Name:______________________________

Princeton University

ORF 363/COS 323
Midterm Exam, Fall 2025

October 9, 2025, from 1:20 pm to 2:40 pm

Instructor:
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AIs:
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PLEASE DO NOT TURN THIS PAGE. START THE EXAM ONLY WHEN YOU ARE INSTRUCTED TO DO SO.

1. You are allowed a single sheet of paper, double-sided, hand-written or typed.
2. Cell phones should be off or in airplane mode. No other electronic devices are allowed.
3. All questions should be answered in the booklet. Please write out and sign the following pledge on the booklet "I pledge my honor that I have not violated the honor code during this examination."
4. Make sure you write your name on the first page of these questions and return the questions to us at the end of the exam. Please don't forget to write your name on the booklet as well.
5. You are allowed to cite results proved in lecture, lecture notes, or problem sets without proof.
6. For the first and third questions, you need to justify your answers to receive full credit.

Problem 1 (32 pts)**:** Find all the local minimizers, local maximizers, global minimizers, and global maximizers of the following function of two variables (or argue if some do not exist):

$$f(x_1, x_2) = x_1^2 + (1 - x_1 x_2)^2.$$

What is the optimal value of the problem of minimizing $f(x_1, x_2)$ over $\mathbb{R}^2$?

Problem 2 (36 pts)**:** Let $f : \mathbb{R}^n \to \mathbb{R}$ be a twice continuously differentiable function, and $\bar{x}$ be a point in $\mathbb{R}^n$. Fill out each entry of the following table with "yes/no" *twice*, once for the question in part (a), and once for the question in part (b). (It's your lucky day! No proofs or counterexamples are required.)

		condition on f		
		quasiconvex	convex	strictly convex
condition on $\bar{x}$	$\nabla f(\bar{x}) = 0$			
	$\nabla f(\bar{x}) = 0$ $\nabla^2 f(\bar{x}) \succeq 0$			
	$\nabla f(\bar{x}) = 0$ $\nabla^2 f(\bar{x}) \succ 0$			

(a) Must $\bar{x}$ (necessarily) be a global minimizer of f?

(b) Must $\bar{x}$ (necessarily) be the unique global minimizer of f?

Problem 3 (32 pts)**:** Yonex offers n different tennis racquets and wants to nudge their prices slightly to reshape demand—for example, to better align it with their supply. The fractional change in demand, denoted by $\delta^{\text{dem}} \in \mathbb{R}^n$, is determined by the change in price $\delta^{\text{price}} \in \mathbb{R}^n$ through the linear relationship $\delta^{\text{dem}} = E\,\delta^{\text{price}}$. Here, the matrix $E \in \mathbb{R}^{n \times n}$, which is called the price-elasticity matrix in economics, is assumed to be known (e.g., estimated through customer surveys). Yonex wants the change in demand to be close to a given target change $\delta^{\text{tar}} \in \mathbb{R}^n$, while also keeping price adjustments small. To acheieve this, it considers the following optimization problem, where $\lambda > 0$ is a given parameter:

$$\min_{\delta^{\text{price}} \in \mathbb{R}^n} \; \|E\delta^{\text{price}} - \delta^{\text{tar}}\|_2^2 \;+\; \lambda \|\delta^{\text{price}}\|_2^2. \tag{1}$$

(a) Is (1) a convex optimization problem? Justify your answer.

(b) Show that (1) has a unique optimal solution. Then explicitly write down the optimal solution in terms of E, δ^{tar}, and λ.