

# GRAVITY'S RAINBOW: MODELING THE WORLD TRADE NETWORK

MICHAEL D. WARD AND JOHN S. AHLQUIST

**ABSTRACT.** The gravity model, long the empirical workhorse for modeling international trade, has recently been put on firmer theoretical foundations (Anderson and van Wincoop, 2003; Helpman, Melitz and Rubinstein, 2008). These theoretical models emphasize how economic conditions in the rest of the world affect bilateral trade. Nevertheless, standard applications of the gravity model ignore the network dependencies in bilateral trade data, assuming that dyadic trade is independent, conditional on a hierarchy of covariates over country, time, and dyad. More nuanced implementations attempting to employ proxies for inherently unobservable “multilateral resistance terms” suffer from measurement error and omitted variable problems. We propose a strategy designed to account for and estimate second- and third-order dependencies in the data. We estimate this model using bilateral trade data from 1990-2008, which substantially outperforms standard accounts in terms of both in- and out-of-sample predictive heuristics. We illustrate the model’s usefulness by tracking specific trading propensities.

JEL: C10, C11, C23, C52, C82, F10, F13, F17

Keywords: gravity model, trade, networks, latent factors

For over four decades scholars have used the gravity model to describe international trade. The earliest suggestions of this approach are found in Linneman (1962); Poyhonen (1963); Linneman (1966). Based on the analogy to Newtonian physics, the gravity model supposes that the flow of goods and services between two locales is proportional to their combined economic “masses” and inversely related to economic and physical distance. Aside from its intuitive appeal, the model’s popularity has largely been driven by its flexibility, ease of statistical estimation, and respectable in-sample fit. A gravity model can typically explain between one half and two thirds of the the (in sample) variation in bilateral international commerce and is often considered a benchmark for free trade.

Few political economists believe that international commerce occurs within a free-trade system, absent the friction and lubrication created by individuals, firms, governments and institutions. As a result, many scholars have introduced into the gravity model a variety of factors thought to increase or decrease the bilateral flow of goods and services. For example, the gravity model has is at the core of a lively empirical debate centering on the “effect” of the GATT/WTO and other FTAs on

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bilateral trade flows (Rose, 2004, 2007; Tomz, Goldstein and Rivers, 2007; Subramanian and Wei, 2007; Goldstein, Rivers and Tomz, 2007; Baier and Bergtrand, 2007).

In its many applications, the gravity model is usually estimated in log-linear form under the assumption that bilateral trade is IID, conditional on covariates, among which analysts frequently include country, dyad, and/or time fixed effects. Adjudicating between competing models typically centers on evaluating the sign and “significance” of various regression parameters, perhaps coupled some discussion about in-sample fit ( $R^2$  and the like). Recent debates about model specification have focused on the proper way to incorporate “multilateral resistance terms” and account for the preponderance of non-trading dyads so as to recover “unbiased” reduced form parameters and derive elasticities and general equilibrium comparative statics (Anderson and van Wincoop, 2003; Baier and Bergtrand, 2009; Helpman, Melitz and Rubinstein, 2008). Recent work devotes considerably less space to the dynamic evolution of the world trading system. Virtually no attention has been paid to the model’s predictive performance out-of-sample under various specifications and estimation strategies.

Our fundamental claim is that when modeling international trade, the assumption of conditional independence is difficult to sustain *even in the presence of directed country or dyad fixed effects*. We arrive at this conclusion from two directions. First, as intuition would suggest and recent theoretical advance has formalized, bilateral trade is not independent of the production, consumption, and trading decisions made by firms and consumers in third countries. Anderson and van Wincoop (2003) derive the gravity equation from a general equilibrium framework based on monopolistic competition among geographically differentiated products. Helpman, Melitz and Rubinstein (2008) extend this logic to account for zero and asymmetric trade flows in a model including heterogeneous firms. In both these models countries’ (weighted) costs of trading with all other countries in the world affect the observed trading relationships between any pair  $i$  and  $j$ . Second, these new theoretical models rely on very strong assumptions<sup>1</sup> and/or “replace the unobservable theoretical trade cost variable... with an *observable* variable” (Baier and Bergtrand 2009, p. 80 emphasis in original). But there is reason to believe that these proxies are incomplete and likely measured with considerable error. We propose to treat these multilateral, higher-order dependencies in the data as latent, estimable constructs using a dynamic specification of the bilinear mixed effects, or “latent space,” network model (Hoff, Raftery and Handcock, 2002; Hoff, 2005a). We show that this model not only recovers evidence of significant network dependence not captured in the standard fixed effects specifications, it also out performs more conventional gravity specifications.

In the next section, we outline recent theoretical and econometric advances surrounding the gravity model, highlighting reasons we should expect there to be higher-order dependencies in the data that are either not captured in standard models or estimated subject to measurement error and omitted variable problems in more nuanced specifications. We then describe the topology of the world trade network, showing that, unlike other social networks, world trade is not well-describe by power

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<sup>1</sup>e.g., bilaterally symmetric trade costs (Anderson and van Wincoop, 2003)

laws. We introduce a dynamic bilinear mixed effects model that captures latent dependency structure in the data. We estimate this model using bilateral export data from 1990-2008, comparing our results to the “gold-standard” gravity model in the international trade literature. Finally, we illustrate the model’s usefulness by tracking the trading propensities of several important exporters over this period.

## 1. THE THEORETICAL NECESSITY FOR A MODEL OF NETWORK RELATIONS

Economic reasoning suggests that the network of world trade is highly interdependent. If, over some time horizon, there is only a finite quantity of goods produced, only some of which are profitably exchanged across geographic and political boundaries then goods available to buyers in one jurisdiction depends on what is—and is not—sold to buyers elsewhere and at what prices. Bilateral transactions are necessarily embedded in a larger web of trade.

This intuition informs the *ex post* theoretical exposition of the gravity model. Ever since Anderson (1979), theories of cross-border trade have used trade in products differentiated by their point of origin to emphasize “multilateral prices.”<sup>2</sup> These price differences imply “multilateral resistance terms” (MRTs) in which trade between any two countries depends on the costs of trading between those two relative to the (average) cost of trading with any other trading partner. Anderson and van Wincoop (2003) provide the current state-of-the-art, endogenizing prices in a general equilibrium framework. A widely-recognized weakness in their model, however, is its reliance on the assumption of symmetric bilateral trade costs. Helpman, Melitz and Rubinstein (2008) point out that most gravity model applications include only dyads with positive trading relationships, ignoring the many non-trading relationships that exist. They extend the Anderson-van Wincoop model using location-specific firms of heterogeneous productivity, allowing them to derive a gravity model capable of accounting for the (possibly asymmetric) set of trading partners (the extensive margin) and trade volume with these partners (the intensive margin). By using the information contained in non-trading dyads, the Helpman et al. model uses firm heterogeneity to circumvent the symmetric trade costs assumption of Anderson-van Wincoop.

Empirical efforts to account for MRTs have taken a variety of forms but all can be viewed as attempts to hold the “rest of the world” constant so as to sustain a conditional independence assumption at the dyadic level. Anderson and van Wincoop (2003) estimate a complete system of  $N^2$  equations using nonlinear least squares on cross-sectional trade data, but relying on the very strong assumption that trade costs within a dyad are symmetric. Subsequent work (Feenstra, 2002; Baier and Bergstrand, 2007) typically employs the less burdensome strategy of country fixed effects based on the notion that what we are really interested in is a country’s “remoteness” from the rest of the world, rather than the multilateral dependencies implied by the theory. Helpman, Melitz and Rubinstein (2008) likewise rely on exporter and importer fixed effects in their two-stage estimation procedure. Alternatively, Baier and Bergstrand (2009) propose an exogenous approximation to the endogenous MRTs of Anderson and van Wincoop. Their solution is to use borders and geographic distance to calculate the “GDP-share-weighted (geometric) average of the gross trade costs facing importer  $j$  across all exporters  $i$ ” (p. 80) and likewise

<sup>2</sup>See also Bergstrand (1985).

for exporter  $i$  across all importers  $j$ . The resulting equation can be estimated using OLS. Whether one relies on the fixed effects strategy or something more complicated appears to be driven by the extent to which interest centers on evaluating the “effect” of a border on trade flows. Unless the “border puzzle” is of primary interest, fixed effects specifications appear to be the current default.

None of these empirical strategies is completely satisfactory. As already mentioned, the Anderson-van Wincoop NLS solution relies on strong assumptions that appear to have little support in the data (Helpman, Melitz and Rubinstein, 2008). The Baier and Bergtrand (2009) approximation permits consideration of asymmetric bilateral trade costs but appears to work poorly in cases where economically small countries are physically proximate to larger trading partners. Both Anderson and van Wincoop (2003) and Baier and Bergtrand (2009) acknowledge the likelihood of measurement error, theoretical misspecification, and omitted variable problems. Fixed effects have their own clear limitations: they only address (time-invariant) unobserved country-level factors, failing to capture unobserved dyadic relationships, much less higher-order dependencies. They preclude the estimation of parameters for country-level covariates, which can be of considerable interest.<sup>3</sup>

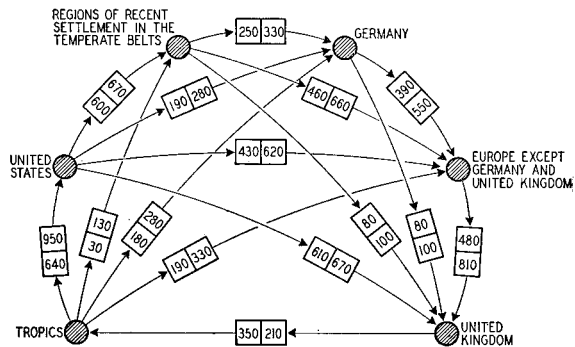
Both intuition and current theoretical work recognizes multilateral dependency in world trade flows. Extant empirical strategies, however, typically fail to capture the higher-order dependencies in the data. Some additional studies that employ various types of gravity specifications include Anderson (1979), Deardorff and Stern (1990), Maskus and Ramazani (1993), Oguledo and MacPhee (1994), Deardorff (1998), Deardorff and Stern (1998), Bliss and Russett (1998), Barbieri and Schneider (1999), Anderson and Marcouiller (2002), Anderson and van Wincoop (2003), Goenner (2003), Rose (2004), Plümper and Krempel (2003) and Goenner (2004).

## 2. THE TOPOLOGY OF THE WORLD TRADE NETWORK

We are hardly the first to recognize the network structure of world trade. There is a long history of examining world trade networks, dating back at least to the 1940s when a Swedish economist, Folke Hilgerdt, began working on the topic for the League of Nations. He published widely including the 1943 *American Economic Review*. His map of the world trade network, circa 1928, may have been the first such mapping, and reveals that Germany, the USA, and the UK have not relinquished their roles as pre-eminent trading partners in the ensuing eight decades. This chart is reproduced in Figure 1.

More recently, a wide variety of scholars have begun looking at the network topology

The System of Multilateral Trade, as Reflected by the Orientation of Balances of Merchandise in 1928.



Note. Balances in millions of dollars, calculated from adjusted frontier values of trade (imports valued c.i.f., exports f.o.b.). Both import and export balances are shown; the smaller of the two figures in each square represents the import balance.

FIGURE 1. Hilgerdt's (1943) map of the world trade network in 1928.

<sup>3</sup>All these same criticisms extend to dyadic fixed effects (and covariates) in a panel data or time-series cross-section setting.

of world trade (Garlaschelli and Loffredo, 2005; Fagiolo, Reyez and Schiavo, 2007*b,a*; Bhattacharya et al., 2008; de Benedictis and Tajoli, 2009; Garlaschelli and Loffredo, 2004) and these have provided a newer evaluation, but a similar picture as that produced at the end of the Second World War.

According to the IMF there was \$16 trillion in exports during 2008, the most recent year with relatively complete data. This excludes (probably) about \$255 billion in Taiwanese exports and re-exports. In terms of number of trade partners (excluding Taiwan, and a few other cases) of the 181 reporting countries in 2008, 92 export goods to at least 100 other countries. About 40 percent of countries export something to almost every other reporting country, and every country exports to at least 20 partners. The biggest traders—Germany, the US, and China—export an average of \$7-8 billion to each of its trading partners, while even the smallest of traders average almost a one billion to their partners. As illustrated in these statistics, the world trade network is quite concentrated. As shown in Table 1, this concentration has grown considerably since 1950.

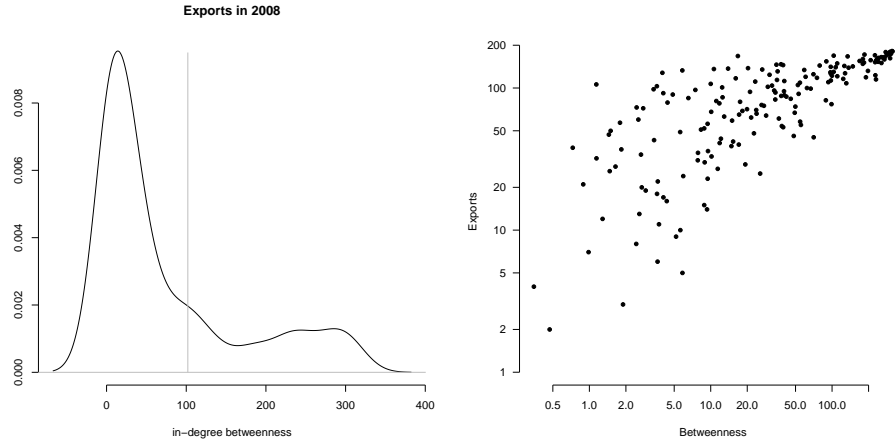
TABLE 1. *Some network statistics about reported exports in 2008.*

	1950	2008
Countries reporting trade	60	181
Total number of trade flows	1649	19234
Total volume of exports	1585	15296
Countries making up 50% of exports	23	9
Share of exports in the top 1% of flows	29%	45%
Graph density	.47	.59

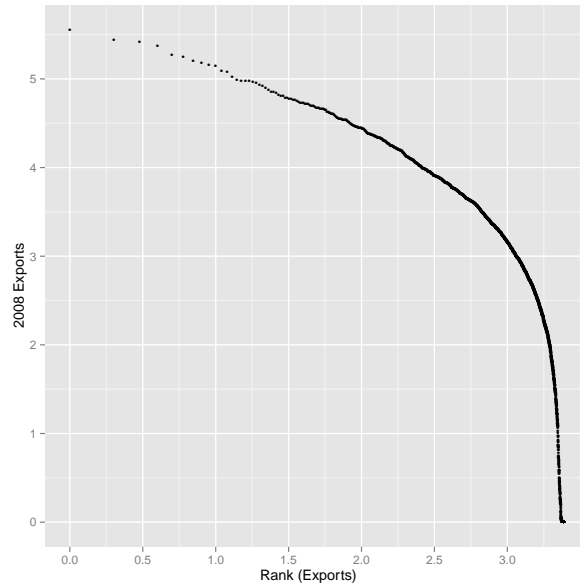
Note: Authors' calculations using IMF data on reported trade. See appendix for information on data.

However, this is a very dense network of exchanges, one that is characterized by large nodes as well as small ones. There are 8169 mutual dyads in 2008, but 2896 asymmetric dyads also exist, along with over 5000 null linkages. Because of the connectivity of this graph, the second order linkages saturate the graph; each node is reachable in two hops, or fewer. Power law estimation suggest that the power law exponent is  $2.23 \pm 0.007$ , close to estimates reported in the literature of “econophysics.” A plot of the betweenness of the network of international commerce in 2008 and a plot of the betweenness and the rank of exports is given in Figure 2. In general, trade is not well portrayed as a Pareto (or Zipf) distribution because unlike things that are distributed as a power law—i.e., rare large events and ubiquitous smaller ones—trade networks are characterized by large flows through a sizable number of nodes. If you look at the top 50 exporters in 2008, the density of the graph is 0.99, and the level of connectedness is 1.0, with an efficiency of 0.03: virtually all the top 50 exporting countries export to one another.

The classic rank-size plot from network analysis is shown in Figure 3 and illustrates that the distribution of trade is neither Zipf, nor power-law distributed, because there is nowhere on this curve straight enough for even the most welcoming network analysts to declare that it fits the expected distribution. Despite recent evidence to

FIGURE 2. *Betweenness and Exports in 2008*

the contrary, networks don't necessarily seem to cause everything, and most network analysis of trade has taken a different tack.<sup>4</sup>

FIGURE 3. *No Power Law in Exports 2008*

In what follows, we return to a more traditional approach to modeling international commerce—the gravity equation—and we develop a dynamic network approach

<sup>4</sup>Another interesting network oriented analysis that focuses on trade is Lupu and Traag (2011) which draws on the community detection work pioneered by Peter Mucha and others Mucha et al. (2010).

that allows us to embrace rather than ignore the network dependencies that exist in that network. Thus, we undertake a network analysis of the gravity model of trade, but avoid both the network focus on size distributions as well as its emphasis on static descriptive analyses of complete networks.

### 3. MODELING NETWORK DEPENDENCIES

The standard gravity model typically employed in statistical studies of bilateral trade between countries  $i$  and  $j$  has the log-linear form:

$$\begin{aligned}\log Y_{i,j} &= \beta_0 + \beta_1 \log G_i + \beta_2 \log G_j + \beta_3 \log D_{i,j} + \log \epsilon_{i,j} \\ \log \epsilon_{i,j} &\sim N(0, \sigma_{ij}^2)\end{aligned}$$

where  $G_i$  represents a variable (or set of variables) measuring  $i$ 's economic "mass" (e.g., GDP, population) and  $D_{ij}$  captures trade "frictions", most commonly geographic distance. In recent years the  $D$  term has been expanded to include other dyadic variables (common language, common colonial heritage, WTO joint membership) and country-level terms (landlocked, island, democracy, etc.). The standard gravity set up assumes a spherical disturbance and is commonly estimated using OLS.<sup>5</sup>  $Y$  is variously measured as bilateral trade or directed imports or exports. We focus on bilateral exports. The preponderance of non-trading dyads pose an estimation problem for the log-linear model. The most common approach historically has been to exclude non-trading dyads from the analysis altogether. Some have added a small constant to all trade flows to allow for estimation (a strategy we follow here). Helpman, Melitz and Rubinstein (2008) propose a selection model for non-zero trading relationships and a second stage gravity model for trade volumes.

Confidence intervals and  $p$ -values for the regression parameters have typically been obtained assuming that the error terms across sets of countries ( $i$ ,  $j$ , and  $k$ ) are independent, conditional on covariates. In particular:

$$E(\epsilon_{i,j}\epsilon_{j,i}) = E(\epsilon_{i,j}\epsilon_{i,k}) = E(\epsilon_{i,j}\epsilon_{k,j}) = E(\epsilon_{i,j}\epsilon_{k,i}) = 0.$$

These assumptions suggest that residual trade imports of the US from Canada will be uncorrelated with the unexplained imports of Canada from the US. Similarly, residuals from US→Canadian exports will be uncorrelated with residuals from Japan→US trade. In the same way all Canadian export will be seen to be uncorrelated as will Canada's exports and imports to other specific partners. Singly each of these assumptions is questionable, but taken together they are especially dubious. There is good reason to expect that a wide range of countries will import certain goods from specific, large exporters. It is also to be expected that many countries will export to specific, large importers. We also expect that if a country has certain trade agreements with another then they will have both large residual exports to and imports from that country. Thus, there should be a large amount of covariation that is attributable to the sender, the receiver, and the dyad that the standard formulation ignores. What happens if these covariances are not ignorable?

To address these issues, we assume that the errors  $\{\epsilon_{i,j}, i \neq j\}$  have a distribution that is exchangeable under identical permutations of the indices  $i, j$  of the importers and exporters. The further assumption of normality implies the residuals can be

<sup>5</sup>Santos Silva and Tenreyro (2006) point out the log formulation has some drawbacks that prevent one from accurately modeling pairs of countries that do not trade with one another. There also are a variety of "robust" variance approaches employed.

represented in terms of a decomposable linear random effects model. We decompose these effects into importer  $r_i$ , exporter  $s_j$  and dyadic  $\gamma_{i,j}$  components:

$$\begin{aligned}
 (1) \quad \epsilon_{i,j} &= r_i + s_j + \gamma_{i,j} \\
 (2) \quad \begin{bmatrix} r_i \\ s_i \end{bmatrix} &\sim N_2 \left( \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_r^2 & \sigma_{rs} \\ \sigma_{rs} & \sigma_s^2 \end{bmatrix} \right) \\
 (3) \quad \begin{bmatrix} \gamma_{i,j} \\ \gamma_{j,i} \end{bmatrix} &\sim N_2 \left( \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_\gamma^2 & \rho\sigma_\gamma^2 \\ \rho\sigma_\gamma^2 & \sigma_\gamma^2 \end{bmatrix} \right).
 \end{aligned}$$

This defines the following moments for the  $\epsilon_{i,j}$ 's which we estimate rather than ignore:

$$\begin{aligned}
 E(\epsilon_{i,j}^2) &= \sigma_r^2 + \sigma_s^2 + \sigma_\gamma^2 \\
 E(\epsilon_{i,j}\epsilon_{j,i}) &= \rho\sigma_\gamma^2 + 2\sigma_{rs} \\
 E(\epsilon_{i,j}\epsilon_{i,k}) &= \sigma_r^2 \\
 E(\epsilon_{i,j}\epsilon_{k,j}) &= \sigma_s^2 \\
 E(\epsilon_{i,j}\epsilon_{k,i}) &= \sigma_{rs},
 \end{aligned}$$

where  $\sigma_r^2$  represents dependence among dyadic observations having a common importer,  $\sigma_s^2$  represents dependence among measurements having a common exporter, and  $\rho$  represents correlation of measurements within a dyad, i.e. reciprocity.

We employ a regression framework based on the social relations model for analyzing these dependencies in dyadic data for international commerce. The social relations model, emerging from psychologists' interest in separating the independent and interactive effects of groups versus individuals, provides a way of acquiring unbiased estimates of actor and partner effects and their interrelationships. Fundamentally, we decompose variance in relational data into sender and receiver effects as well as permit within-dyad correlations via the analysis of variance (ANOVA) protocol (Warner, Kenny and Stoto, 1979; Wong, 1982). The idea of further decomposing the variance in the context of dyadic data was developed to permit the statistical analysis of normally distributed dyadic data using additive effects (Gill and Swartz, 2001; Li and Loken, 2002). These ideas have been extended to a generalized linear model that incorporates third-order dependence via a bi-linear effect (Hoff, 2005a) similar in spirit to that first introduced by Gabriel (1998).

The question remains: what does "higher-order dependencies" mean? Second-order dependence refers to what is often described as reciprocity in the context of directed relationships. The prevalence of reciprocity in directional network data challenges the basic assumption of observational independence. We have already identified how we propose to model reciprocity.

To discuss fully third-order dependence, some formalism is helpful. Third-order dependence includes (a) transitivity, (b) balance, and (c) clusterability (Wasserman and Faust, 1994). *Transitivity* follows the familiar logic of "a friend of a friend is a friend." Consider a triad of countries  $\{ijk\}$ . This triad is composed of three dyads  $\{ij\}$ ,  $\{jk\}$ , and  $\{ki\}$ . In a network context, there are six possible links if the data are directional and three if data are non-directional. In particular, for directed binary relations, triad  $ijk$  is transitive if whenever  $y_{ij} = 1$  and  $y_{jk} = 1$ , we also observe that  $y_{ik} = 1$ . A triad  $ijk$  is said to be *balanced* if all pairs of actors relate to one another in an identical fashion, specifically:  $y_{ij} \times y_{jk} \times y_{ki} > 0$ . The idea is that if



the relationship between  $i$  and  $j$  is “positive” then both will relate to another unit  $k$  identically. If  $y_{ij}$  is positive, then to observe balance,  $y_{jk}$  and  $y_{ki}$  are either both positive or both negative. A triad is clusterable if it is balanced or the relations are all negative. A clusterable triad can be divided into groups where the measurements are positive within groups and negative between groups.

In other words, knowing something about the relationship between  $i$  and  $j$  as well as between  $j$  and  $k$  may reveal something about the relationship between  $i$  and  $k$ , even when we do not directly or perfectly observe it. The take-away is that treating dyads  $\{ij\}$ ,  $\{jk\}$ , and  $\{ki\}$  as independent from each other, which is routine in empirical analyses of international economic data, usually ignores important patterns. Hoff, Raftery and Handcock (2002) note:

In some social network data, the probability of a relational tie between two individuals may increase as the characteristics of the individuals become more similar. A subset of individuals in the population with a large number of social ties between them may be indicative of group of individuals who have nearby positions in this space of characteristics, or “social space.” Note if some of the characteristics are unobserved, then a probability measure over these unobserved characteristics induces a model in which the presence of two individuals is dependent on the presence of other ties.

The “social space” summarizing these unobserved characteristics is another “image” of the third-order dependence in these dyadic data. Stated differently, once the higher-order dependencies are taken into account, the dyadic data can be analyzed by techniques such as regression that assume conditional independence.

Hoff, Raftery and Handcock (2002) and Hoff (2005a) develop models to estimate and map these latent positions such that positions in a latent space represents these dependencies. We employ the asymmetric bilinear mixed effects specification where  $Y_{n \times n}$  is an asymmetric matrix of dyadic (log) exports in which  $y_{ij}$  is exports that  $i$  sends to  $j$ . This is formalized:

$$\begin{aligned} (y_{ij}, y_{ji})' &\sim N_2((\theta_{ij}, \theta_{ji})', \Sigma_\gamma) \\ \theta_{ij} &= \beta' x_{ij} + s_i + r_j + u_i' v_j \\ u_i &\sim N_k(0, \Sigma_u) \\ v_i &\sim N_k(0, \Sigma_v) \\ \Sigma_u &= I_k(\sigma_{u_1}^2, \dots, \sigma_{u_k}^2)' \\ \Sigma_v &= I_k(\sigma_{v_1}^2, \dots, \sigma_{v_k}^2)' \end{aligned}$$

The dyadic error variance,  $\Sigma_\gamma$ , is as expressed in equation 3. Exporter and importer effects are modeled as  $s_i$  and  $r_j$  as in expression 2.  $I_k$  is the  $k \times k$  identity matrix. Most importantly for our analysis, the  $k$ -vectors  $u_i$  and  $v_j$  describe  $i$  and  $j$ 's positions in the latent “exporter space” and “importer space,” respectively.

Conditional on the inclusion of a latent dimension that captures the dependence of the observation on one another, the dyadic data can be treated as independent, and the coefficients can be estimated by well known techniques. If the data are truly independent, estimations under these two scenarios will be equivalent, indeed identical. However, if there is clustering or dependence of the observations on one another in the manner specified above, they will diverge substantially.

In this research we undertake an additional step. Previous work on this topic (Ward and Hoff, 2007, 2008) has shown the importance of accounting for latent factors in bilateral trade models, and illustrated the persistence of such factors as relevant explanatory factors for a period of over 10 years in out-of-sample tests. However, to date no one has been able to dynamically include updated latent factors in such models. Our empirical examination in this article includes the bilinear factors from prior years as lagged dyadic variables in a very simple dynamic model of bilateral trade.<sup>6</sup> These model changes are portrayed in Figure 4.<sup>7</sup>

We estimate the model using annual data, 1990-2008. Bilateral exports come from the IMF's Direction of Trade statistics as well as specific country sources for Taiwan, and *CIA World Factbook* estimates for Cuba, Myanmar, and North Korea. Rather than include a list of covariates running in to the dozens, as is now common in empirical studies, we define the basic gravity model with size (i.e., the log of GDP in real 2005 \$US) and geographic distance<sup>8</sup> We impute missing covariate values for countries in existence in any particular year using copula methods (Hoff, 2007). This simple gravity model is embedded in the latent factor model described above.

We undertake a fully Bayesian estimation of model parameters using diffuse but proper conjugate priors and a Markov chain Monte Carlo (MCMC) algorithm, sampling parameters values from their posterior distributions.<sup>9</sup>

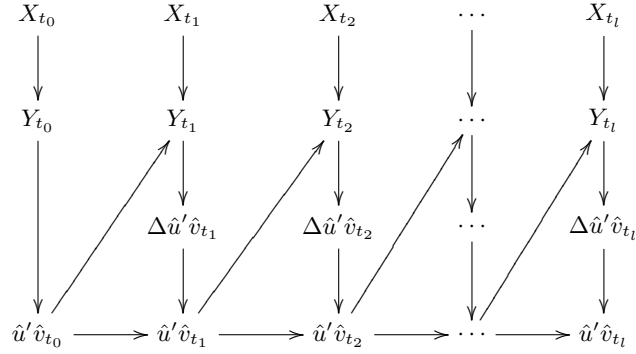


FIGURE 4. *Modeling dynamic networks with lagged latent positions*

<sup>6</sup>This requires one minor innovation. Whenever the estimated latent positions for year  $t$  do not provide an estimate for country  $i$  because it did not exist, we have to impute the latent position of  $i$  at  $t$ , when we include the lag at  $t + 1$ . To accomplish this, we employ a matrix regression of the latent dimensions conditional on the known covariates, then use the retrieved estimates to generate estimated, prior latent positions for countries, such as the Czech Republic and Slovakia, that were not previously part of the sample because they did not exist as independent entities.

<sup>7</sup>Westveld and Hoff (2011) develops an multi-slice approach to these kind of data, based on modeling the correlation structure over several dimensions.

<sup>8</sup>Proximity is the minimum distance between countries as reported in Gleditsch and Ward (2001).

<sup>9</sup>For computational ease we parameterize the bilinear model above putting regressors specific to a dyad and those specific to an exporter or an importer in different design matrices. We build a Markov chain in  $\{\beta, \Sigma_{sr}, U'V, \Sigma_U, \Sigma_V, \Sigma_\gamma\}$  (where  $U$  and  $V$  are  $K \times n$  matrices of latent vectors) that eventually samples from the desired target posterior distribution  $p(\beta, \Sigma_{sr}, U'V, \Sigma_U, \Sigma_V, \Sigma_\gamma | Y)$ . The full conditionals for the regression terms  $(\beta, U'V)$  are multivariate normal, and the covariance terms are modeled as inverse-Wishart conditional distributions. Details on the full conditionals are found in Hoff (2005b). Software to estimate this model is available at [www.stat.washington.edu/hoff/Code/GBME/gbme.r](http://www.stat.washington.edu/hoff/Code/GBME/gbme.r) as well as included in the  $\mathcal{R}$  package `latentnet`, available at <http://cran.r-project.org>.

#### 4. RESULTS

We ran the MCMC for 60 thousand iterations, saving every 50<sup>th</sup> iteration.<sup>10</sup> We discard the first 1000 iterations as burn-in. Figure 5 displays the MCMC chain mixing for a single year (1991) with lagged latent positions included. The mixing of these chains is stable and posterior means are given by a red line through the scan plots.<sup>11</sup>

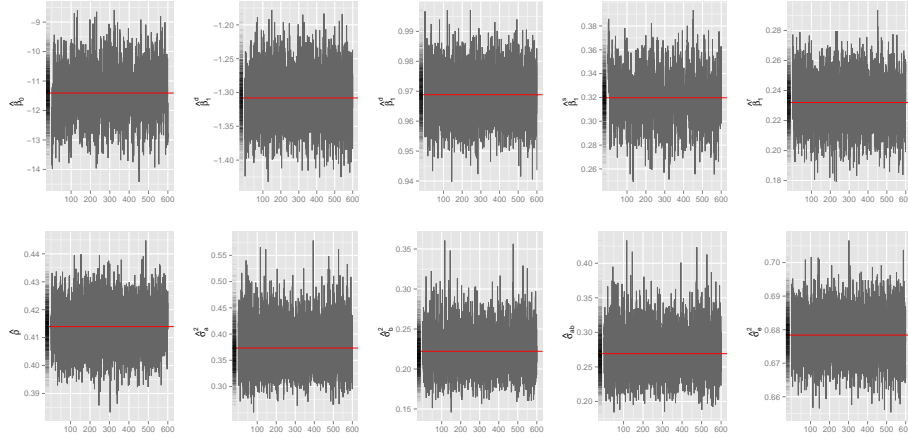


FIGURE 5. *Mixing of the MCMC Chains, for 1991.*

The basic results are strong. An informal perusal of gravity model results in the empirical literature puts most  $R^2$  in the 0.4-0.7 range while the Helpman et al. two stage procedure generates  $R^2$  around 0.7 for 1986 data. The in-sample predictive fit for the latent space model, shown in Figure 6, ranges between 0.8 and 0.85, far exceeding typical results and suggesting that the higher order dependencies are important for modeling trade.

To more carefully evaluate out-of-sample performance, we set up a fairly difficult test. We compare our model to specifications presented in Goldstein, Rivers and Tomz (2007). Their models use log bilateral imports as the response variable and are pooled estimations over 1946-2004, enabling them to estimate a variety of country, time, and dyadic fixed effects in addition to including over a dozen covariates. Their models exclude non-trading dyads from analysis and are estimated in \$US 1967. Based on Figure 7 it is clear that comparing our model to one that excludes non-trading dyads stacks the deck against us. We fit their basic model (Table 1 on p. 53) with slight modification. We include both importer and exporter country fixed effects, but omit year effects. We then perform an out-of-sample prediction exercise. For each year  $t$  in 2000-04 (2004 is the last year in their data) we fit their model pooling data from 1990 to  $t - 1$ . We combine these parameter estimates with year  $t$ 's covariate data to generate predicted values year  $t$ . We compare these predictions to the one-year-ahead prediction (with no covariate updating) from our GBME results, i.e., we assume that  $\hat{y}_{t-1}$  is our best prediction for  $y_t$ . Our comparison criterion is root-mean-squared prediction error. Results, presented in Table 2 show

<sup>10</sup>We have conducted these analysis multiple times, with different seeds, and starting values, for different numbers of iterations. The results are robust to these sensitivity experiments.

<sup>11</sup>All the years show the same pattern of mixing.



FIGURE 6.  $R^2$  Values for Each Year. 95% credible intervals given between  $\nabla$  and  $\triangle$ .

that the GBME predictions are between 15% and 50% better than the gravity model benchmark.

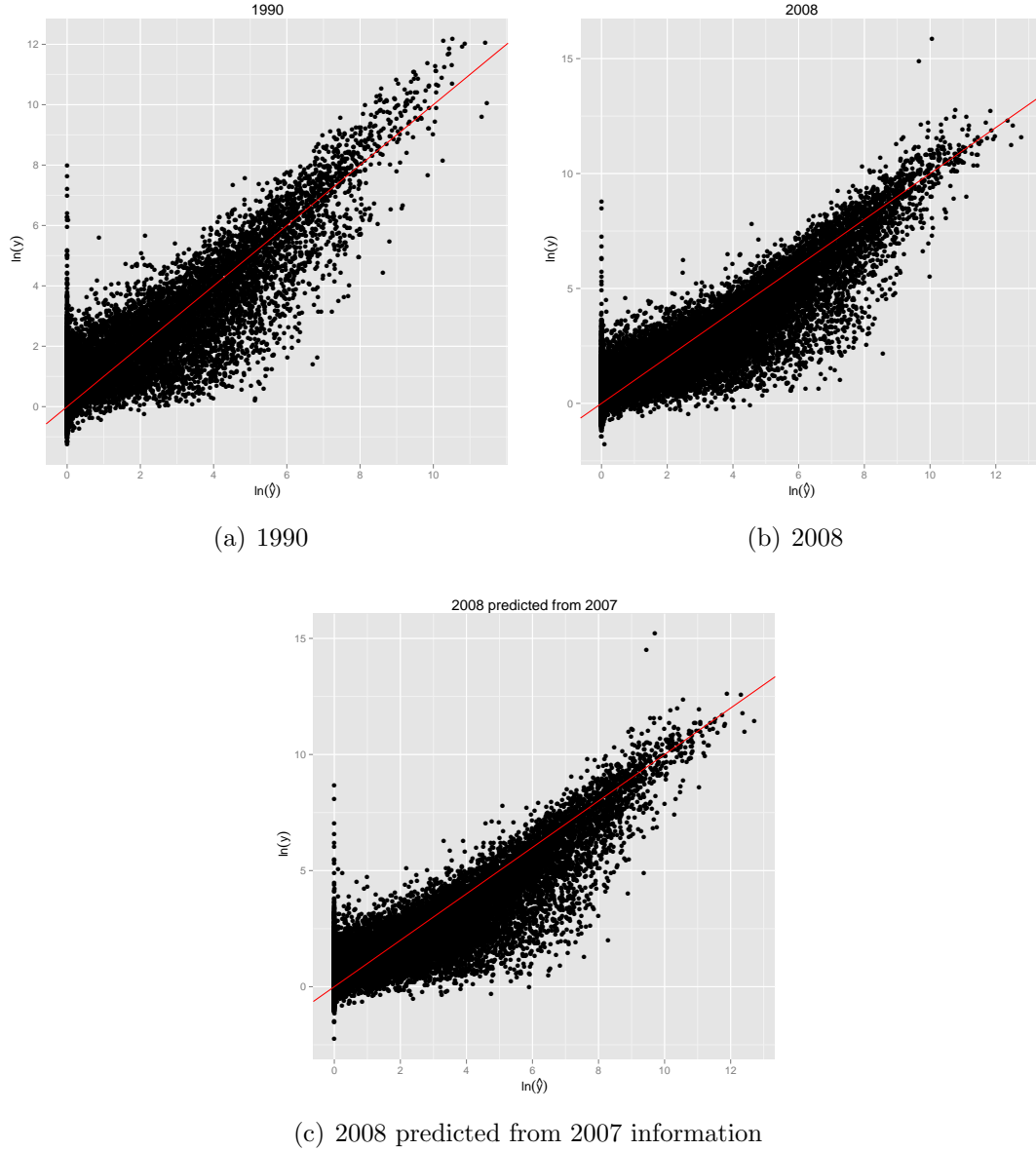
TABLE 2. Out-of-sample predictive performance. Table entries are Root Mean Squared Errors

	Goldstein Rivers Tomz	GBME
2000	1.26	0.85
2001	1.25	0.86
2002	1.24	0.86
2003	1.26	0.84
2004	1.03	0.89

Figure 7 illustrates the in-sample fits of logged exports for 1990 and 2008 along with the out-of-sample fit for 2008, using estimates and covariate values from 2007. The upper row of plots shows that the in-sample fit is quite good and does not change much from the beginning of our sample to the end of our sample.<sup>12</sup> Using the estimates from 2007 along with covariates available in 2007, we present in the bottom frame of Figure 7 the predictions implied by the model and the estimates for dyadic trade in 2008. The basic pattern for out-of-sample fit is the same as it is for the in-sample fit, which is generally evidence that the estimated model is capturing the data generating process rather than just reflecting over-fitting.

To get a better feel for results, Figure 8 displays posterior means and 95% Bayesian credible intervals for parameter estimates for 1991-2008. Parameters are precisely estimated and with signs in the expected directions. The lagged latent position

<sup>12</sup>Intermediate years show no divergence from this strong pattern.

FIGURE 7. *In-sample fit in 1990 and 2008.*

has a coefficient that is close to 1.0 suggesting a stability in the underlying network structure of dependencies. Countries that are close to one another tend to trade more with each other, and the GDP of the exporter and importer both are associated with larger amounts of trade, though the importer slightly more so. Importantly, we also recover very precise reciprocity coefficients, implying that countries tend to export to one another.

These basic findings emerge for every year that we examined, and if you consider a pooled estimate of the coefficients, you see the same basic pattern emerging, a pattern which is expected from the gravity model: countries that are close geographically and in terms of the size of the economy tend to trade more with one another than pairs of countries that are distant or have dissimilar sized economies. That is old news; but what is real news is that these results persist strongly in a

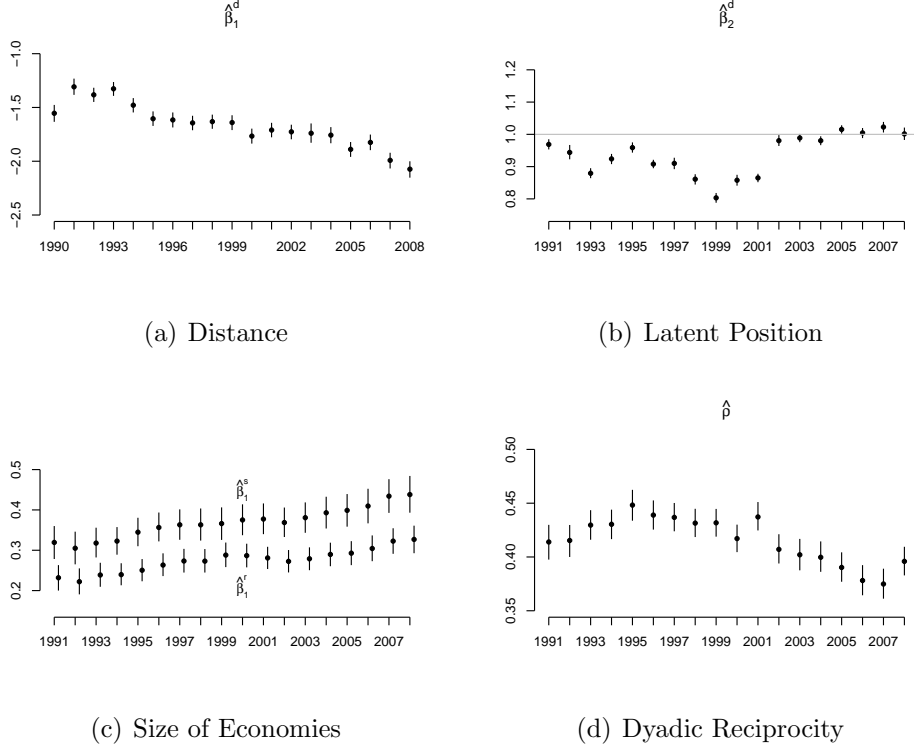


FIGURE 8. *Coefficient Plots for 1991-2008, show very precise estimates.*

model that takes into account the first, second, and third order dependencies directly rather than through a hodge-podge of ad hoc variables.

The values of the posterior means of the coefficients, estimated over time are presented in Figure 8. What is striking about these is that not only are all the coefficients estimated precisely in every year, but for the most part they don't change very much over time. The size effect of the exporter is always bigger than the size effect of the importer, but the two sets of coefficients are broadly similar. What is noticeable is the upward drift in these two coefficients, which each effectively double in size over the two decades examined. This can be interpreted as evidence of the increase of the importance of trade in the modern economy, such that exports are more highly tied to the size of the domestic economies in recent years.<sup>13</sup>

**4.1. Exploring the Exporter and Importer Random Effects.** As others have noted, it is unlikely that even the most expansive model will be correctly specified. Moreover there are theoretical reasons to expect correlation between a trade flows from one country to all its various trading partners. Importer and exporter effects are now considered *de rigueur*. Figure 9 displays these estimated effects from the GBME specification for 1990 and 2008. The black line in each panel represents the assumptions of the gravity model, given the variance of the estimated random effects. The gray band is the 95% confidence band within which we expect 95% of the cases to appear. The blue line reflects the actual distribution of the random

<sup>13</sup>All data are normalized to 2005 US dollars.

effects. While the GBME estimates are close to the assumed normal distribution there are some noteworthy cases in the tails.

For exporters, the largest of the 1990 random effects belong to rich trading nations (in order, from the highest): Japan, Germany, France, the UK, the US, and the Netherlands, while the smallest exporter effects are found in Algeria, Iraq, Iran, Cambodia, Myanmar, and Afghanistan. By 2008 the largest exporter effects are exhibited by Germany, the Netherlands, Belgium, and China (the largest), while the largest importer effects are in Belgium, the Netherlands, China, the US, and Germany. China had the 20th largest exporter effect in 1990, but had grown to the largest (12% larger than the nearest exporter) in 2008.

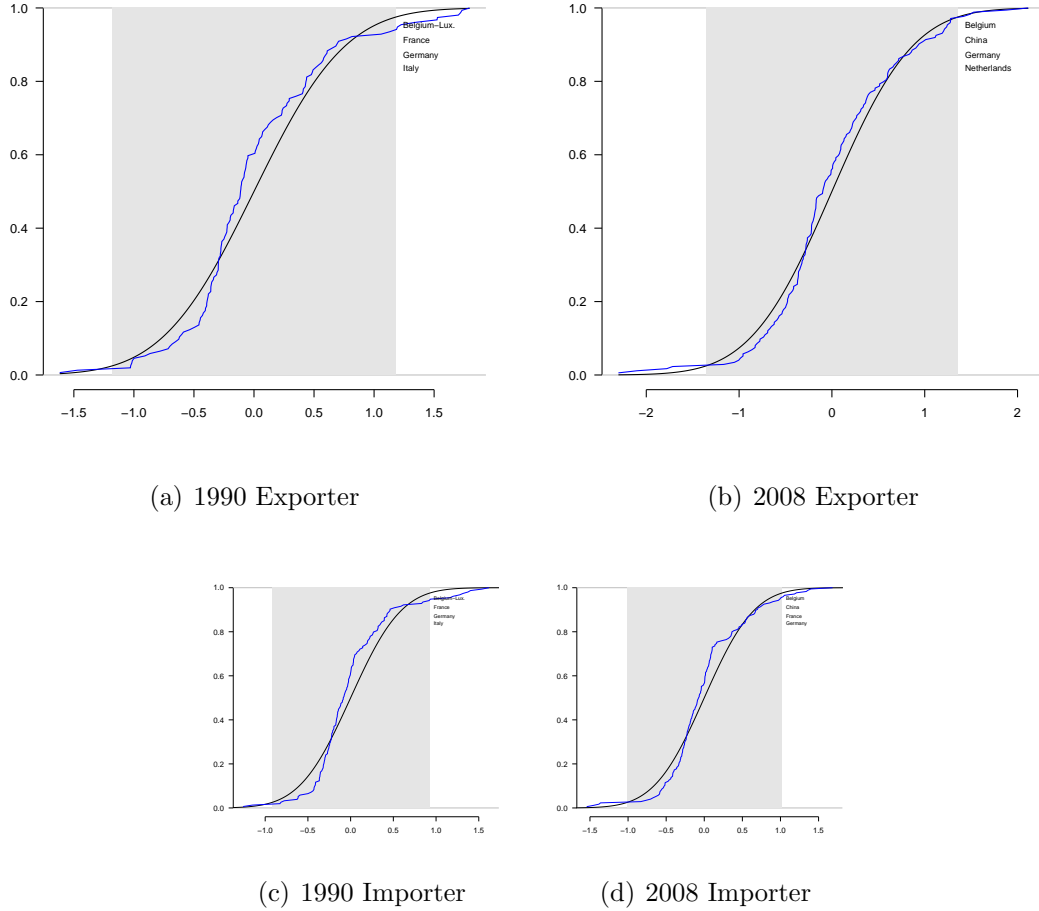


FIGURE 9. *Exporter and Importer Random Effects for 1990 and 2008. Gray bands illustrate the  $\pm 1.96\sigma$  bands*

**4.2. Exploring the Higher-Order Terms.** The higher-order variance terms are part of the real payoff to the GBME approach. In the absence of network structure in the data these terms should be close to zero, collapsing our model to a simple Gaussian gravity model. In figure 10 we plot the error variance ( $\sigma_\gamma^2$ ), reciprocity ( $\rho$ ), latent exporter variance ( $\sigma_{u_1}^2, \sigma_{u_2}^2$ ) and importer variance terms ( $\sigma_{v_1}^2, \sigma_{v_2}^2$ ). The points are color coded such that darker points represent more recent years. All terms

are quite far from zero, implying strong higher-order dependencies characterize these data. In fact the variance of the latent dimensions is about the same or greater than the model error variance; thus, much of the explained variance can be attributed to the dependencies, not to the gravity model itself.

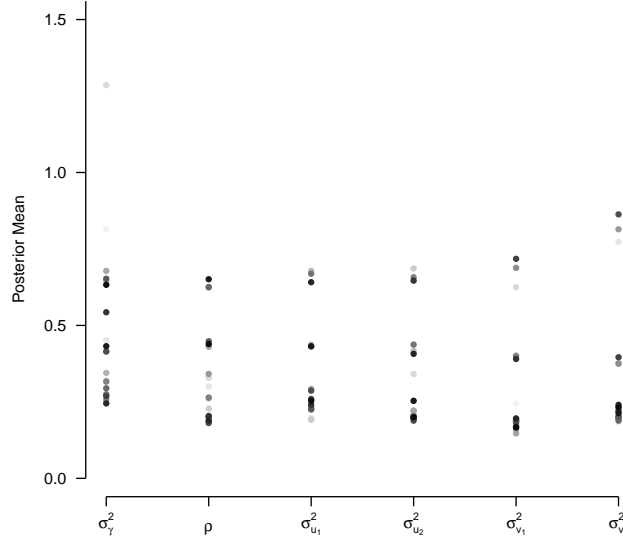


FIGURE 10. *Higher-order variance terms.*

**4.3. Exploring Latent Space.** Having recovered evidence of significant third-order dependence in international trade, what does the latent space look like? To see that that latent positions,  $u_i'v_j$ , captures third-order dependence recall that  $u_i'v_j = \|u_i\| \|v_j\| \cos \phi$ , where  $\phi$  is the angle between the vectors.<sup>14</sup> Thus  $\|u_i\|$  describes  $i$ 's latent propensity to export and  $\|v_j\|$  describes  $j$ 's propensity to import and the relative orientation of the two vectors describes the two countries' propensity trade with one another. Countries with  $u$  and  $v$  in the same direction tend to export and import more with one another beyond what is expected under the gravity model alone. It is worth emphasizing that the latent space describes the underlying structure of multilateral dependence in the data not captured in the systematic regression terms or even the unit-level random effects. Rather than a set of country-specific attributes the latent space can be thought of as a global or network-wide object through which we examine the prevalence and dynamic evolution of the dependencies in the data.

Figure 12 displays the estimated latent network export positions for countries in each year from 1990 to 2008. These positions have been rotated to a common orientation of 1997 for comparison.<sup>15</sup> In order to provide some context for the plot, countries are colored based on their region of the world according to the legend in the lower right panel. We note that latent positions are fairly stable over time both

<sup>14</sup>Many distance norms are feasible.

<sup>15</sup>Latent positions are invariant to linear transformations of rotation, reflection, and scale.



for exporters and importers. However, early in the 1990s, mostly Northern, rich economies were grouped close together in these latent tendencies to trade with one another. By the same token countries in the south, especially Asian countries were also clustered. By 2008, this regionally-based clustering in the latent space had diminished considerably, though perhaps not completely. This evolution through time provides an interesting window into the process of “globalization,” at least in international trade. Increasingly deep trading relationships have not been simply deepening of trade between, e.g., the developed economies of Western Europe and North America but rather we see growth in the “extensive margin” of international trade across much of the world. It is noteworthy that the gravity model alone seems to capture the patterns of trade with African countries, which is overwhelmingly determined by their level of economic output. In particular, African nations are generally not drawn in to denser trading networks as globalization marches forward, unlike other regions of the world, especially Asia.

## 5. CONCLUSION

Empirical studies of international trade typically use variations on the gravity model, relying on conditional independence assumptions across dyads (or even dyad-years). Based on recent theoretical work we argue that these assumptions are difficult to sustain, even when using the current “fixes” of country fixed effects and a large array of *ad hoc* variables ranging from the existence of common borders, through similar colonial histories, on to common languages, alliance patterns, and we are tempted to suggest shared colors in flags. These models have rarely been used for any kind of cross-validation or forecasting.

We argue that network data of this kind should exhibit higher-order dependencies such as reciprocity and clusterability. We show that the network structure of international trade does not follow the power law pattern that seem to characterize some other socially-generated networks. The presence of such dependence implies misspecification and a high likelihood of bias in current models. Building on recent advances in network analysis we model the world trading system without assuming a particular network structure. We construct a dynamic bilinear mixed effects model using a simple gravity specification with only GDP and distance as covariates. We recover significant evidence of both second-order (reciprocity) and third-order (clusterability) network dependencies in world trade. The inclusion of dynamic latent positions contributes to more accurate predictions of international trade patterns as well a dynamic picture of the evolution of the world trading system. We show that the network oriented, general bilinear mixed effects trade model, based on a simple gravity specification, predictively out-performs standard specifications that ignore the interdependencies in international trade.

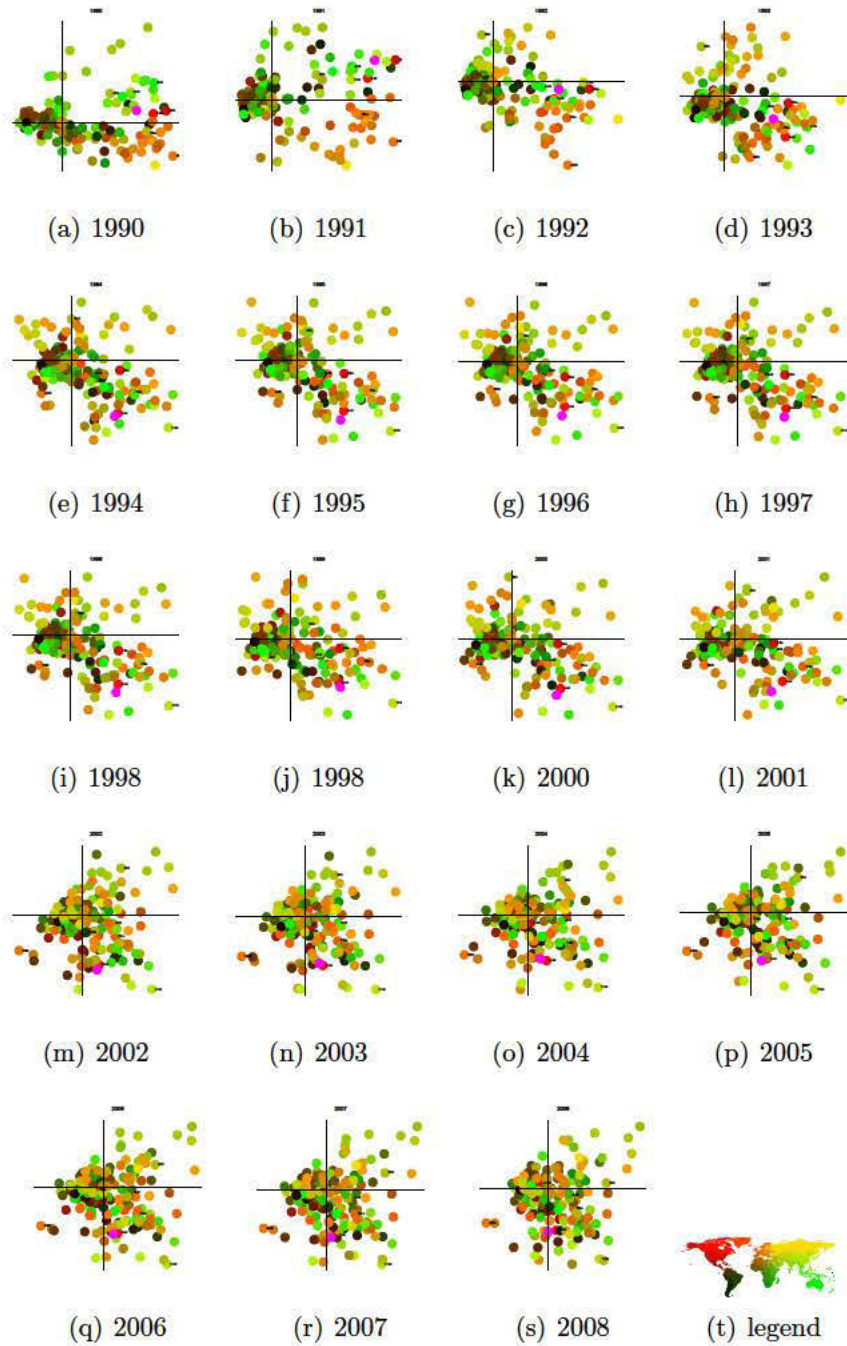


FIGURE 11. *The latent export positions of countries 1990-2008.*

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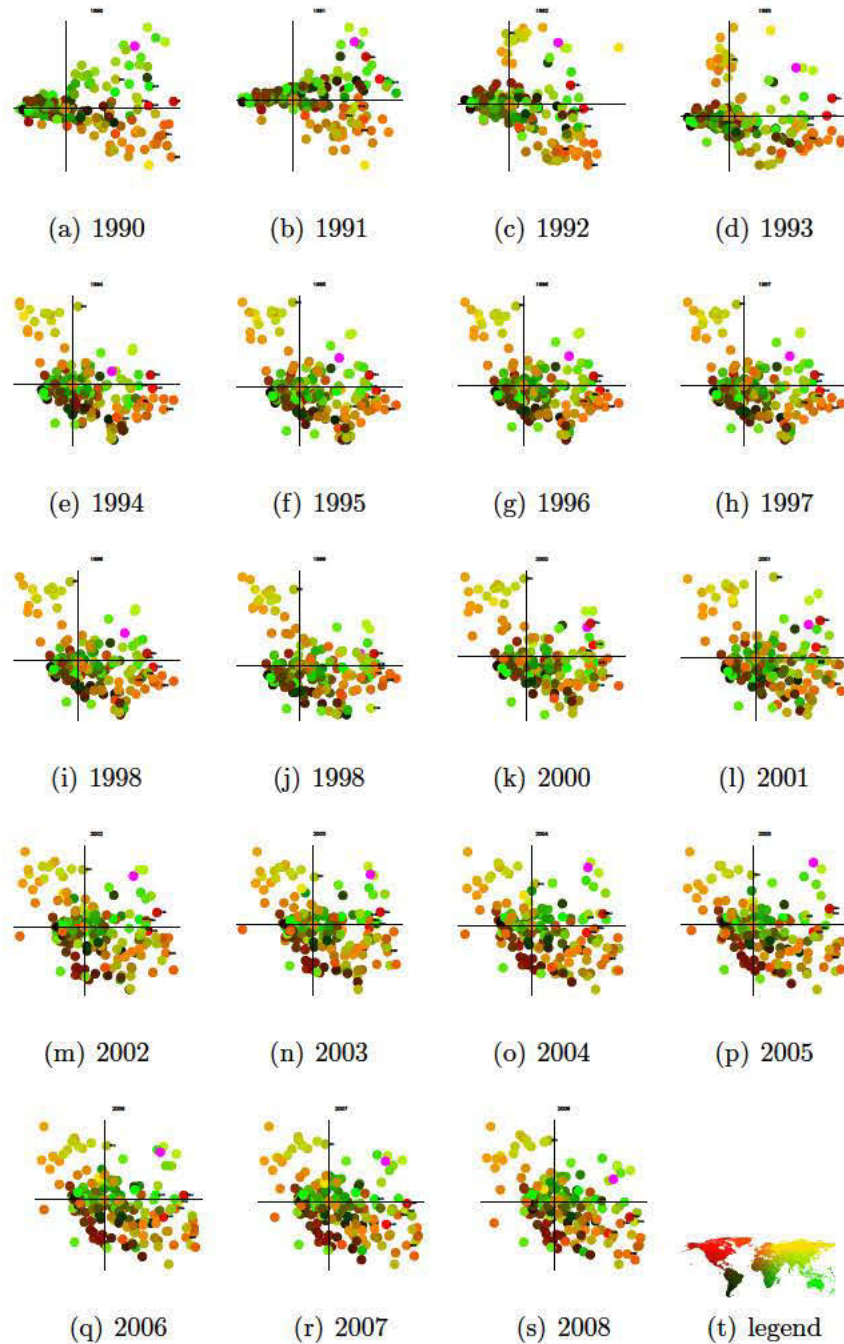


FIGURE 12. *The latent import positions of countries 1990-2008.*

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WARD: DEPARTMENT OF POLITICAL SCIENCE, DUKE UNIVERSITY, DURHAM, NC USA  
E-mail address: michael.d.ward@duke.edu

AHLQUIST: DEPARTMENT OF POLITICAL SCIENCE, FLORIDA STATE UNIVERSITY, TALLAHASSEE, FL USA  
E-mail address: jsahlquist@fsu.edu