzing steady, turbulent developing flow of heavily particle-laden mixtures in vertical ducts.

These equations have to be supplemented with boundary conditions at the walls and at the open ends of the duct. In the present study, we employ the following conditions at the walls (located at $y = \pm B$):

\[
\bar{u_{sy}} = \bar{u_{sy}} = K = \bar{u_{sz}} = \frac{dK}{dy} = 0; \quad \rho_s \dot{\phi}_5 \epsilon = 2 \mu_o \left( \frac{dK}{dy} \right)^2 \quad (29)
\]

Note that the simplified system of eqs 21–28 involves only first derivatives of the dependent variables in the
axial direction which, at least in principle, allows us to examine the acceleration and transition zones in cocurrent flows as an initial value problem, where we start with the known conditions at the inlet and march downstream. Such an approach should work nicely whenever the flow pattern evolves smoothly to the fully developed solution without forming regions of flow recirculation. If internal recirculation is encountered during the process of flow development, the scheme may fail catastrophically or it may yield a solution which must be carefully examined to be sure that it is physically meaningful. Even though such an uncertainty exists about the solution generated by viewing the system of equations as an initial value problem, it is worthwhile to pursue this approach as it is much simpler than that for the corresponding boundary value problem. In this manuscript, we will consider only solutions of the initial value problem.

**Laminar Flow Limit.** As particles and gas enter a vertical duct cocurrently, the wall resistance can be expected to slow them down in the near-wall region. This will then lead to an accumulation of particles near the wall. Simultaneously, the particles in the center of the duct will be accelerated by the high-velocity gas stream, giving rise to a rapid decrease in particle concentration with axial distance. These two kinematic effects combine to produce large transverse gradients in particle concentration in the acceleration zone. In order to examine the importance of turbulent fluctuations in the details of flow development in the acceleration zone, it is useful to consider the laminar limit of the above model. It will be shown through a comparison of the predictions of the turbulent and laminar flow models that turbulence does not play a significant role in the acceleration zone. The laminar flow limit is obtained by setting $K$ and $\nu_T$ to zero in eqs 21-26 and dropping the overbars.

**Table 3. Relationship between $\phi_{so}$, $\nu_s$, and $Re$**

<table>
<thead>
<tr>
<th>$\phi_{so}$</th>
<th>$\nu_s$</th>
<th>$Re$</th>
<th>$\phi_{so}$</th>
<th>$\nu_s$</th>
<th>$Re$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>0.0093</td>
<td>172.4</td>
<td>0.20</td>
<td>0.0427</td>
<td>705.2</td>
</tr>
<tr>
<td>0.10</td>
<td>0.0195</td>
<td>358.4</td>
<td>0.25</td>
<td>0.0562</td>
<td>841.6</td>
</tr>
<tr>
<td>0.15</td>
<td>0.0305</td>
<td>540.1</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The relevant wall boundary conditions for the laminar case are

Under fully-developed flow conditions, the velocities \( u_s \) and \( u_g \) vanish for all \( y \); it then follows from (31) that the gas phase pressure is uniform over the cross section. Introducing this into (35), one finds that the particle concentration will also be uniform over the cross section. It is well-known from experiments that this is not the case in high-velocity gas-particle flow in vertical ducts. Therefore, the results obtained by solving the laminar flow model are useful only to the extent that they can help clarify features of the turbulent flow development process.

Figure 6. Lateral variation of (a) \( \phi_s \), (b) \( u_{sy} \), and (c) \( u_{sz} \) in the transition zone for cocurrent downflow, in the laminar limit. \( \phi_{so} = 0.15 \). See Tables 2 and 3 for particle and gas properties and the relationship between \( \nu_s, \phi_{so} \), and \( \text{Re} z^* \).

\[ \nu_s = 0.002 \; (--) ; \quad z^* = 0.0762 \; (--) ; \quad z^* = 0.4962 \; (--) ; \quad z^* = 1.496 \; (--) . \]

Figure 7. Lateral variation of (a) \( \phi_s \), (b) \( u_{sy} \), and (c) \( u_{yz} \) in the transition zone for cocurrent upflow, in the laminar limit. \( \phi_{so} = 0.15 \). See Tables 2 and 3 for particle and gas properties and the relationship between \( \nu_s, \phi_{so} \), and \( \text{Re} z^* \).

\[ \nu_s = 0.0021 \; (--) ; \quad z^* = 0.0185 \; (--) ; \quad z^* = 0.486 \; (--) ; \quad z^* = 1.486 \; (--) . \]

3. Results and Discussion

Laminar Flow Model. Before discussing the results obtained by solving the two-phase laminar flow model, we will consider briefly the corresponding single-phase incompressible flow problem. In the entrance region, the continuity and momentum balance equations can be simplified to obtain

\[ \frac{\partial u_s^*}{\partial z^*} + \frac{\partial u_g^*}{\partial y^*} = 0 ; \quad \frac{\partial p^*}{\partial y^*} = 0 \]

\[ u_s^* \frac{\partial u_g^*}{\partial z^*} + u_g^* \frac{\partial u_s^*}{\partial y^*} = - \frac{\partial p^*}{\partial z^*} + \frac{\partial^2 u_s^*}{\partial y^*^2} \]
where the gravitational potential has been absorbed into the pressure gradient term and all the variables have been rendered dimensionless as follows:

\[
y^* = y/L; \quad z^* = z/L, \quad L/B = Re = BU/\nu
\]

\[
u_s^* = u_s/L; \quad \phi_u^* = u_y/L; \quad \rho_u = \rho U^2
\]

We consider inlet velocity profiles which can be represented as \( u_z = (n + 1)/n[1 - y^2] \) with \( n \) being an adjustable exponent. For all values of \( n \), the fully-developed solution has the well-known parabolic form, \( u_z^* = \frac{1}{2}[1 - (1 - y^2)^{3/2}] \). Two integrations were carried out for \( n = 5 \) and \( n = 8 \) to investigate whether the value assumed for \( n \) influences the order-of-magnitude value of \( z^* \) at which the flow becomes nearly fully-developed. We simply note that, in both cases, by the time \( z^* \approx 0.05 \), the flow was essentially fully-developed.

In what follows, we present the results obtained with the laminar two-phase flow model for a specific set of conditions. The half-width of the duct, \( B \), is 20 cm, and the lateral variation of the particle and gas phase velocities at the inlet are of the form

\[
u_{s\infty} = [1 - y^2] (m/s), \quad \phi_{s\infty} = (5[1 - y^2] + 0.5) (m/s)
\]

At the inlet, it is also assumed that the concentration of particles (\( \phi_{s0} \)) is uniform over the cross section, and the value of \( \phi_{s0} \) is left as a parameter. The superficial velocities (in m/s) of the particle and gas phases are then given by \( U_\nu = (5/6)(\phi_{s0}) \) and \( U_\phi = (25/6)(1 - \phi_{s0}) + 0.5 \), respectively. The mixture superficial velocity, \( U \), is \( 5(1 - \phi_{s0}) + 0.5 \) (m/s). The gas and particle properties assumed for the purpose of these calculations are summarized in Table 2. Let us define a Reynolds number for the laminar problem by

\[
Re = \frac{\rho \nu_s^* UB}{\mu_{s\infty}(\nu_s)} \quad \text{where} \quad \nu_s = \frac{U_s}{U}
\]

\( \nu_s \) is a good estimate of the particle concentration in the duct under fully-developed laminar flow conditions. As in the case of single-phase flow, one can define a characteristic length in the axial direction, \( L \), as \( L = B Re \), and then it is straightforward to cast the two-phase laminar flow equations in dimensionless form using \( B \) and \( L \) as characteristic lengths in the \( y \) and \( z \) directions, respectively, \( U/B \) and \( U \) as the characteristic velocities in the \( y \) and \( z \) directions, respectively, and \( \rho \nu_s g L \) as the characteristic scale for the gas pressure. The dimensionless equations will not be presented here, but can be found elsewhere (Dasgupta, 1996).

The parabolic partial differential equations describing developing flow were converted to a system of ordinary differential equations (in \( z^* \)), through discretization of the variables in the \( y^* \)-direction by a Galerkin method using a piecewise cubic Hermite polynomial approximation in \( y^* \), which was integrated using a fully-implicit, Euler method. The details of the analysis are described elsewhere (Dasgupta, 1996).

Figure 1 presents axial variation of laterally averaged values of \( \phi_{s\infty} \), \( \phi_{s\infty,y} \), and dimensionless axial pressure gradient (\( \nu_s \nu_s \gamma \)) for steady developing flow in the absence of gravity for five different values of \( \phi_{s0} \). Changing \( \phi_{s0} \) changes \( \nu_s \) and \( Re \), see Table 3. The two plots on the left show the rapid changes occurring in the acceleration zone, while those on the right present the behavior in the transition zone. It is clear that most of the changes in these variables occur in the acceleration zone. Thus, if one tries to determine whether flow has become fully-developed by measuring changes in the axial pressure gradient, which is a frequently adopted criterion in experiments, it will be concluded that the flow is essentially fully-developed after the acceleration zone. Figures 2 and 3 show the lateral variation of particle concentration and the two axial velocities at several different axial locations in the acceleration and transition zones, respectively, for one of the cases shown in Figure 1. The solid lines in Figure 2 show the conditions at the inlet. In the acceleration zone, the particle concentration decreases rapidly at the center of the duct, as one would expect; see Figure 2a. In the near-wall region, the particle concentration increases at first and then begins to decrease. This decrease can be traced to a lateral flux of particles induced by the lateral gradient in the particle phase pressure (associated with particle concentration); see eq 12. The rapid increase in the axial velocity of the particles in the acceleration zone is illustrated in Figure 2b. Correspondingly, the gas velocity decreases in the acceleration zone; see Figure 2c. Although we had set the gas pressure to be uniform over the cross section at the inlet, a small lateral variation of gas pressure is predicted to develop in near-wall region of the acceleration zone (not shown). This variation becomes negligible very quickly, so that by the time \( z^* = 0.0001 \) it is hardly noticeable. It is clear from Figures 3a–c, which extends the results to larger values of \( z^* \), that both the lateral variation of particle concentration and the two velocities evolve...
gradually in the transition zone. The velocity profiles reach their asymptotic form by \( z^* \approx 0.14 \), while the particle concentration profile requires a value of \( z^* \) almost equal to unity. As seen in Table 3, the characteristic axial length for this case, \( L \), is over a hundred meters, so that the acceleration zone (\( z^* \approx 0.002 \)) is only about 20 cm long, while the transition zone is over 10 meters in length (\( z^* > 0.1 \)).

The same set of calculations has been repeated for both cocurrent downflow and cocurrent upflow. Figures 4 and 5 show \( \left[ \phi_s \right]_{avg} \) and \( \left( \partial \phi_s / \partial z \right)_{avg} \) as functions of axial distance, respectively. In each of these figures, the top two plots correspond to cocurrent upflow, while the bottom two are for cocurrent downflow; the plots on the left are magnifications of the acceleration zone. One again sees that these variables change rapidly in the acceleration zone, while their changes in the transition zone are minimal. It can be seen readily from Figures 1 and 4 that axial variation of \( \left[ \phi_s \right]_{avg} \) in the acceleration zone is nearly identical for all three cases considered (cocurrent upflow, cocurrent downflow, and flow in the absence of gravity). Indeed, the lateral variation of particle concentration and the two velocities at various axial locations in the acceleration zone are also nearly identical in these three cases (Dasgupta, 1996).

Gravity does play a significant role in the manner in which the flow pattern evolves in the transition zone. Figures 6 and 7 show the concentration and velocity profiles at different axial locations in the transition zone for cocurrent downflow and cocurrent upflow, respectively, and these can be contrasted with each other and with Figure 3 for flow in the absence of gravity. It is clear from Figure 3b that the velocity of the particle phase in the wall region steadily decreases because of

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**Figure 9.** Lateral variation of \( \phi_s \) at different axial locations predicted by the turbulent flow model (−−−) and its laminar limit (−−−−) for cocurrent flow in the absence of gravity. (a) \( z^* = 0.0, 0.000057, \) and 0.0012; (b) \( z^* = 0.0117; \) (c) \( z^* = 0.0302; \) (d) \( z^* = 0.0756; \) (e) \( z^* = 0.0974; \) (f) \( z^* = 0.134. \)
the wall resistance, while in the center of the duct the velocity increases gradually. The only effects of the relatively higher concentration of particles in the wall region are to increase the particle phase viscosity and pressure there. In cocurrent upflow, the weight of particles in the near-wall region is in excess of the available pressure gradient, and therefore, the upward velocity of the particles in this region decreases more rapidly, when compared to the case of flow in the absence of gravity (compare Figures 3b and 7b). Indeed, at some intermediate stage during the flow development, the particle phase velocity in the wall region in the cocurrent upflow example falls below the fully-developed value (a feature not seen in the case of flow in the absence of gravity), even becoming slightly negative, indicating the presence of a region of internal recirculation. Note that downflow in the wall region is absent in the fully-developed, cocurrent upflow in the laminar limit (Figure 7b), so that the observed internal recirculation is associated with the process of flow development only. As mentioned earlier, the results obtained by viewing the developing flow as an initial value problem should be examined carefully to ensure that they are physically meaningful. To do this, we calculated each term appearing in the model equations, including those discarded through order-of-magnitude analysis to make sure the terms that had been dropped were indeed small and that the computed flow patterns did not change when we tightened the tolerance requirements in integration and the number of nodes used in the spatial discretization (in the y-direction).

In cocurrent downflow, the weight of the particles in the near-wall region is again higher than the available pressure gradient, which tends to partially offset the
tendency of wall resistance to slow down the motion of the particles. Indeed, a small maximum develops in the velocity at an intermediate value of $y^*$ (away from $y^* = 0$) at some intermediate stage during the process of flow development. It should be noted that such a maximum does not have to exist. Thus, in flow through narrower ducts, the lateral segregation of particles induced by the kinematic effect in the acceleration zone relaxes more rapidly toward the uniform distribution and the evolution of the (axial) velocity profile with axial distance (i.e., $u_s$ as a function of $y^*$ at different values of $z^*$) resembles that for the case of flow in the absence of gravity.

The changes in the velocity of the gas phase parallel to those seen in the particle phase velocity (compare Figures 3b and 3c, 6b and 6c, and 7b and 7c), which is indeed expected. In all three cases (cocurrent upflow, cocurrent downflow, and flow in the absence of gravity), the lengths of duct needed for flow development are comparable, $z^* \approx 0.5 - 1.0$. As the characteristic axial length is the same in all three cases, one sees that flow development requires typically the same dimensional length as well. In any case, this duct height is larger than those typically used to study riser flows, which suggests that flow patterns recorded in many of the experimental studies may not reflect fully-developed flow.

In the laminar flow model, the variation of the transverse distribution of particle concentration with axial distance in the transition zone is also affected by the dependence of the particle phase pressure on particle concentration. Thus, for example, increasing the value of the proportionality constant $C_p$ to 2 (from 1) decreases the estimated length of the transition zone by a factor of 3 (Dasgupta, 1996).

**Turbulent Flow Model Calculations.** In order to integrate the $K - \epsilon$ model equations for steady developing flow presented earlier, we must prescribe initial conditions for both $K$ and $\epsilon$. In the examples presented below, we have retained the inlet velocity profiles for the gas and particles described earlier (eq 37) and assumed that

\[
K = K_o [3(1 - y^*)^2 - 2(1 - y^*)^3] \quad \text{for } y^* \geq 0; \quad \epsilon = \epsilon_o
\]

where $K_o$ and $\epsilon_o$ are taken to be 0.1 and 0.005 m$^2$/s$^3$, respectively. The particular functional form chosen for $K$ satisfies the requirement that both $K$ and its first derivative with respect to $y^*$ vanish at $y^* = 1$. Furthermore, this expression has positive curvature at $y^* = 1$ so that the $\epsilon$ boundary condition can be satisfied there. The specific choice of value for $K_o$ mentioned above is equivalent to assuming that the magnitude of the velocity fluctuations at the inlet is about one-tenth of the superficial velocity of the feed mixture. The order-of-magnitude of $\epsilon_o$ is consistent with that estimated from the $\epsilon$ boundary condition (eq 29).

Before presenting some specific examples, it is useful to summarize the general trends observed in the simulations. The predictions of the turbulent flow model and the laminar model are essentially indistinguishable in the acceleration zone. In both cases, an appreciable buildup of the particles occurs in the near-wall region early in the acceleration zone, because of
the kinematic effect discussed earlier. This sets up a significant driving force for a lateral flux of the particles toward the center of the duct, as $\rho_s$ increases with particle concentration, and the extent of segregation decreases significantly with downstream distance (still in the acceleration zone). Although the turbulent fluctuations have not had much influence in this zone, both the intensity of fluctuations, as characterized by the magnitude of $K$, and the rate of their dissipation, $\epsilon$, increase rapidly.

Further downstream, now in the transition zone, both $K$ and its lateral variation have become large enough that the process of segregating the particles toward the wall region (turbophoresis) begins once more. Simultaneously, $\epsilon$ increases, which reverses the trend of increasing $K$ (with axial distance). It was found that for all the cases investigated of flow in the absence of gravity and of cocurrent downflow, the intensity of the fluctuations gradually decreased with axial distance, suggesting that the solution was evolving toward the laminar limit. In the cocurrent upflow examples analyzed, integration of the problem as an initial value problem could be extended only up to $z/B$ of about 10, after which it failed catastrophically.

The axial variation of $[\phi_s]_{avg}$ and $(\partial \rho_s/\partial z)_{avg}$, as predicted by the turbulent flow model and the laminar limit of this model, are presented in Figure 8 for cocurrent flow in the absence of gravity. In this and all the subsequent examples, we have set the half-width of the duct to 10 cm and $\phi_{so}$ to 0.20. The axial characteristic length, $L$, is then equal to 35.26 m. It is clear from Figure 8 that in the acceleration zone, the turbulent flow model and its laminar limit are essentially indistinguishable. In both models, $[\phi_s]_{avg}$ decreases rapidly with axial distance in the acceleration zone. The turbulent flow model predicts that $[\phi_s]_{avg}$ will reach a minimum

Figure 13. Lateral variation of $\phi_s$ at different axial locations predicted by the turbulent flow model (---) and its laminar limit (----) for cocurrent downflow. (a) $z^* = 0.00017$ and 0.0011; (b) $z^* = 0.011$; (c) $z^* = 0.030$; (d) $z^* = 0.084$; (e) $z^* = 0.163$; (f) $z^* = 0.283$. 

value and then recover slightly, while this recovery is not seen with the laminar model. The variation of axial pressure gradient with $z^*$ reflects the same characteristics.

The lateral variation of $\phi_s$ at various axial locations, as predicted by the turbulent flow model and the laminar limit, are shown in Figure 9. The corresponding lateral variation of $u_{z2}$ at various axial locations are shown in Figure 10, while the $K$ and $\epsilon$ profiles for the turbulent flow model are shown in Figure 11. The $\phi_s$ and $u_{z2}$ profiles for the two models in the acceleration regime are essentially indistinguishable, both during the initial segregation caused by the kinematic effect, and the subsequent weakening of the extent of segregation resulting from the particle phase pressure; see Figures 9a and 10a. As discussed in the earlier section, the extent of segregation is predicted by the laminar model to decrease monotonically in the transition regime; see the broken lines in Figure 9. In contrast, the turbulent flow model predicts that the particles will segregate toward the wall again in the transition regime (Figures 9b and 9c). At further downstream distances the extent of segregation gradually decreases (Figures 9d and 9e). Although the two velocity profiles separate in the transition regime, they approach each other once again further downstream; see Figure 10. In the acceleration regime, both $K$ and $\epsilon$ increase rapidly; see Figure 11. It is clear from a comparison of Figures 9b and 11a that the resegregation of particles in the transition regime is indeed driven by turbophoresis, i.e., lateral gradient in $K$. It can also be seen from Figure 11a that after a period of initial increase the magnitude of $K$ gradually decreases with axial distance (in the transition regime), and this coincides with the weaken-

Figure 14. Lateral variation of the axial velocity of the particles at different axial locations predicted by the turbulent flow model (—) and its laminar limit (---) for cocurrent downflow. (a) $z^* = 0.00017$ and $0.0011$; (b) $z^* = 0.011$; (c) $z^* = 0.030$; (d) $z^* = 0.084$; (e) $z^* = 0.163$; (f) $z^* = 0.283$. Ind. Eng. Chem. Res., Vol. 36, No. 8, 1997 3387
ing of the extent of segregation seen in Figures 9d–f. Figure 11b reveals that the magnitude of $\epsilon$ continues to remain large in the transition regime and that this is responsible for the gradual weakening of the turbulent fluctuations. Clearly, at further downstream distances, the turbulent flow model will collapse to the laminar limit.

In order to verify whether this decay of fluctuations is a consequence of the fact that the Reynolds number, as defined by (38), is too low, calculations were performed by decreasing the value assumed for the laminar viscosity of the particle phase; however, the decay of fluctuations persisted. It is not clear at present whether this initial intensification of turbulence followed by a decay is a real phenomenon or an indication of a deficiency of the $K-\epsilon$ model for the two-phase flow; we are inclined to suspect the latter. In the absence of data on fluctuation statistics, it is difficult to identify what this deficiency is and is even harder to propose a remedy. Ideally, one should generate such fluctuation statistics experimentally; however, this is a formidable problem as tomographic techniques that can measure fluctuations without perturbing the flow are just beginning to become available. An alternate approach would be to generate such statistics through transient integration of the volume-averaged equations, which is more easily done than the experiments. Unfortunately, papers describing results generated through such transient integrations do not report the values of correlations needed for a time-averaged model for flow. It is hoped that the present manuscript will prompt researchers engaged in study of heavily particle-laden flow to report fluctuation statistics generated through experiments and/or computations. It must be emphasized, even with this uncertainty about the adequacy of the $K-\epsilon$ closure, that the present set of calculations plays a useful role by revealing unequivocally the role of turbophoresis in driving the particles to the wall region.

An illustration of the predictions of the turbulent flow model and its laminar limit for cocurrent downflow is provided in Figures 12–15. The axial variation of $\phi_s$ at different axial locations is shown in Figure 12, which is qualitatively similar to Figure 8 (for flow in the absence of gravity). The lateral variation of $\phi_s$ at different values of $z^*$, shown in Figure 13, reveals clearly the segregation induced in the acceleration zone by the kinematic effect and that due to turbophoresis in the transition zone. The lateral variation of $u_{z\phi}$ at different values of $z^*$, shown in Figure 14, develops at some intermediate value of $y^*$ in the case of the turbulent flow model (see Figures 14c–f). The maximum in $u_{z\phi}$ that develops at some intermediate value of $y^*$ in the case of the turbulent flow model (see Figures 14c–f) is a direct consequence of the accumulation of particles in the wall region. The intensity of turbulent fluctuations and the rate of its dissipation, $\epsilon$, increase rapidly in the acceleration zone (Figure 15). In the transition zone, $\epsilon$ continues to remain large, which gives rise to a gradual decay in the turbulence intensity.

Figure 16 shows $\phi_s$ as a function of $y^*$ at different axial locations in cocurrent upflow in order to illustrate that the initial segregation due to kinematic effect is once again partly dissipated by the action of particle
phase pressure and that particles segregate toward the wall at a later stage through turbophoresis. The integration could not be continued much further beyond \( z^* = 0.023 \). A close look at the reason for this failure of the integration revealed that \( K \) was becoming extremely small close to the wall and strong downflow was developing there (Dasgupta, 1996). In any case, this partial integration again confirms the role of turbophoresis in driving the segregation.

4. Summary

Although the time-averaged values of the axial pressure gradient and the cross-sectional average concentration of particles become roughly constant after the acceleration zone, the lateral variation of the time-averaged particle concentration and the velocities of the two phases continue to evolve over much longer distances. The length of the flow development zone expressed as a multiple of duct half-width is proportional to a Reynolds number defined by eq 38. As seen from Table 3, this Reynolds number increases with the flux of particles, and therefore, the flow patterns develop over longer lengths of duct at higher fluxes of particles. This suggests that the radial variation of particle concentration and velocities reported in many of the experimental studies on riser flow may not be representative of fully-developed solutions. Therefore, a comparison of the predictions of the models for fully-developed flow with experimental data, particularly those at high-particle flux, should be made with caution. Turbulent fluctuations play a negligible role in the acceleration zone. Nonuniform distribution of particles evolving in this zone is primarily a kinematic effect.

However, the nonuniform distribution of particles observed in experiments at large distances away from the entrance is not due to the persistence of this kinematic effect. Instead, this segregation is due to turbophoresis, and this can be seen clearly in the model calculations. Calculations performed with a speculative \( K - \epsilon \) closure suggest that in gas-particle flow through vertical ducts the turbulent fluctuations intensify at first and subsequently decay. Experimental data on fluctuation statistics in such flows are needed to assess if this is a real phenomenon or a deficiency of the model.

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