Role of wall friction in fluidization and standpipe flow

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Abstract

Fluidization–defluidization experiments have been carried out in beds of three different diameters, using XL glass beads and air as the fluidizing medium. The variation of pressure drop and bed height manifested a hysteretic behavior, which was more pronounced in smaller tubes. Analysis of the results using a one-dimensional model proposed by Jackson revealed that the observed effect of tube diameter may be attributed to wall friction. From this analysis, we extracted a quantitative estimate of the variation of the compressive yield stress of an assembly of particles as a function of particle volume fraction. The standpipe performance data reported by Srivastava et al. [Powder Technol. 100 (1998) 173.] for these glass beads were analyzed on the basis of the estimated compressive yield strength. It was found that the support provided by the standpipe wall could be estimated quantitatively from the standpipe holdup data and the compressive yield strength determined separately from our fluidization–defluidization experiments. © 2002 Elsevier Science B.V. All rights reserved.

Keywords: Fluidization; Defluidization; Circulating fluidized bed; Standpipe; Wall friction

1. Introduction

Circulating fluidized beds (CFB) have been widely used over many decades for fluid catalytic cracking (FCC), and for roasting and combustion applications. The commercial success of FCC has led to the use of CFB in other industrial applications such as Fischer–Tropsch synthesis in the SASOL synthetic gasoline process, oxidation of n-butane to maleic anhydride and calcination of aluminum hydroxide to high-grade alumina [1]. A CFB loop typically consists of a riser, cyclones, fluidized bed and a solids return system, usually comprising a standpipe and a valve. Risers and standpipes are known to exhibit a rich variety of flow behavior. Experimental measurements reveal that particle concentration is non-uniform over the cross-section of a vertical riser with particles being concentrated near the walls [2]. Standpipes are known to exhibit instabilities under certain operating conditions, which lead to inadequate pressure buildup in the pipe [3,4].

Although there have been several studies on gas–particle flow instabilities in the individual components of the CFB circuit, much less is understood about the particle circula-
tions fluidized beds can display a mechanical structure, whose strength decreases with increasing bed expansion. Furthermore, these authors attributed the existence of sur-
presure at minimum fluidization (i.e. the additional pres-
sure drop in excess of the weight of the bed which is
required to initiate the expansion of a packed bed) to
particle–particle and particle–wall interactions. Tsinontides
and Jackson [9], in experiments involving fluidization
and defluidization of cracking catalyst particles in air,
observed a marked hysteresis in the fluidization character-
istics, and attributed it to a yield stress in the granular
assembly resulting from inter-particle and particle–wall
interactions.

There is no doubt that stresses resulting from cohesive
and/or frictional interactions play an important role in
determining the mechanical behavior of granular assemblies
such as those found in fluidized beds and standpipes. Yet,
quantitative information on the magnitude of such stresses is
scanty, thus limiting quantitative understanding of the com-
plex flow behavior observed in circulating fluidized bed
experiments.

Jackson [10] has argued that information about the nature
and magnitude of contact stresses arising due to frictional
interactions can be obtained through simple measurements
of bed height and pressure drop during the process of
fluidization and defluidization. In the present study, we
have performed such fluidization–defluidization experi-
ments in beds of different diameters in order to estimate
the magnitude of contact stresses. The particles used in this
study are XL glass beads, the same as those used in our
earlier CFB experiments [6]. These data provide strong
evidence for the effect of wall stresses on fluidization and
defluidization characteristics. In this paper, we demon-
strate that the simple 1-D model proposed by Jackson is able to
capture several aspects of our data. We have also extracted
from these data a quantitative estimate for the yield stresses
in assemblies of these particles.

Armed with the yield stress information, we have ana-
lyzed the standpipe data reported by Srivastava et al. [6]. We
find that a simple one-dimensional model of granular flow
can capture the general trends observed in our circulating
fluidized bed experiments.

This paper is organized as follows. Section 2 outlines the
1-D model for the effect of wall friction described by
Jackson [10,11]. The experimental system and procedure
employed in our fluidization–defluidization experiments are
presented in Section 3. Section 4 describes the fluidized bed
data and analysis. In Section 5, we examine the standpipe
data of Srivastava et al. [6]. Section 6 summarizes the major
results of this study.

2. Theory

An analysis of the role of contact stresses on the
mechanical behavior of fluidized beds of non-cohesive
particles has been described recently by Jackson [10,11].
The present treatment is a minor modification of his
model. Let us consider a bed of particles in a vertical tube of
diameter $D$ subject to a uniform upward flow of a gas at
a superficial velocity $\bar{u}$. Following Jackson, we consider
the classical Janssen’s one-dimensional analysis of stress
distribution in a packed bed and write the balance of forces
in the vertical direction acting on the assembly of particles
as

$$\frac{d\sigma_y}{dy} = \rho_p \bar{u} - \beta(\nu) \frac{\bar{u}}{1-\nu} \pm \frac{4}{D} \mu \sigma_s$$

(1)

where $y$ is the vertical coordinate measured from the upper
surface of the bed (with the $y$-axis pointing downwards). Here, $\sigma_s$ is the cross-sectional average of the $\gamma_y$-component
of the compressive stress in the particle phase, transmitted
through sustained contacts between particles. The term on
the left-hand side of the equation represents the gradient in
this stress. The first term on the right represents the gravita-
tional force acting on the particles where $\rho_p$ is the density
of the particles, $\nu$ is the volume fraction of particles and $g$ is
the specific gravity force. The second term on the right
represents the drag exerted by the fluid on the particles,
where $\beta$ is the drag coefficient. The third term on the right
represents the frictional force exerted on the particle assem-
bly by the walls of the container. Here, $j$ is the Janssen’s coefficient (usually assumed to be a constant) which is the ratio between the normal stress exerted by the particle assembly on the walls and $\sigma_s$, while $\mu$ is the coefficient of wall friction. The sign of this term depends on the direction in which friction is acting. Therefore, the negative sign applies when the bed is compacting during the process of defluidization while the positive sign applies when the bed is at incipient (compressive) yield everywhere. In other words, $\sigma_s = \sigma_s^c$. In this case, Eqs. (1) and (3) can be combined to obtain

$$\frac{dp}{dy} = \beta(\nu) \frac{\bar{u}}{1 - \nu}$$

where $p$ is the is the fluid phase pressure. The form for $\beta$ is assumed to be

$$\beta(\nu) = \frac{\rho_p v_g}{v_t} \frac{1}{(1 - \nu)^{-1}}$$

where $v_t$ is the terminal velocity of a single particle and $n$ is the Richardson–Zaki [12] index.

Non-cohesive granular material can support only compressive stresses, and the compressive yield stress $\sigma_s^c$ can be expected to be a monotonically increasing function of $\nu$. Various expressions have been proposed for the functional dependence of $\sigma_s^c$ on $\nu$, see for example Atkinson and Bransby [13], Schaeffer [14], Prakash and Rao [15] and Johnson et al. [16]. As discussed in a greater detail later, the following simple functional form for $\sigma_s^c$, considered by Jackson [10], was found to be adequate for capturing our experimental results

$$\sigma_s^c(\nu) = \begin{cases} F \frac{(\nu - \nu_{min})}{(\nu_{max} - \nu)} & \nu_{min} < \nu < \nu_{max} \\ 0 & \nu < \nu_{min} \end{cases}$$

where $F$ is a constant. This functional form assumes that sustained frictional contact between multiple neighbors does not occur at values of $\nu \leq \nu_{min}$. Furthermore, the compressive yield stress is postulated to diverge as $\nu \rightarrow \nu_{max}$. (In practice, one can pack the particles at volume fractions exceeding $\nu_{max}$ without crushing the particles by mechanical means such as tapping. The above form assumes that such higher packing levels are not attainable in simple fluidization–defluidization processes.)

2.1. Defluidization

Consider a defluidization experiment in which the gas flow rate is reduced in small increments, allowing the bed to equilibrate at each gas flow rate. In such an experiment, the bed height generally decreases monotonically with gas flow rate, so that the bed is progressively compressed by its own weight. An important assumption in the model is that the local volume fraction of the particles is such that the bed is at incipient (compressive) yield everywhere. In other words, $\sigma_s = \sigma_s^c$. In this case, Eqs. (1) and (3) can be combined to obtain

$$\frac{d\nu}{dy} = \left( \rho_p v_g - \rho_p v_g \frac{1}{v_t} \frac{1}{(1 - \nu)^{\nu - 0.4}} \right) \left( \frac{d\phi_s^c}{d\nu} \right)$$

This equation can be integrated numerically, starting at $\nu = 0$ where $\nu = \nu_{min}$ (corresponding to the condition that $\sigma_s = \sigma_s^c = \sigma_s^c = 0$ at the top of the bed) and terminating at the bottom of the bed at $y = H$ such that

$$\int_0^H \nu(y) dy = m$$

where $m$ is the specified mass loading of particles per unit cross-sectional area. Once the particle volume fraction profile in the bed is determined, it is straightforward to integrate Eq. (2) to find the overall pressure drop across the bed. In this manner, one can determine the pressure drop and bed height as functions of $\bar{u}$ to generate the theoretical defluidization curve.

The solids volume fraction and stress profiles at the end of the defluidization cycle can be found from Eqs. (5) and (6) by letting $\bar{u} = 0$. Let us now consider the process of reflooding from this point onwards.

2.2. Fluidization cycle

When the gas velocity is increased gradually from zero, the pressure drop across the bed increases steadily while the height of the bed remains unaltered until a critical gas velocity, $\bar{u}_c$, is reached. In this regime, as we increase $\bar{u}$ ($\bar{u} < \bar{u}_c$), a larger and larger fraction of the weight of the bed is supported by the upward drag exerted by the gas on the particles. This will necessarily change the stress profile, $\sigma_s(y)$, in the particle phase while the solids volume fraction profile, $\nu(y)$ remains unaltered. Jackson [10] argued that as $\bar{u}$ is progressively increased $\sigma_s$ at the bottom of the bed must correspondingly take on lower and lower values. He further argued that when $\bar{u} = \bar{u}_c$, $\sigma_s$ at the bottom of the bed becomes zero. At this point, the bottom of the bed loses contact with the distributor (provided no cohesive interaction between the particle assembly and the distributor exists), and the entire bed is lifted by the fluid. As the bed rises, the particles at the bottom rain down and eventually recom pact to form a new bed.

The value of $\bar{u}_c$ can be determined as follows. We combine Eqs. (1) and (3) and write

$$\frac{d\sigma_s}{dy} - \frac{4}{D} \mu \sigma_s = \rho_p v_g - \rho_p v_g \frac{1}{v_t} \frac{1}{(1 - \nu)^{\nu - 0.4}} \bar{u}$$

$$\frac{d\phi_s^c}{d\nu} = \frac{d\phi_s^c}{d\nu}$$
with friction acting to oppose the bed expansion. Here, \( \nu(y) \) is the particle volume fraction profile determined from defluidization calculations with \( \bar{u} = 0 \). Thus

\[
\sigma_s(y = H) e^{-JH} - \sigma_s(y = 0) = \rho_p g \int_0^H \nu e^{-Jy} dy - \rho_p \bar{u} g \bar{u} \int_0^H \frac{\nu}{1 - \nu} e^{-Jy} dy \tag{8}
\]

where \( J = (4/D) \mu \bar{u} \). At the point at which the bed rises, \( \sigma_s \) is zero both at the top and at the bottom of the bed, so the left hand side of Eq. (8) vanishes when \( \bar{u} = \bar{u}_c \). Therefore, we have

\[
\bar{u}_c = \frac{\int_0^H \nu e^{-Jy} dy}{\int_0^H \frac{\nu}{1 - \nu} e^{-Jy} dy} \tag{9}
\]

For any \( \bar{u} \leq \bar{u}_c \), one can readily integrate Eqs. (2) and (7) to determine the stress and pressure profiles, respectively. In this manner, we determine how the overall pressure drop across the bed varies with \( \bar{u} \) (i.e. the fluidization curve).

Jackson [10] assigned the same value for the Janssen’s coefficient \( j \) for both fluidization and defluidization, so the role of wall friction was captured by a single parameter \( J \) in the entire fluidization–defluidization cycle. If cylindrical symmetry holds, then along the centerline of the cylindrical bed the shear stress will be zero and the vertical and horizontal normal stresses, \( \sigma_v \) and \( \sigma_h \), respectively, will be the principal stresses. It seems reasonable to expect that during defluidization the vertical normal stress (\( \sigma_v \)) will be larger than the horizontal normal stress (\( \sigma_h \)) (as the bed is gradually being compacted when we progressively lower the gas flow rate). Therefore, during defluidization, at least along the centerline, the vertical stress \( \sigma_v \) will be the major principal stress and

\[
\frac{\sigma_h}{\sigma_v} = j_{df} = \frac{1 - \sin \phi}{1 + \sin \phi}
\]

where \( \phi \) is the angle of internal friction for the particles. We have assumed in our analysis that the value of \( j = j_{df} \) estimated in this manner from the angle of internal friction applies for the defluidization branch.

In the fluidization branch, the stress profile determines the gas velocity at which the bed will detach from the distributor. As already pointed out by Jackson [10], just before the detachment of the bed from the distributor (i.e. \( \bar{u} \) slightly smaller than \( \bar{u}_c \)), the drag exerted by the gas on the bed exceeds the weight of the bed when wall friction is present. This suggests that \( \sigma_s \) (along the centerline of the bed) should be the minor principal stress at least in the lower part of the bed, where \( \sigma_s \) decreases as \( y \) increases. This led us to postulate that \( j_l = j_{df}^{-1} \) where \( j_l \) is the Janssen’s coefficient for the fluidization branch. The model employed in our study differs from that of Jackson [10] in this one regard. This change was also motivated by the fact that we could fit our experimental data (described below) to the 1-D model better when we assumed that \( j_l = j_{df}^{-1} \).

In order to understand the role of wall friction on the fluidization and defluidization behavior, and also evaluate the adequacy of the model described here, we performed fluidization and defluidization experiments in tubes of different diameters. These are described below.

### 3. Experimental

The physical properties of particles used in the experiments, namely XL glass beads, are summarized in Table 1. These are exactly the same particles used by Srivastava et al. [6] in their circulating fluidized bed experiments. The angle of internal friction for the particles was determined via measurements in a Jenike shear cell apparatus.

Fluidization experiments were performed in three different Plexiglas tubes (0.5\(^{\circ}\), 1.0\(^{\circ}\) and 2.0\(^{\circ}\) ID). Following Tsinontides [17], the gas distributors were constructed by sandwiching two layers of fibrous material between two micronic mesh screens. Distributors were rejected as unsatisfactory if any obvious maldistribution of the feed gas was detected. Dry air was further dried by passing it over a packed bed of drying agent and used in all the experiments described below. At each experimental condition, the volumetric flow rate of air exiting at the top of the fluidized bed was measured and was used to compute the superficial velocity of the air through the bed.

The bed height was determined by means of a tape measure attached to the outside of each tube. The pressure drop across the distributor and bed was measured by means of a U-tube manometer filled with red liquid of specific gravity 0.827. The pressure drop across the distributor alone, as a function of the gas flow rate, was determined through separate tests in the empty tubes. The pressure drop across the bed was obtained by subtracting the latter from the former.

The fluidization and defluidization experiments were performed as follows. A fairly deep bed, roughly 30 cm in height, was formed by depositing a known mass of glass beads into the tube. The bed was allowed to bubble gently for a period of time by operating the column at an air flow rate which is slightly larger than that corresponding to the point of minimum bubbling. The air flow rate was then reduced progressively in small steps (all the way down to zero flow rate), allowing the bed to equilibrate at each condition and recording the pressure drop, the bed height.

<table>
<thead>
<tr>
<th>Physical properties of XL glass beads</th>
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<tr>
<td>( d_{50} )</td>
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<tr>
<td>( \rho_p )</td>
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<td>( \phi )</td>
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the air flow rate and visual observations of the state of the bed. In this manner, the defluidization branch was mapped out. The fluidization branch was mapped out in a similar manner by progressively increasing the air flow rate in small steps from zero to the minimum bubbling point.

4. Fluidization–defluidization results

Figs. 2a, 3a and 4a show the measured pressure drop across the bed in the 0.5", 1.0" and 2.0" tubes, respect-

![Graph](image1)

Fig. 2. Dimensionless pressure drop (a) and bed height (b) as functions of the superficial gas velocity, for XL glass beads in a vertical tube of 0.5" diameter; *, increasing gas flow during fluidization, +, decreasing gas flow during defluidization. The solids curves represent model predictions. Bed height \(H_0\) at zero glass flow rate = 29.6 cm.

![Graph](image2)

Fig. 3. Dimensionless pressure drop (a) and bed height (b) as functions of the superficial gas velocity, for XL glass beads in a vertical tube of 1" diameter; *, increasing gas flow during fluidization, +, decreasing gas flow during defluidization. The solids curves represent model predictions. Bed height \(H_0\) at zero glass flow rate = 28.9 cm.

![Graph](image3)

Fig. 4. Dimensionless pressure drop (a) and bed height (b) as functions of the superficial gas velocity, for XL glass beads in a vertical tube of 2" diameter; *, increasing gas flow during fluidization, +, decreasing gas flow during defluidization. The solids curves represent model predictions. Bed height \(H_0\) at zero glass flow rate = 28.0 cm.

tively, as a function of the superficial velocity of the fluidizing air for a complete defluidization–fluidization cycle. Figs. 2b, 3b and 4b show the corresponding bed height data. The smooth curves in these figures represent the model predictions (discussed later), while the points refer to experimental data. The mass of particles loaded per unit cross-sectional area, \(m\), is 36.3 g/cm² for all three tubes. The pressure drop has been normalized with respect to \(mg\), the weight of the bed per unit cross-sectional area. A normalized pressure drop of 1.0 thus corresponds to the case where the pressure drop across the bed exactly balances the weight of
particles. The height of the bed is normalized with respect to height $H_0$, which is the height of the bed at zero gas flow rate.

It is instructive to first discuss the qualitatively similar features among the three figures in so far as the experimental results go. As the superficial velocity ($u$) is increased from zero in the fluidization branch, the pressure drop increases linearly with it. In ideal systems, the bed will become fluidized at a gas superficial velocity $U_{mf}$ commonly known as the minimum fluidization velocity, where the pressure drop first equals the weight of the bed per unit cross-sectional area ($mg$), and for all $u > U_{mf}$, the pressure drop is exactly equal to $mg$. However, in our system, the pressure drop overshoots $mg$ (see Figs. 2a, 3a and 4a) and continues to increase linearly with $u$ until $u = \tilde{u}_c$. This phenomenon, where $\tilde{u}_c > U_{mf}$, occurs when there exist cohesive forces between particles and between the particles and the distributor plate, or when frictional forces at the walls of the bounding tube, or a combination of both [9,10]. The percent overshoot in the pressure drop ($\Delta P/mg^{-1}$) is plotted in Fig. 5 against the inverse of the tube diameter ($D$). The trend suggests that the overshoot goes to zero as $D \to \infty$. Thus, it appears reasonable to anticipate that the pressure drop overshoot seen in our experiments may be rationalized using a model that considers only wall friction effect.

Beyond $\tilde{u}_c$, the pressure drop in the fluidization branch decreases abruptly and this is accompanied by a jump in the bed height. As the gas velocity is further increased, the bed expands without any visible signs of bubbling until the point of minimum bubbling, $\tilde{u}_{mb}$, is reached.

In the defluidization branch, as one decreases $\tilde{u}$ from $\tilde{u}_{mb}$, the pressure drop data remain close to the values recorded in the fluidization experiments initially. With further decreases in $\tilde{u}$, the defluidization and fluidization branches separate with the former always lying below the latter. As $\tilde{u}$ is progressively brought down to zero, pressure drop readings as well as the height of the bed decrease smoothly down to values corresponding to zero flow of gas.

There is thus a marked hysteresis between the fluidization and defluidization curves in the beds in all the three tubes. The features seen here are very similar to those observed by Tsinontides and Jackson [9] in their experiments with cracking catalyst. We found the hysteresis and the pressure drop overshoot to be more pronounced as tube diameter was decreased (compare Figs. 2a, 3a and 4a).

Fig. 4. Dimensionless pressure drop (a) and bed height (b) as functions of the superficial gas velocity, for XL glass beads in a vertical tube of 2W diameter; *, increasing gas flow during fluidization; +, decreasing gas flow during defluidization. The solids curves represent model predictions. Bed height $H_0$ at zero glass flow rate = 28.9 cm.

Fig. 5. Percentage overshoot in the pressure drop during fluidization as a function of the inverse of tube diameter; *, experimental data points. The solids curve represents model predictions.
These observations are consistent with those of Tsinontides and Jackson [9] and readily suggest that wall friction is important. Another noteworthy feature is the increase in \( \bar{u}_c \) with a decrease in the tube diameter; \( \bar{u}_c \) is 0.167 cm/s in the 2" tube, 0.183 cm/s in the 1" tube and 0.207 cm/s in the 0.5" tube. Tsinontides and Jackson [9], on the other hand, did not note any such dependence of \( \bar{u}_c \) on the confining tube diameter.

Another point of interest is the persistent offset between the pressure drop and \( mg \); the asymptotic value of pressure drop is less than \( mg \) in all the tubes. Such an offset had also been reported elsewhere with different particles; see for example Tsinontides and Jackson [9]. This offset suggests that the beds were not completely fluidized and that a portion of the weight of the bed was still being supported by the distributor plate and/or the frictional stresses acting on the cylindrical walls. The 2" tube has the largest offset of 5% while the 1" and 0.5" tubes have offsets of 4% and 2.5%, respectively. This trend, namely that the offset decreases as tube diameter is decreased, suggests that the offset is not due to wall friction. We conjecture that lateral non-uniformities in bed density become more pronounced as tube diameter increases and is responsible for the observed effect of tube diameter on the offset. The 1-D model described in Section 2 is then a gross simplification and may be used to interpret only some of the features of the experimental data. This model simply cannot capture the observed asymptotic offset or its dependence on tube diameter. Consequently, we did not attempt to analyze the pressure data obtained on the fluidization branch for \( \bar{u} > \bar{u}_c \).

The rest of the fluidization–defluidization data were analyzed in terms of the 1-D model as follows. The Richardson–Zaki coefficient, \( n \), was estimated from the defluidization data by assuming that the particle volume fraction is independent of height and the spatial variation allows us to simplify the parameter estimation process. It then follows that

\[
\frac{\Delta P}{H} \approx \frac{\rho_p \bar{v} g}{v_t} \frac{1}{(1 - \bar{v})^n} \bar{u}
\]

where \( \Delta P \) and \( H \) are, respectively, the pressure drop across the bed and the bed height for a particular value of \( \bar{u} \) and \( \bar{v} = m/\rho_p H \). This can be rewritten as

\[
\log \left( \frac{\Delta P \bar{u}}{\rho_p \bar{v} g} \right) = n \log(1 - \bar{v}) - \log v_t.
\]

The slope of the best-fit line gives the value of \( n \). We found that the data obtained on all three tubes yields a value of \( \approx 5.0 \) for \( n \). We attempted to estimate the effective terminal velocity, \( v_t \), from the intercept. This led to estimates of 7.25, 7.8 and 5.8 cm/s for \( v_t \) from the data obtained in the 0.5", 1.0" and 2.0" tubes, respectively. We suspect that this discrepancy in the estimated value stems from the fact that only a limited region in the \( \bar{v} \) space is probed by the defluidization branch, with \( \bar{v} \) typically being between 0.47 and 0.52, and the estimation of \( v_t \) requires extrapolation to \( \bar{v} = 0 \). Thus, best-fit lines with even slightly differing slopes yield very different values for \( v_t \). For this reason, \( v_t \) was left as an adjustable parameter such that it lies in the range given above.

The other quantity which can be estimated a priori is the limiting value of the solids volume fraction \( v_{\max} \). It is assumed here that this is the solids volume fraction at the bottom of a deep bed upon the end of defluidization. An estimate of this quantity was made by a simple experiment. A known is mass of particles was added to an initially empty bed, the bed was gently fluidized and defluidized, and the bed height was measured. This process was repeated by adding incremental amounts of solid until a fairly deep bed about 40 cm in height was obtained. The incremental changes in bed heights following incremental additions of particles to the bed represent a direct quantitative measure of the solids volume fraction at the bottom of the bed. In this manner, \( v_{\max} \) was estimated to be 0.55. Although the bed could have been packed more densely by subjecting it to mechanical vibrations, the packing state achieved spontaneously in defluidization is the most relevant one for our problem. Having made a priori estimates of \( n \) and \( v_{\max} \), the adjustable parameters remaining in the 1-D model described in Section 2 are \( F \), \( v_t \), \( v_{\min} \) and \( \mu \). The values of these parameters (see Table 2) were estimated by fitting the model to the data. The model predictions obtained with these parameter values are shown in Figs. 2–4 as solid curves. It is clear that the model is able to capture the hysteresis, the overshoots in the pressure and the bed heights upon defluidization reasonably well. All the parameter values used in the model are reasonable, with the exception of the wall friction coefficient, \( \mu \), which is smaller than what we would have expected (\( \approx 0.2 \)).

It is instructive to compare the actual height of the fluidized bed in the fluidization cycle in the region \( \bar{u} > \bar{u}_c \) with the height, \( H_{\text{ideal}} \) of an ideal fluidized bed where the particle volume fraction is independent of height and the

<table>
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<tr>
<td>( n )</td>
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<tr>
<td>( \mu )</td>
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<td>( \beta )</td>
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<td>( \beta )</td>
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<td>( F )</td>
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<tr>
<td>( v_{\max} )</td>
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Table 2: Values of model parameters.
pressure drop balances the weight of bed per unit area of cross-section. The upward supporting force due to the gas drag, per unit cross-sectional area of such an ideal fluidized bed, is

\[ H_{\text{ideal}} = \frac{\rho_g v u}{v_t(1 - \nu) u}. \]

Equating this to the weight of the bed per unit cross-sectional area, \( mg \),

\[ H_{\text{ideal}} = \frac{\rho_g v g}{v_t(1 - \nu)^{\nu_t}} u = mg. \]

However, \( \rho_g v H_{\text{ideal}} = m \), so that \( \nu = m/\rho_g H_{\text{ideal}} \). Inserting this into the force balance above

\[ \frac{\bar{u}}{(1 - m/\rho_g H_{\text{ideal}})^{\nu_t}} = \nu_t \]

or

\[ H_{\text{ideal}} = \frac{m/\rho_g}{1 - (\bar{u}/\nu_t)^{1/n}}. \]

This is also shown in Figs. 2b, 3b and 4b.

The percentage overshoot in the pressure drop during fluidization, predicted by the model for the combination of parameters in Table 2, is plotted in Fig. 5 as a function of \( D^{-1} \) (solid curve in this figure). According to the model, the wall friction effect becomes weaker and weaker as \( D \to \infty \), and the overshoot goes to zero. The model is able to capture the experimental data quite nicely. Interestingly, the theoretical curve seems to flatten out as the tube diameter is decreased. As the diameter is decreased, friction at the wall supports a larger and larger fraction of the weight of the bed as a result of which the bed is more loosely packed. This leads to a decrease in \( \sigma_s \) and a corresponding decrease in the normal stress exerted at the wall. The effect of decrease in the tube diameter is thus countered by the decrease in the normal stress at the wall. (The model predicts that at even smaller values of \( D \), the overshoot will decrease with decreasing tube diameter. At such small tube diameters, however, issues such as static arch formation enters the picture and the model becomes inadequate.)

5. Analysis of standpipe data

Experiments on flow behavior of XL glass beads (same as the ones used in the present study) in circulating fluid beds (CFBs) performed by Srivastava et al. [6] revealed that the shear stress experienced by the particles at the walls of the standpipe played an important role in ensuring stable particle circulation in the CFB loop. We refer the reader to Srivastava et al. [6] for details of the CFB experiments. In these experiments, the aeration gas was injected at equal rates through eight ports in the standpipe. Thus, for practical purposes, one can treat the aeration as being spatially uniform for most of the length of the standpipe (with the exception of a small segment of the standpipe near the slide valve). Fig. 6a–c [6,18] show the cross-sectional average solids volume fraction between ports 4 and 5 (which flanked the Electrical Capacitance Tomography unit) of the standpipe as a function of the external aeration rate for three different slide valve openings. The triangles denote results obtained from ECT measurements, with the filled triangles and open triangles representing steady and unsteady flow conditions, respectively. The pressure drop across ports 4 and 5 can be converted into an equivalent solids volume fraction by dividing by \( \rho_p g h \) with \( h \) being the distance between the two ports. This quantity, measured only during stable operating conditions, is shown as filled diamonds and is a quantitative measure of the support provided by the gas to suspend the weight of the bed. The difference between the solids volume fraction estimated from ECT and pressure drop data, henceforth referred to as \( \Delta \nu \), is a quantitative measure of support provided by shear stress at the wall and axial gradient in the vertical normal stress in the granular phase. It is clear from our fluidized bed experiments and modeling described in the previous sections that the axial gradient in the vertical normal stress in the granular phase is not likely to be the main source of the support, as this would have resulted in a rapid increase in particle volume fraction (within a depth of a few tens of cm) to a value very close to \( \nu_{\text{max}} \) and this was not the case in most of the operating conditions reported in Fig. 6a–c. Thus wall friction is the most likely explanation for the observed \( \Delta \nu \). All the data reported in Fig. 6a–c in the stable operating region are summarized in Fig. 7, which shows a plot of \( \Delta \nu \) against \( \nu \) estimated from the ECT measurements. In spite of the scatter, a trend can be observed, suggesting that a simple relation does indeed exist between these two quantities. A simple analysis of the wall friction in the standpipe, where one uses the frictional yield stress in XL glass beads (i.e. Eq. (5) and the parameter values) estimated through fluidization–defluidization measurements, is described below.

Consider a simple one-dimensional model of granular flow in a tall cylindrical standpipe of radius \( R \) under steady flow conditions. In particular, let us focus on a region away from the entrance or the exit, such as the region where the ECT measurements were made by Srivastava et al. [6]. Even though under such conditions ECT measurements revealed random low-amplitude high-frequency fluctuations in the solids volume fraction arising due to non-uniformities, one may assume to a good approximation that the bed at this location was essentially at a uniform solids volume fraction of \( \bar{\nu} \), which was determined by time-averaging the ECT data. Assuming that the granular material is in compressive yield, the vertical stress \( \sigma_s \) can thus be determined from Eq. (4) and the model parameters estimated from defluidization–fluidization experiments. The normal stress \( \sigma_n \) exerted
on the wall would then be \( j \sigma_s \) where \( j \) is the Janssen’s coefficient. The shear stress at the wall would then be \( \mu j \sigma_s \). A simple force balance under conditions where \( \bar{\sigma} \) is independent of axial position yields

\[
\Delta \nu = \frac{2 \mu j \sigma_s(\bar{\nu})}{\rho_p g R}.
\]

From the slope of a plot of \( \Delta \nu \) against \( \sigma_s(\bar{\nu}) \), the value of \( \mu j \) was estimated to be 0.225. The right hand side of the above equation is shown in Fig. 7 as a solid curve. It is clear that the 1-D model is able to capture the general trend in the standpipe data quite nicely, in spite of the presence of fluctuations recorded by the ECT unit.

In the presence of such fluctuations, it seems reasonable to expect that on an average \( j \) would have a value close to 1, so that the value of \( \mu \) is indeed close to 0.225. It is remarkable to note that this is equal to \( \sin \varphi \). Wall friction coefficient will be equal to \( \sin \varphi \) when a layer of particles remains stuck to the wall, which was indeed the case in the standpipe experiments. Electrostatic effects are known to be important in transport of glass beads in Plexiglas pipes [19,20]. Adhesion of a layer of particles to the wall as a result of electrostatic effects in the standpipe is quite conceivable. This may also explain why a much smaller value of \( \mu \) was required to fit our fluidization–defluidization data, where tribo-electric charging would have occurred to a much smaller extent. Indeed, upon emptying the fluidized bed at the end of our fluidization–defluidization experiments, we observed only a few spots where the particles remained adhered to the wall. In any case, the above argument is at best a speculation and we must conclude
by noting that this difference in the value of $\mu$ between the standpipe and fluidization experiments remains unresolved.

6. Summary

Fluidization and defluidization experiments were conducted in tubes of different diameters in order to understand the role of friction in modifying fluidization–defluidization characteristics. Careful measurements of the gas flow rate, bed height and pressure drop across the bed were made during the process of both fluidization as well as defluidization. A marked hysteresis between the fluidization and defluidization curves in all the beds is observed. The process of fluidization is characterized by the presence of an overshoot in the pressure drop beyond that required to suspend the weight of particles in the bed. The hysteresis and pressure overshoot are more pronounced in the beds of smaller diameter. These features are very similar to those observed by Tsinontides and Jackson [9] with their experiments on cracking catalyst particles. In our study, these features appear to be a consequence of wall friction. Another noteworthy feature is the persistent offset in the pressure drop at high gas flow rates from the theoretical value, which was more pronounced in larger diameter tubes. We believe that this is due to the presence of lateral non-uniformities in the bed density.

An estimate of the magnitude of contact stresses was made by fitting the model of Jackson [10] to the fluidization–defluidization data. The model is able to capture the hysteresis, the pressure drop overshoot and the bed height upon defluidization quite well. The frictional yield stress obtained in this manner was used to analyze the standpipe data reported by Srivastava et al. [6]. A 1-D model based on a simple analysis of wall friction in the standpipe is able to capture the general trends in the standpipe data reasonable well. However, the values of the wall friction coefficient, $\mu$, obtained from the standpipe and fluidization experiments are quite different. It is hypothesized that this could be due to electrostatic effects.

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References