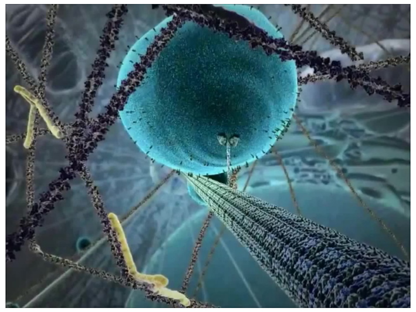
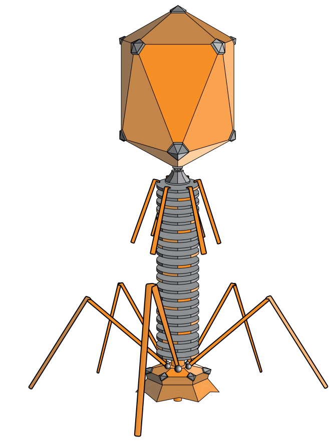
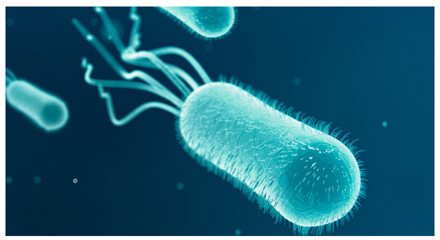
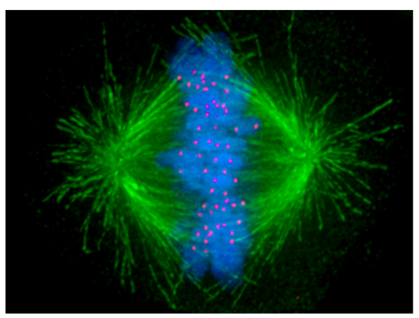
MAE 545 (Spring 2018)

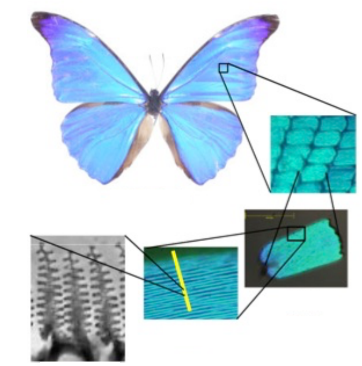




Special Topics - Lessons from Biology for Engineering Tiny Devices







Lectures:

T, Th 11:00 AM-12:20 PM,
EQUAD D221

Office hours:

Th 1:30-3:00 PM,
EQUAD D414
(or by appointment)

Andrej Košmrlj

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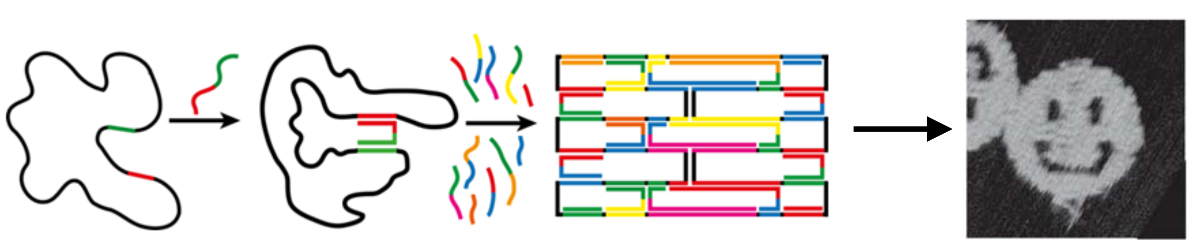


Lecture Notes

- text books: none
- lecture slides will be posted on Blackboard

  http://blackboard.princeton.edu

  course: MAE545_S2018

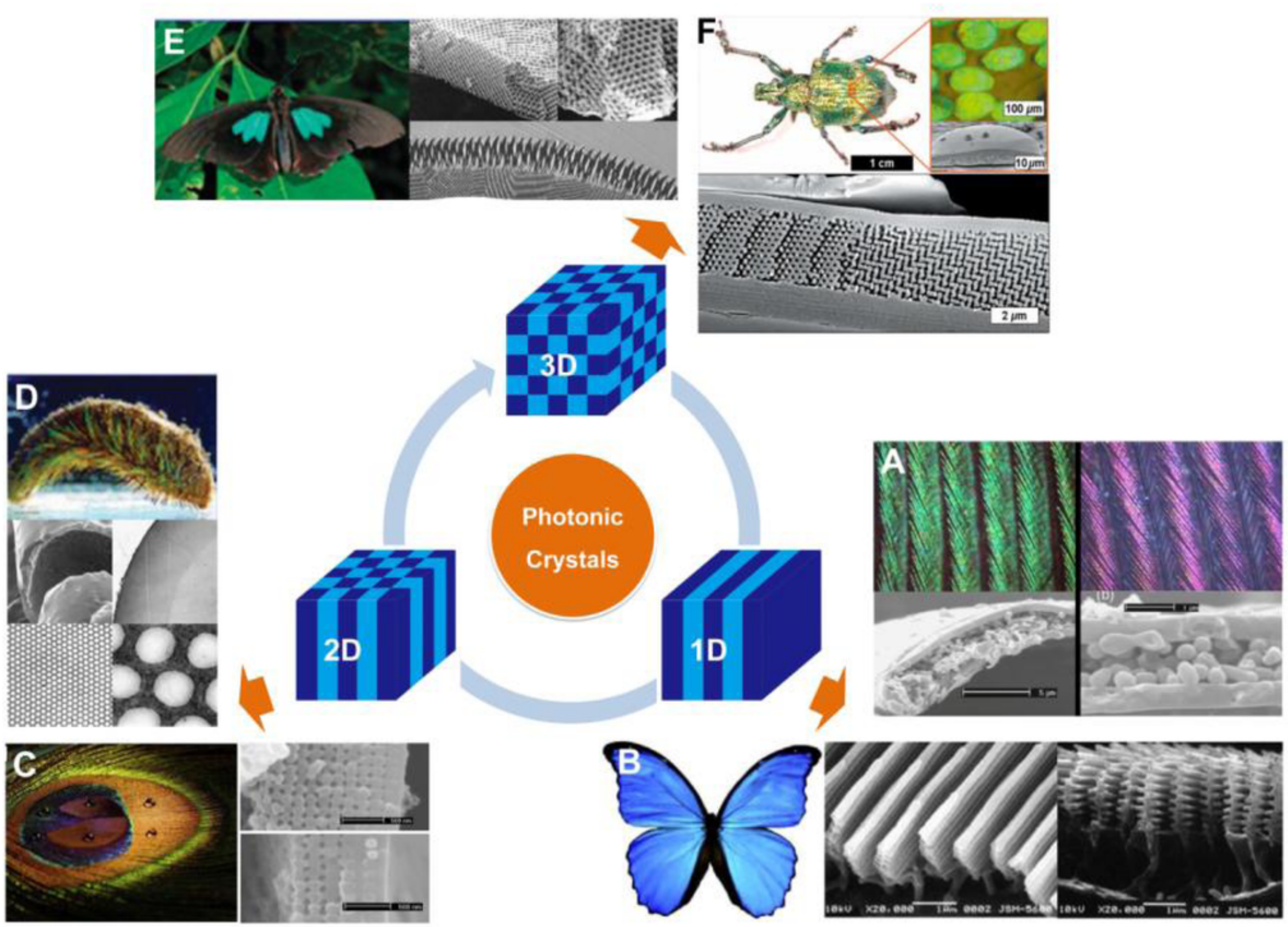
Assignments

- presentation of research paper in class
- final paper (final project)

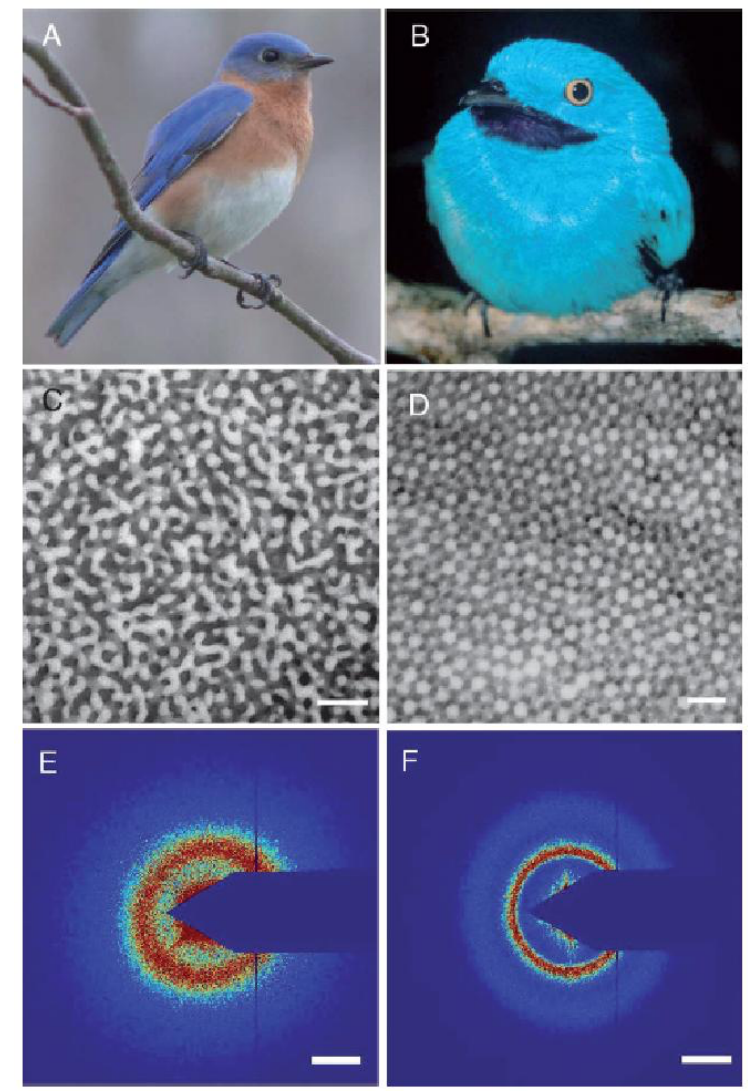
Course overview

Structural colors

Structural colors of animals and plants appear due to the selective reflection of ambient light on structural features underneath the surface.



H. Wang and K-Q. Zhang, Sensors 13, 4192 (2013)



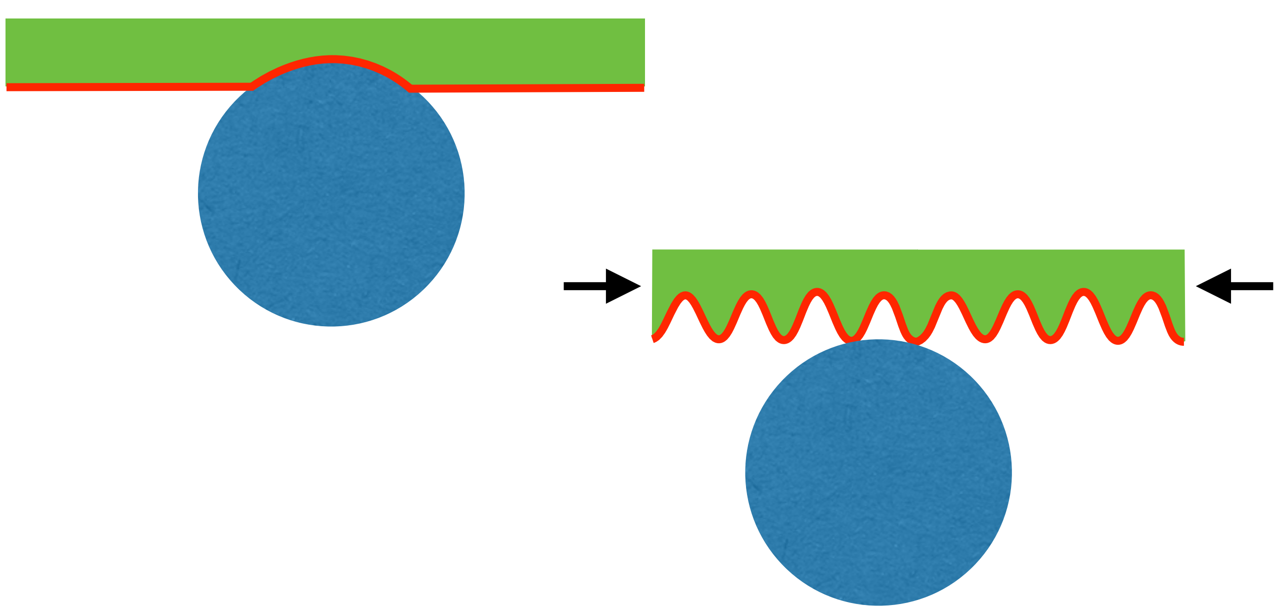
V. Saranathan et al., J. R. Soc. Interface 9, 2563 (2012)

Wrinkling

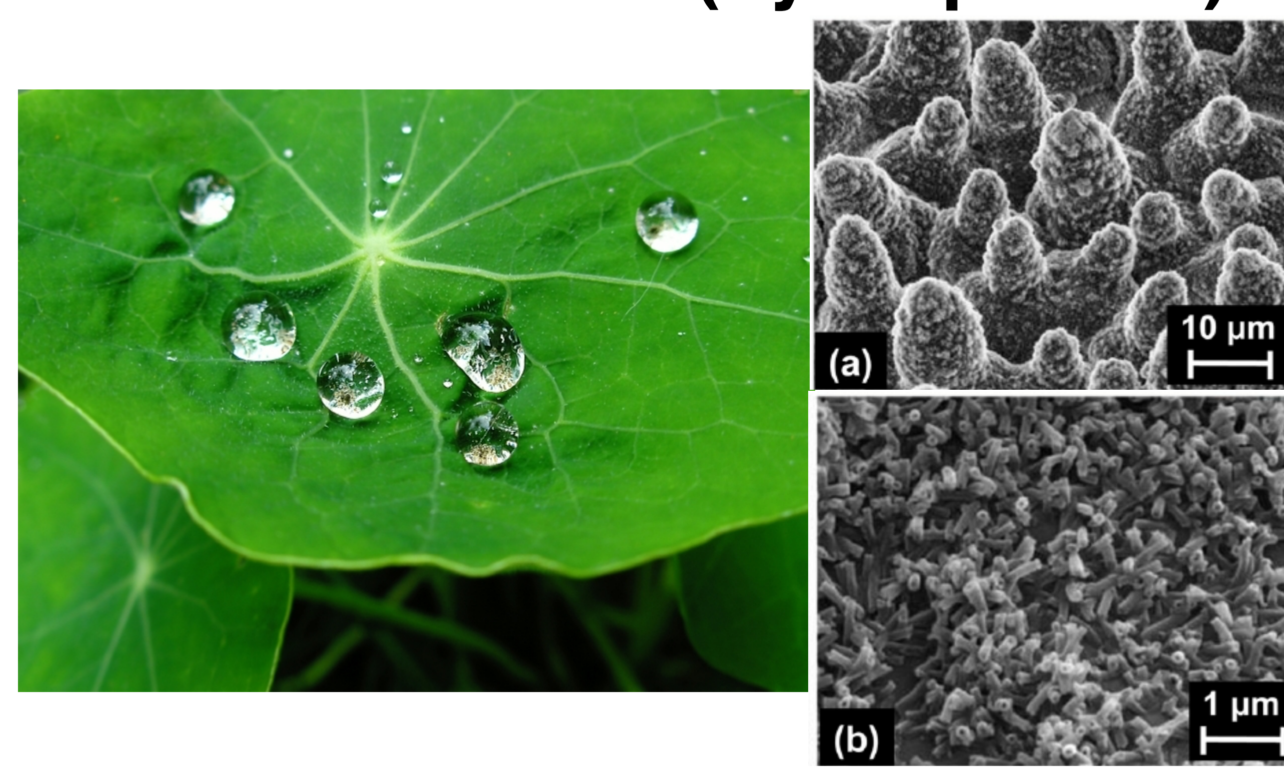
Wrinkling of thin films on soft substrates can be used to make flexible electronics and to tune drag, adhesion and wetting.

Golf balls (reduced drag)

wrinkled surface effectively reduces adhesion



Lotus leaves (hydrophobic)



Growth and forms in nature

Brain

Gut

Beaks

Leaf

Flower

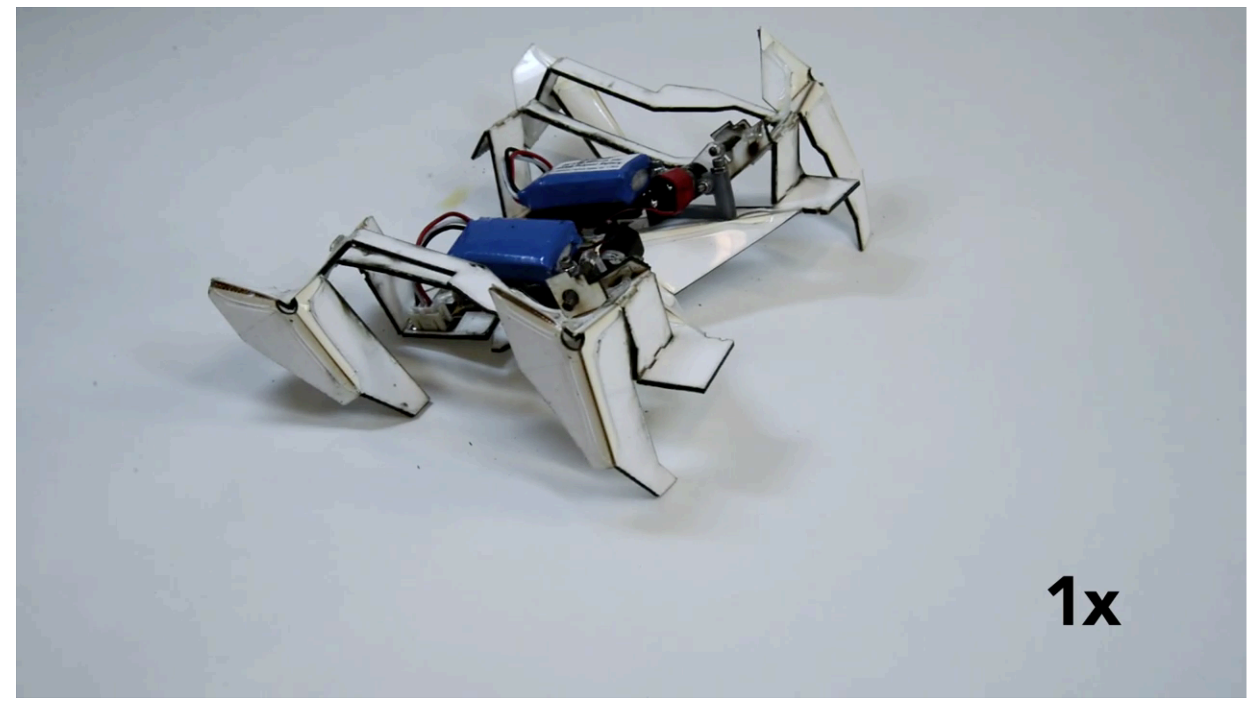
Shells

From transformable shapes to self-folding robots

opening/closing of flowers

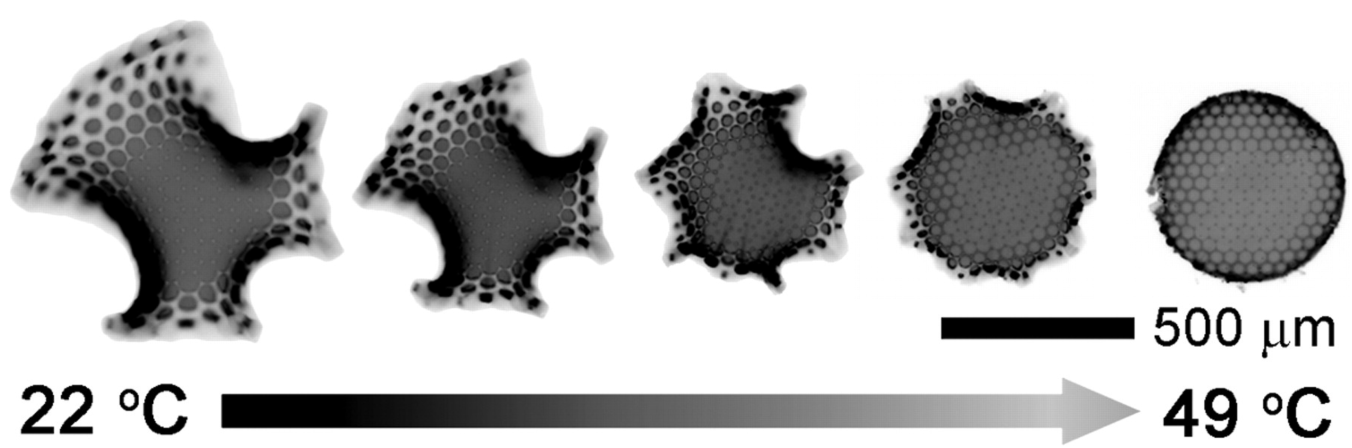
https://vimeo.com/98276732

self-folding robots

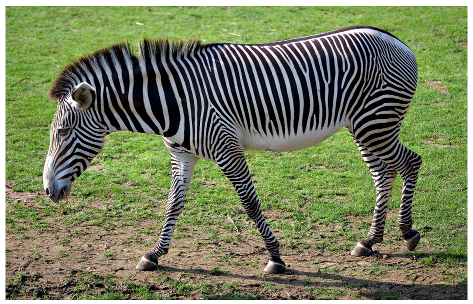


https://www.youtube.com/watch?v=1M-vQdyY6OE

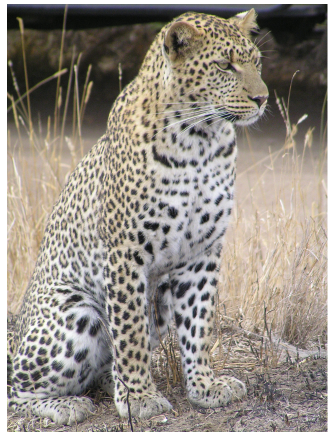
swelling of patterned gels



Patterns in nature



zebra

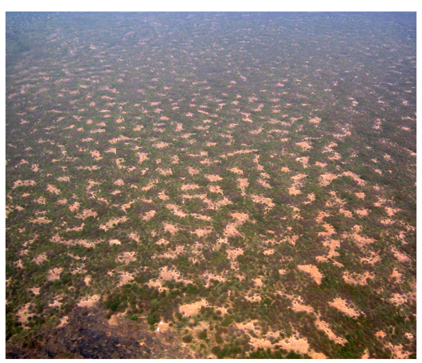


leopard

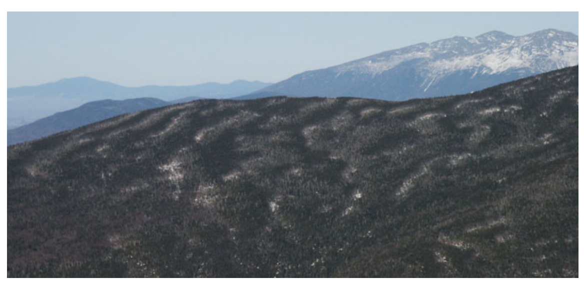
royal angelfish

peacock

giant pufferfish



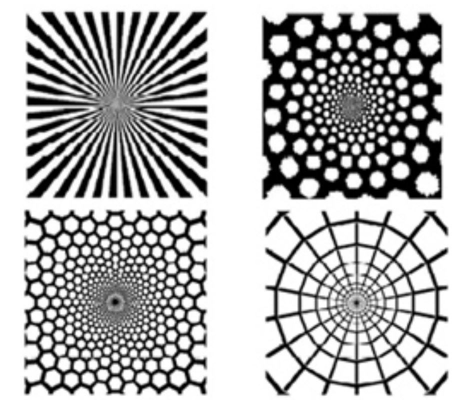
tiger bush



fir waves

mussels

clouds

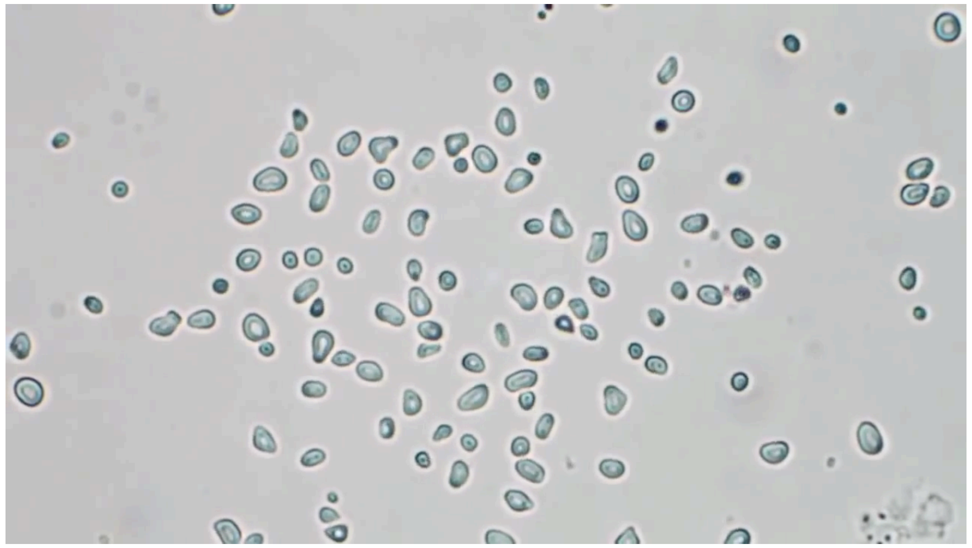


hallucination patterns

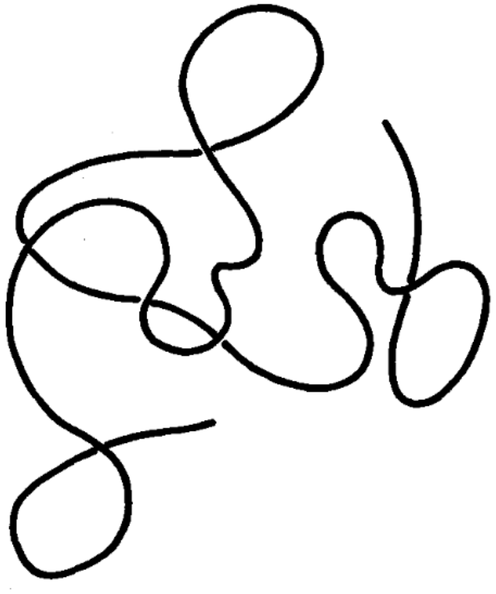
Turing patterns

Random walks and directed swimming

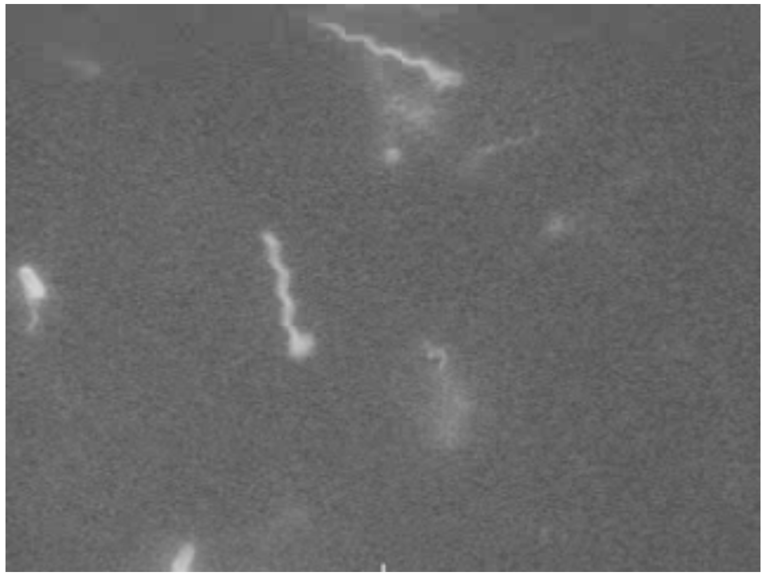
Brownian motion



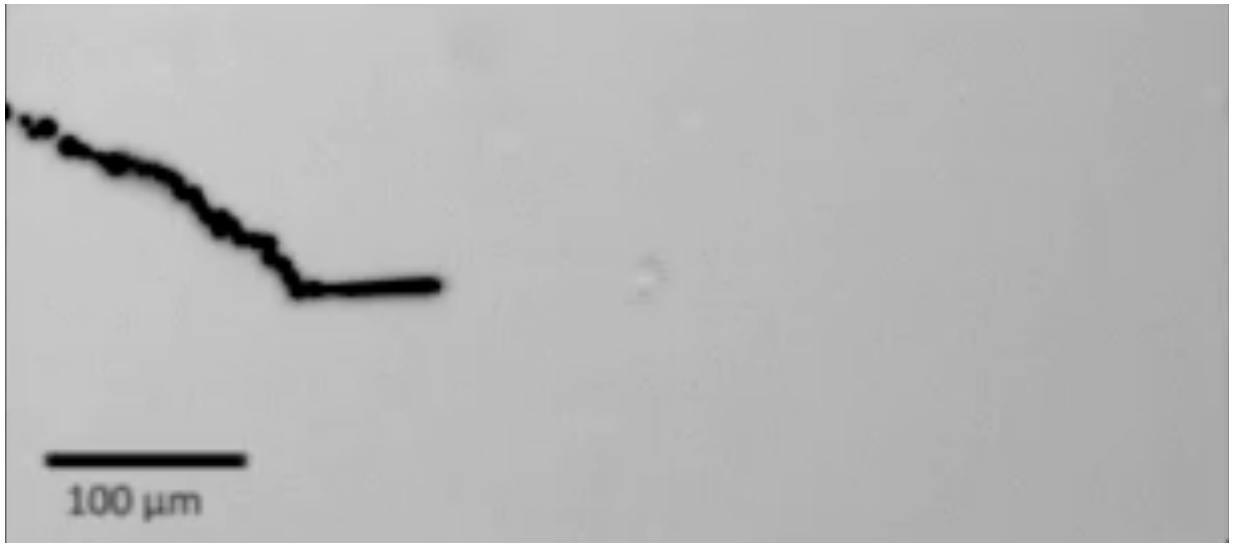
Polymer random coils



Swimming of E. coli



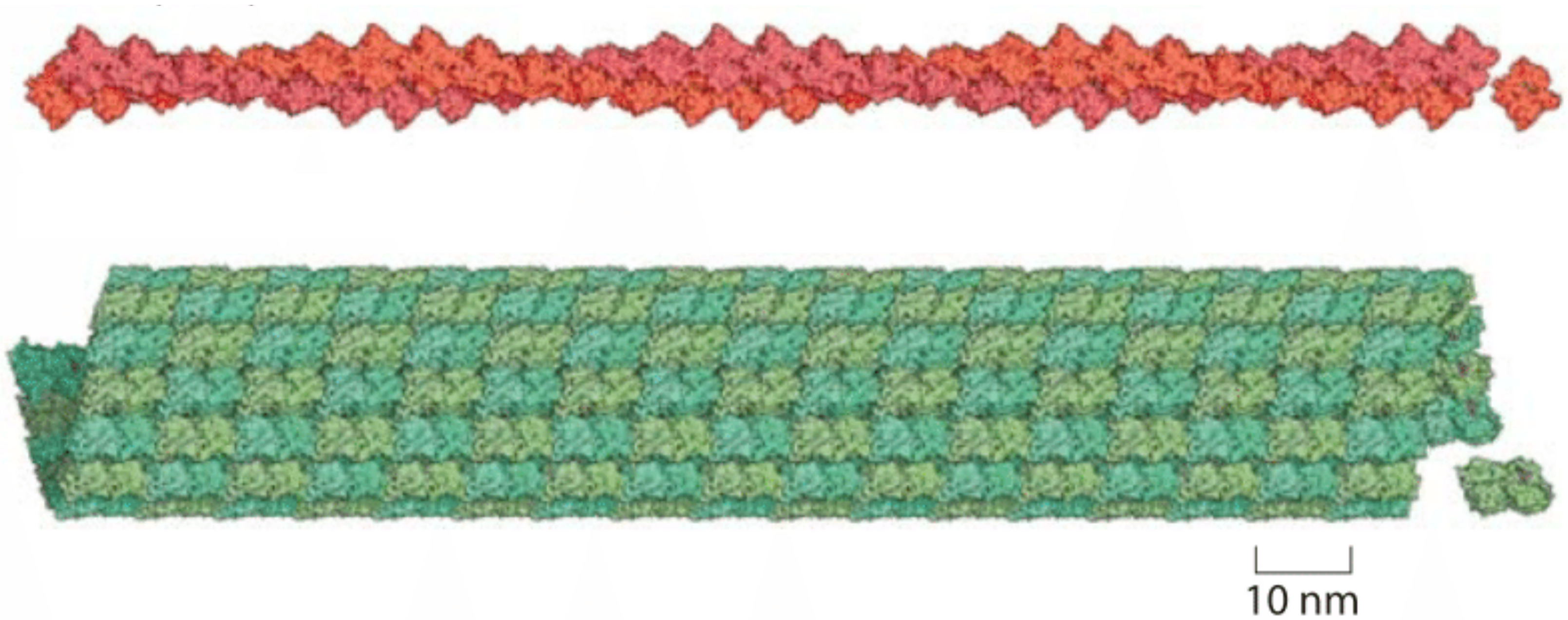
Swimming of microrockets



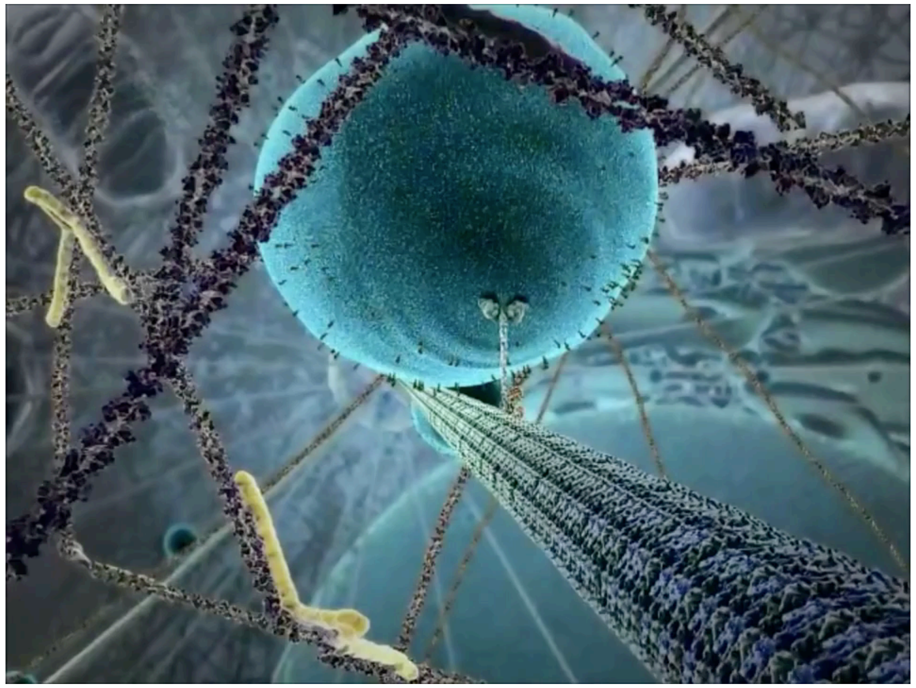
Protein filaments and molecular motors

Actin filament

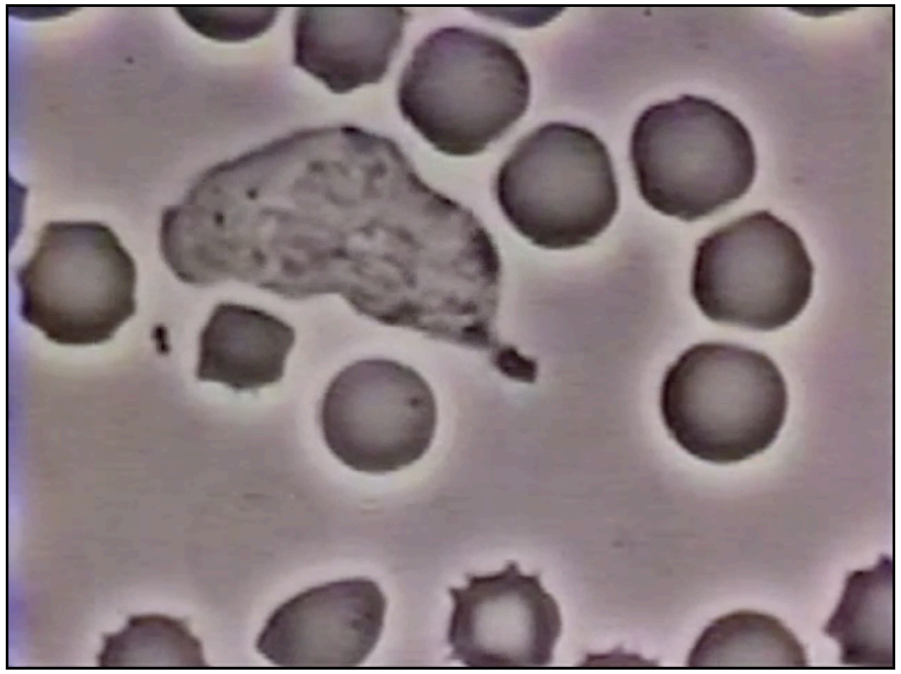
Microtubule



Cargo transport



Crawling of cells



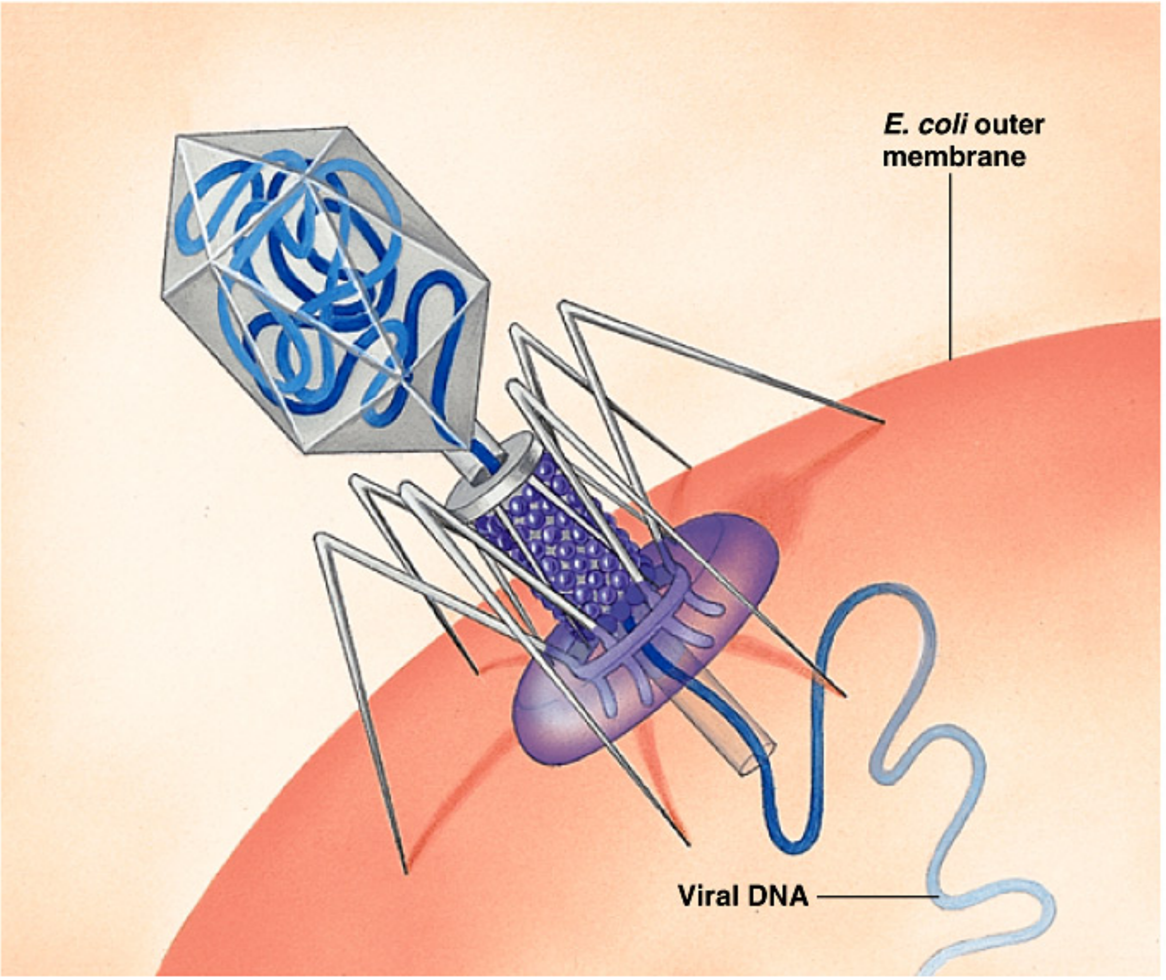
Contraction of muscles

Viruses and drug delivery

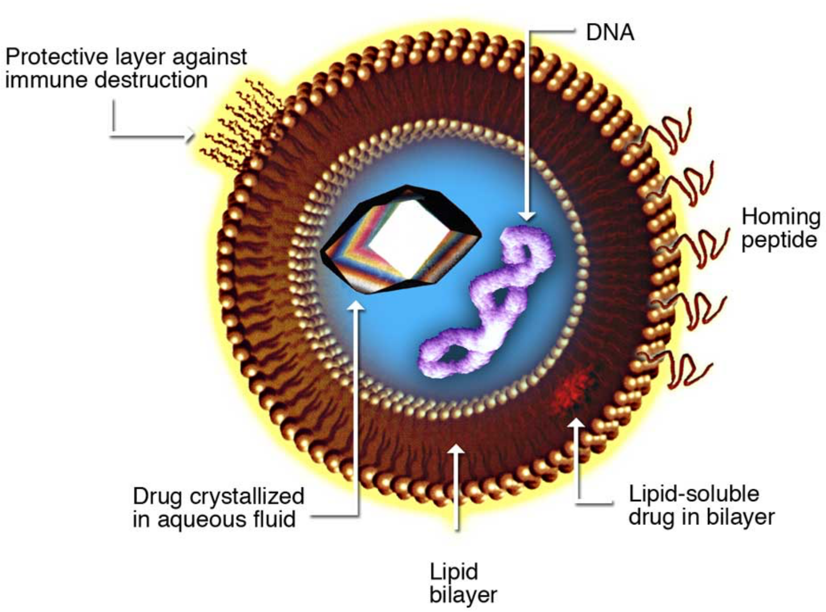
assembly of
viral capsids

packing of viral DNA
inside the capsid

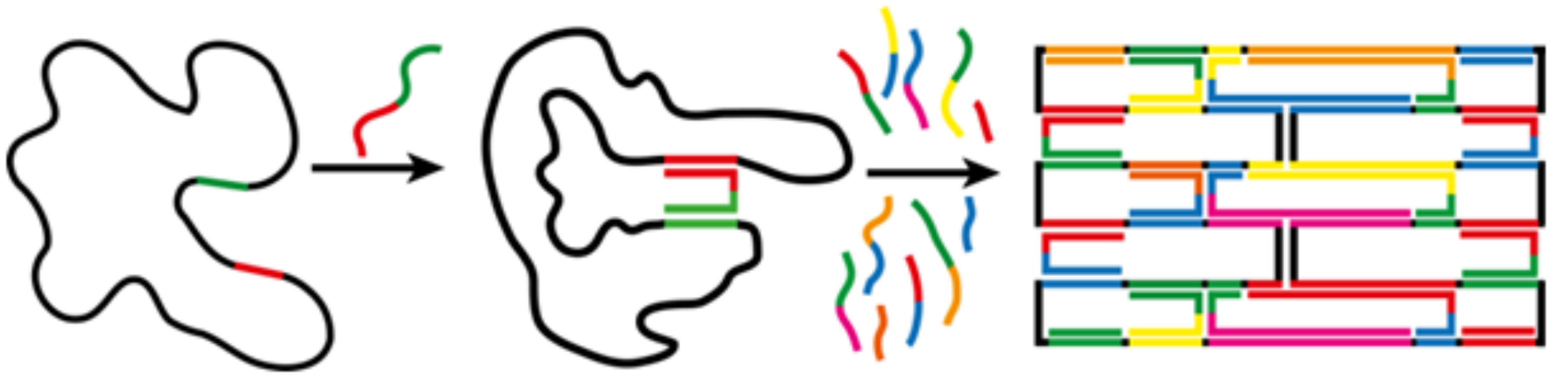
infection of cells

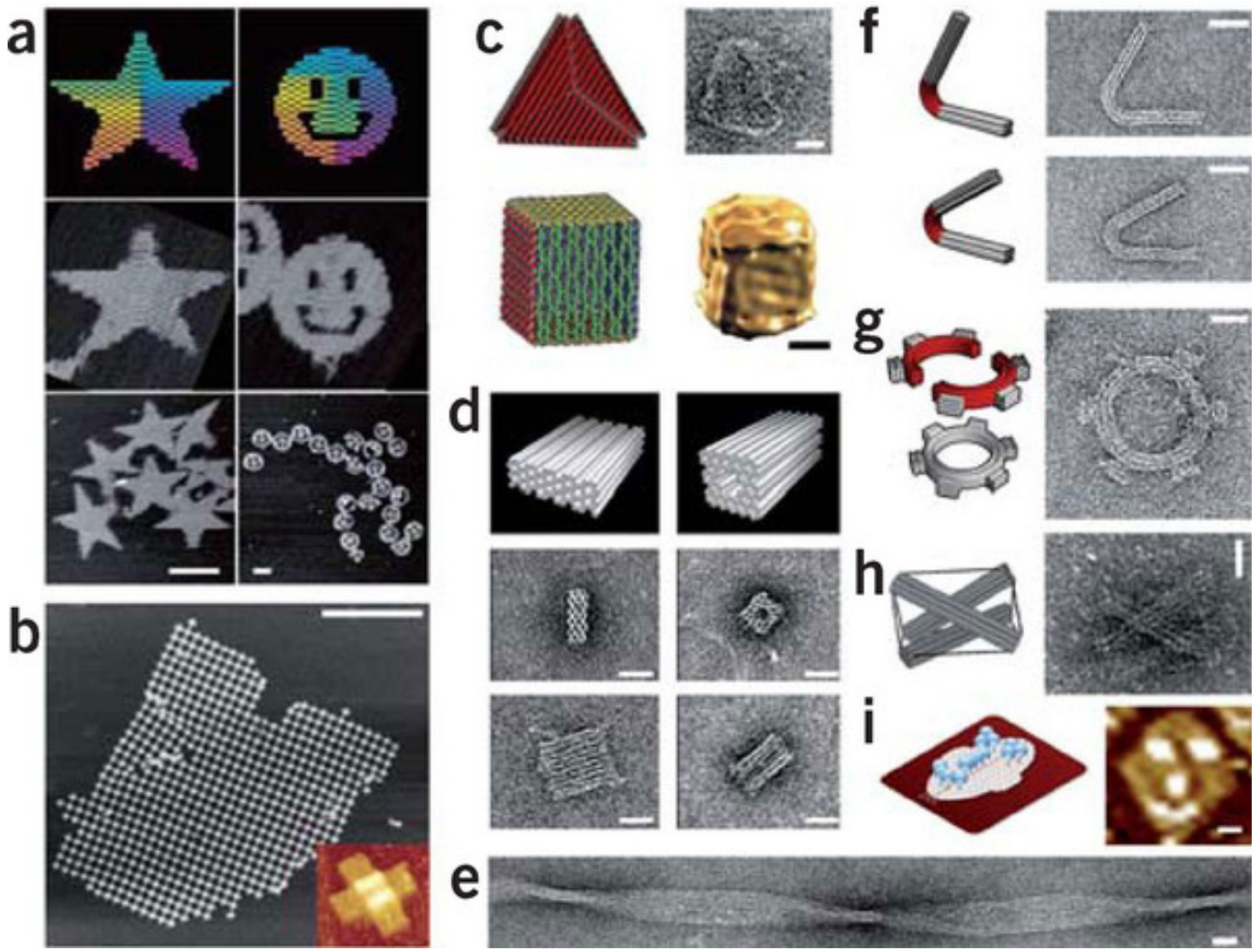


drug delivery



DNA Origami

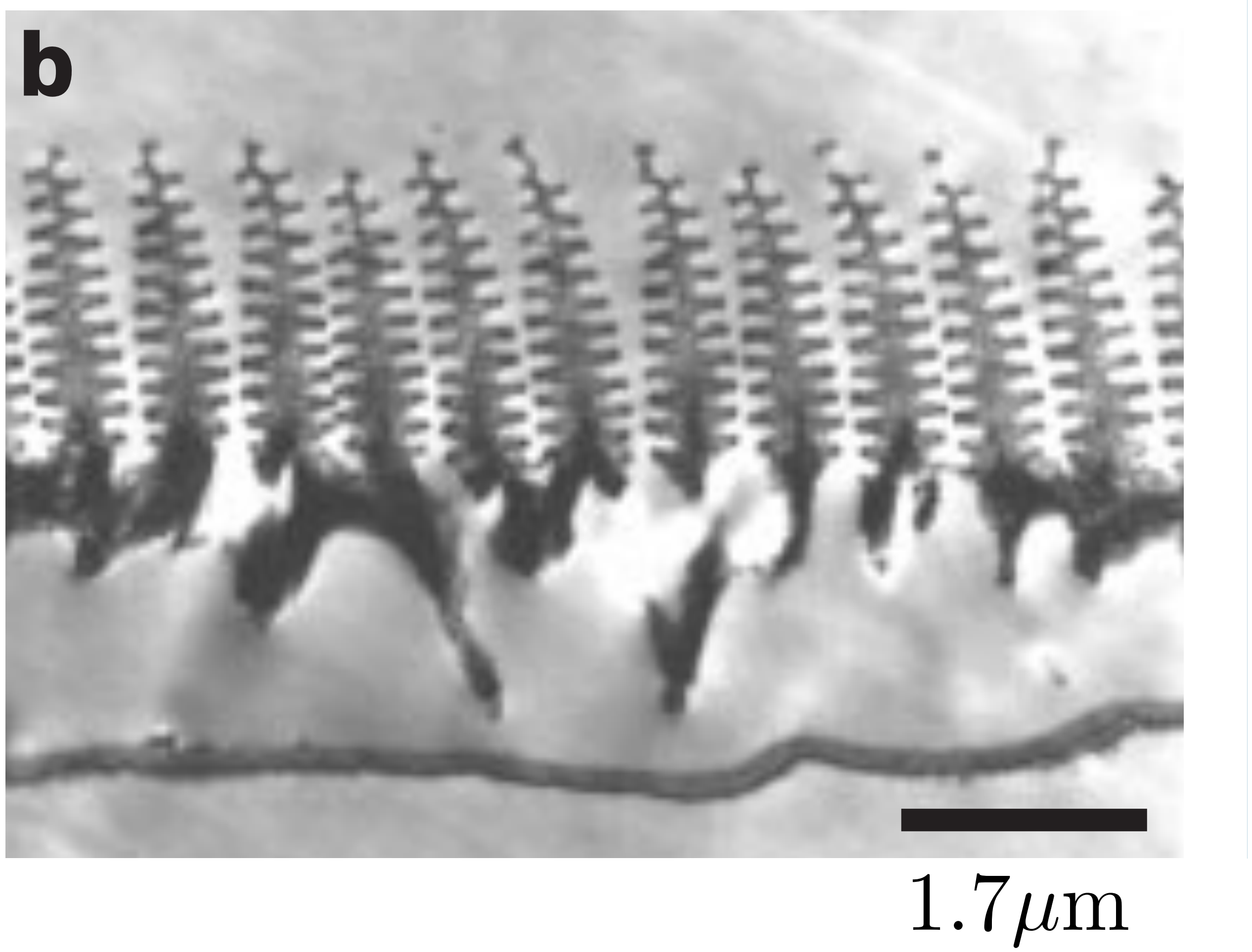




C. E. Castro et al., Nature methods (2011)

MAE 545: Lecture 1 (2/6)

Structural colors

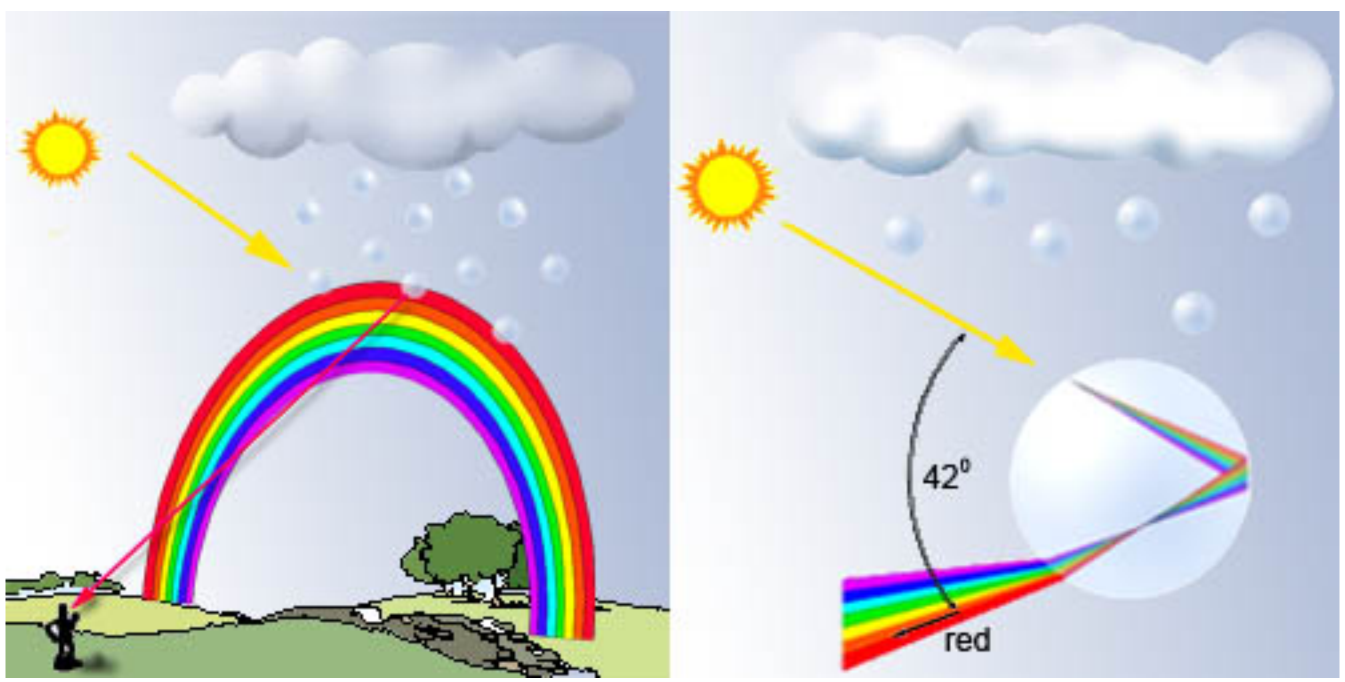


Structural color

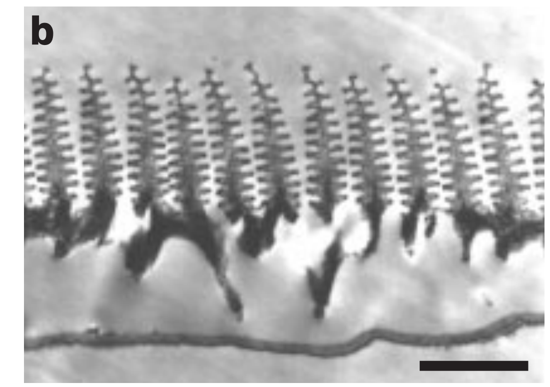
Structural colors of animals and plants appear due to the selective reflection of ambient light on structural features underneath the surface.

White light coming from the sun consists of all colors.

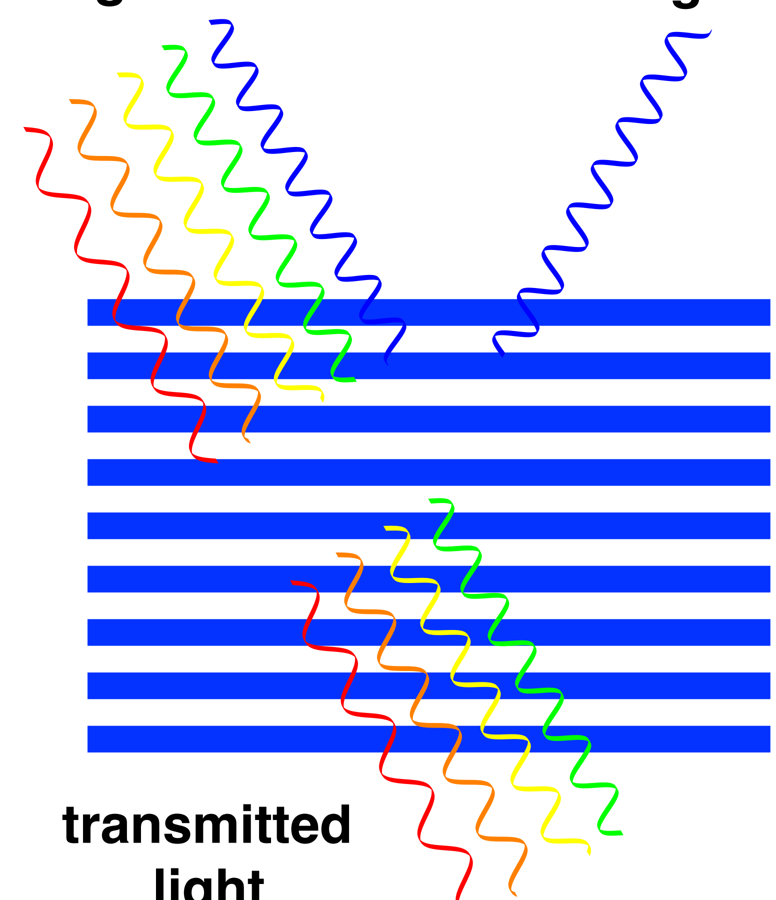
rainbow



structural color



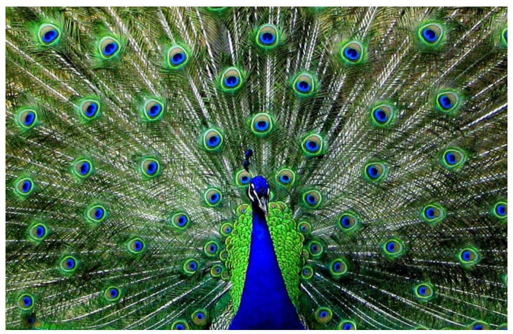
$1.7 \mu m$

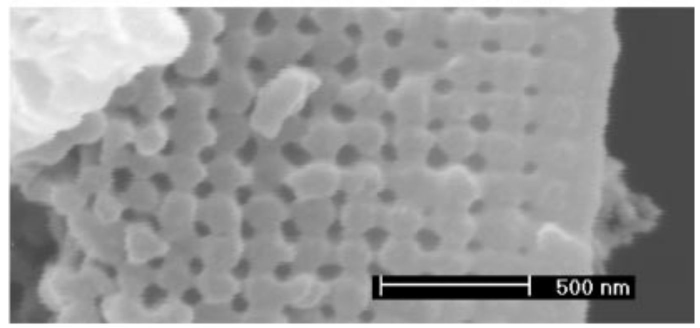


Structural colors

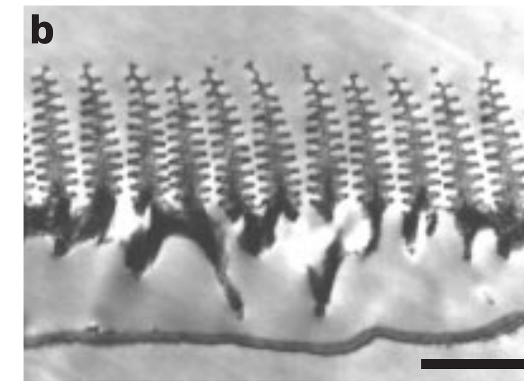
Structural colors of animals and plants appear due to the selective reflection of ambient light on structural features underneath the surface.

Peacock feather eyes



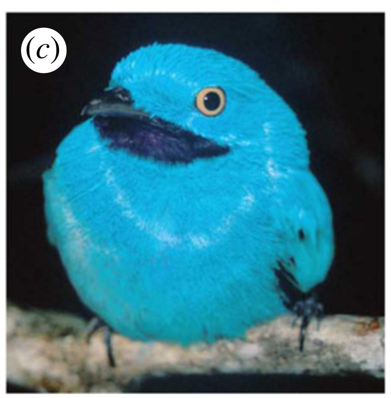


Morpho butterfly

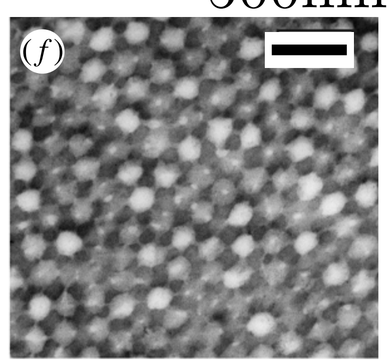


$1.7\mu m$

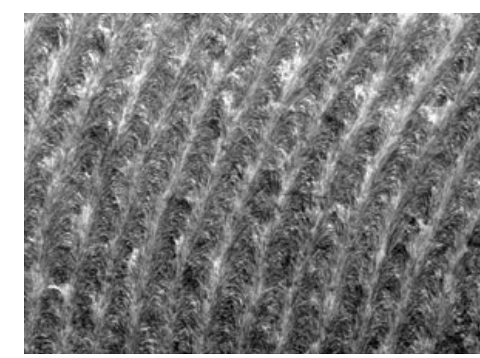
Plum-throated Cotinga



500nm

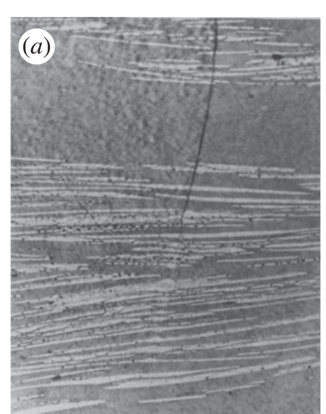


Marble berry



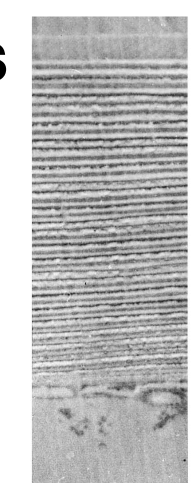
250nm

bleak fish



$1\mu m$

Chrysina aurigans beetle



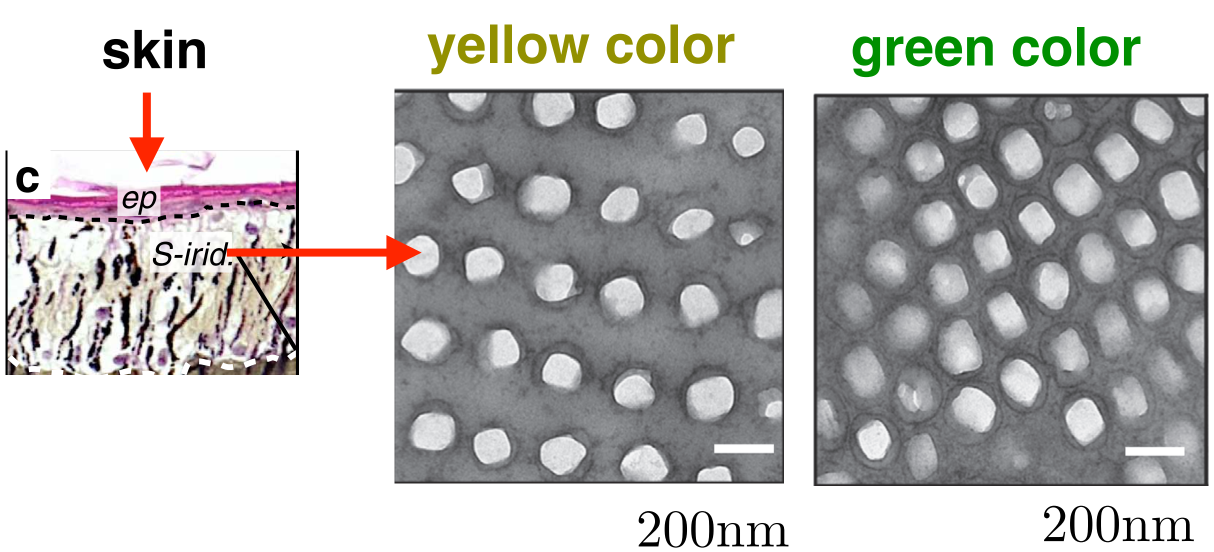
$1\mu m$

Dynamic structural colors

Chameleon (speed 8x)

J. Teyssier et al., Nat. Comm. 6, 6368 (2015)

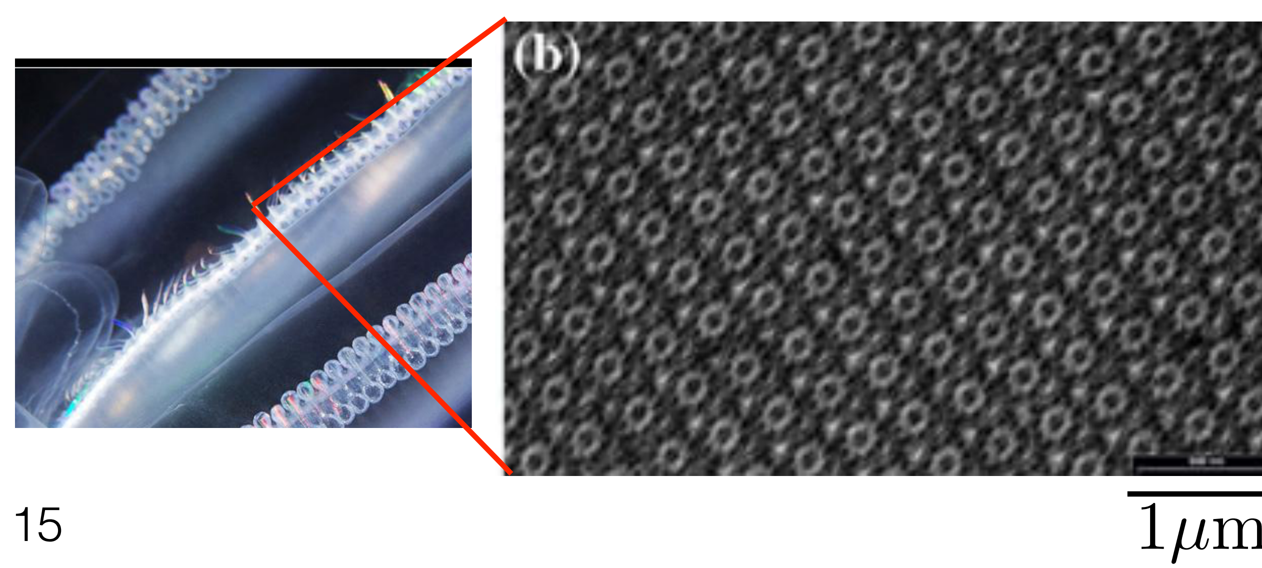
Changes in osmotic concentration lead to the swelling of cells in excited chameleon. This changes the spacing of periodic structure from which the ambient light is reflected.



Comb Jelly (real time)

https://www.youtube.com/watch?v=Qy90d0XvJIE

Rainbow color waves are produced by the beating of cilia, which change the orientation of periodic structure from which the ambient light is reflected.



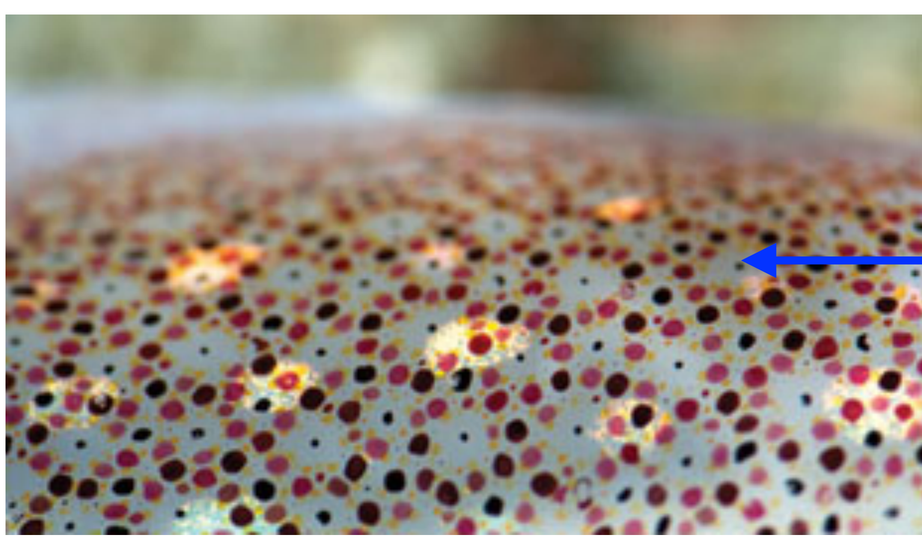
Dynamic colors in cephalopods

octopus

squid

https://www.youtube.com/watch?v=9MB2ltsAPnQ

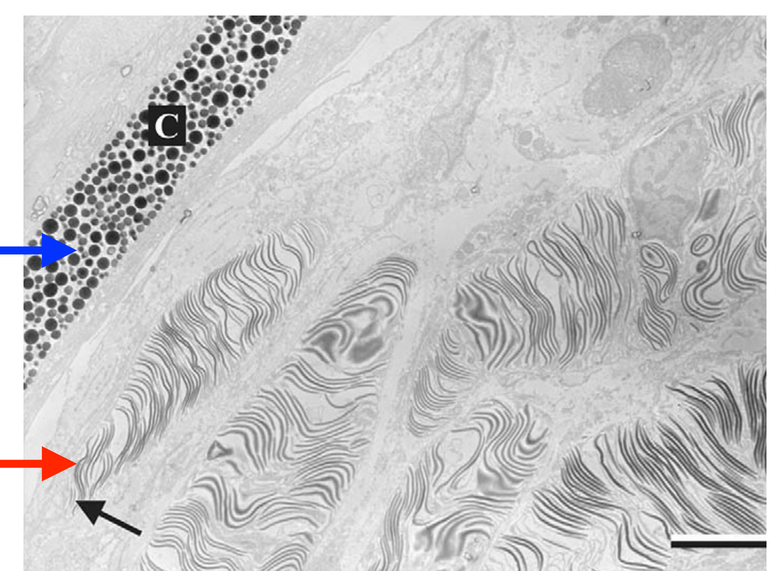
Dynamical color change in cephalopod is achieved by modulation of size and spacing of both the pigment cells and the cells reflecting light.



squid skin surface

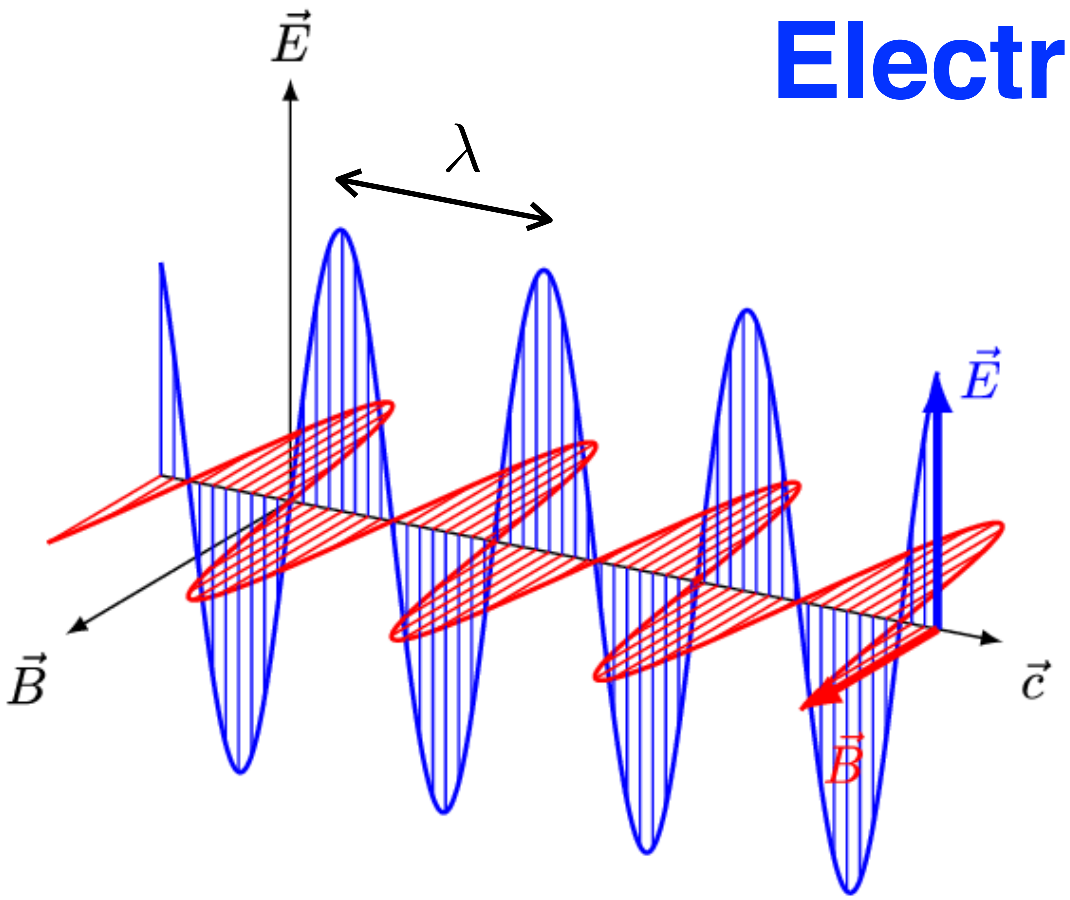
chromatophores (pigment cells)

iridophores (reflecting light)



$7.5\mu m$

Electromagnetic waves



electric field

magnetic field

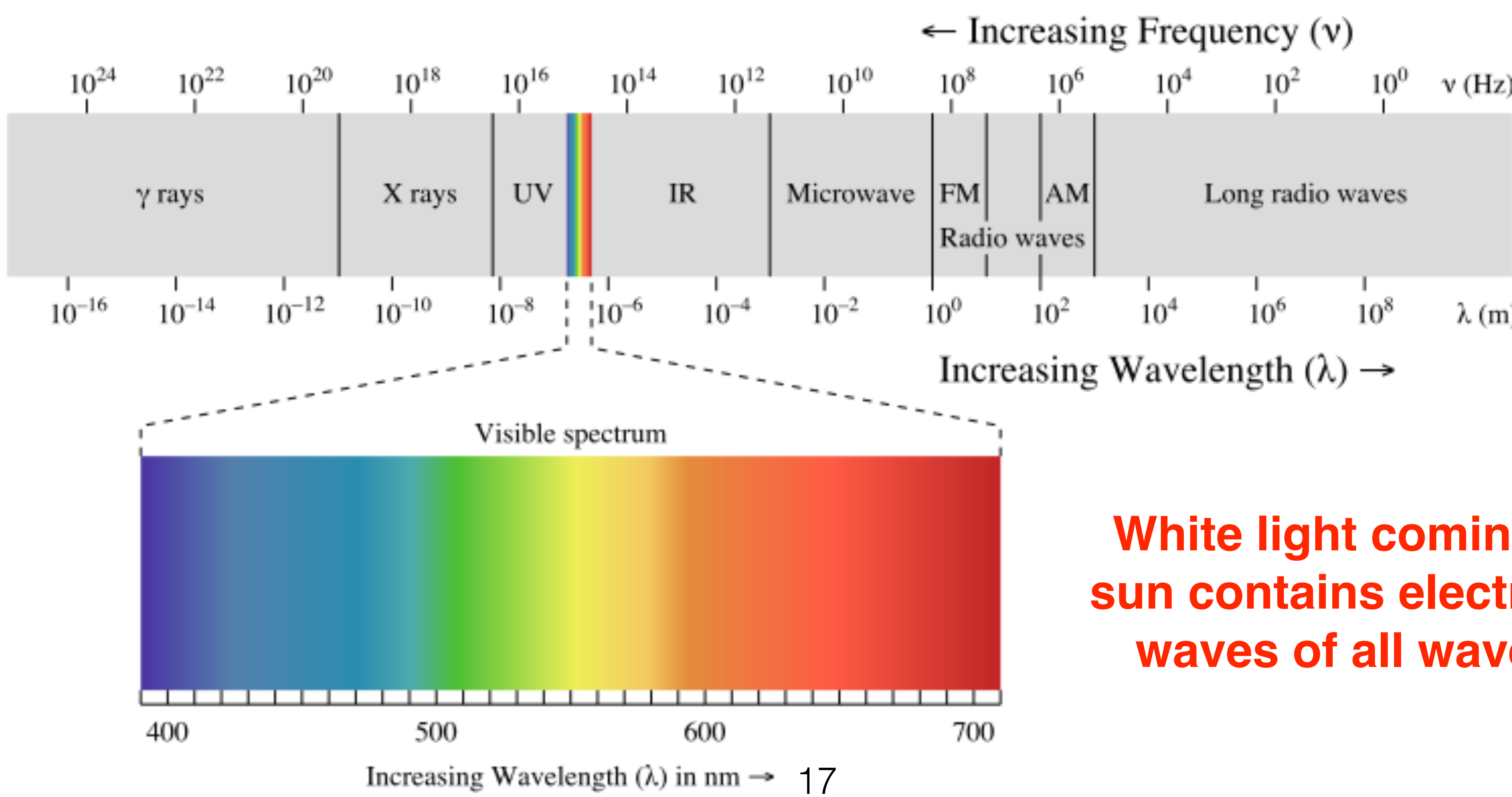
$$c^2 \vec{B}_0 = \vec{c} \times \vec{E}_0$$

speed of light

$$c_0 = \lambda \nu = 3 \times 10^8 \mathrm{m/s}$$

wavelength λ

frequency ν



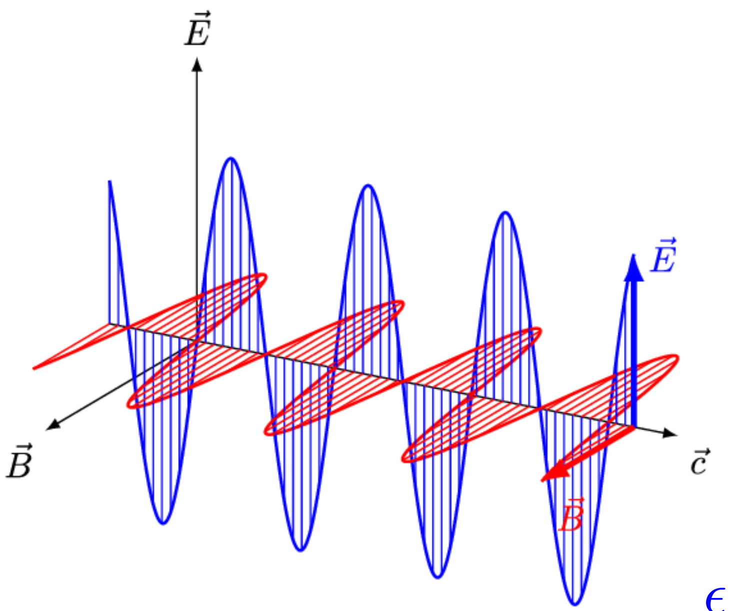
White light coming from the sun contains electromagnetic waves of all wavelengths!

Wave equation

$$\frac{\partial^2 u}{\partial t^2} = c^2 \nabla^2 u$$

Solutions are traveling waves with velocity c.

electromagnetic waves

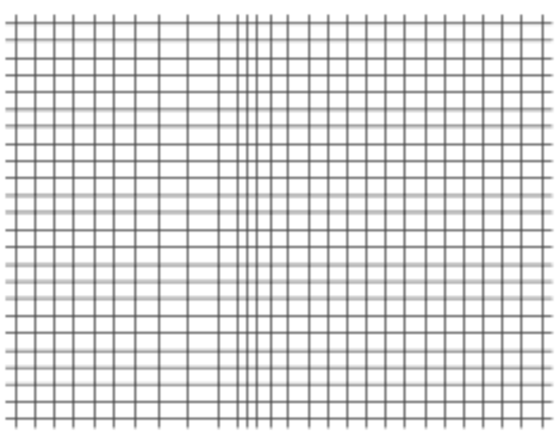


$$c = \frac{1}{\sqrt{\epsilon\mu}}$$

ϵ **permittivity**

μ **permeability**

sound waves

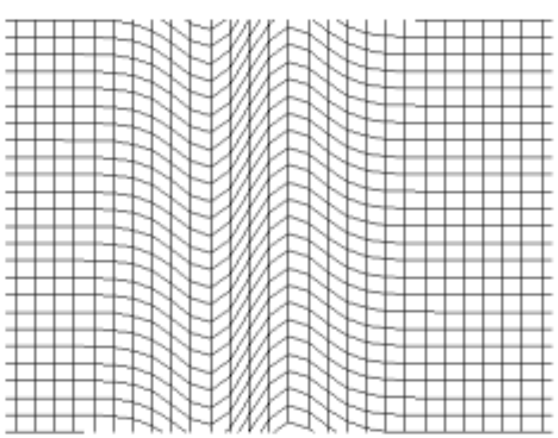


$$c = \sqrt{\frac{K}{\rho}}$$

K **bulk modulus**

ρ **mass density**

shear waves



$$c = \sqrt{\frac{\mu}{\rho}}$$

μ **shear modulus**

ρ **mass density**

waves in ropes under tension

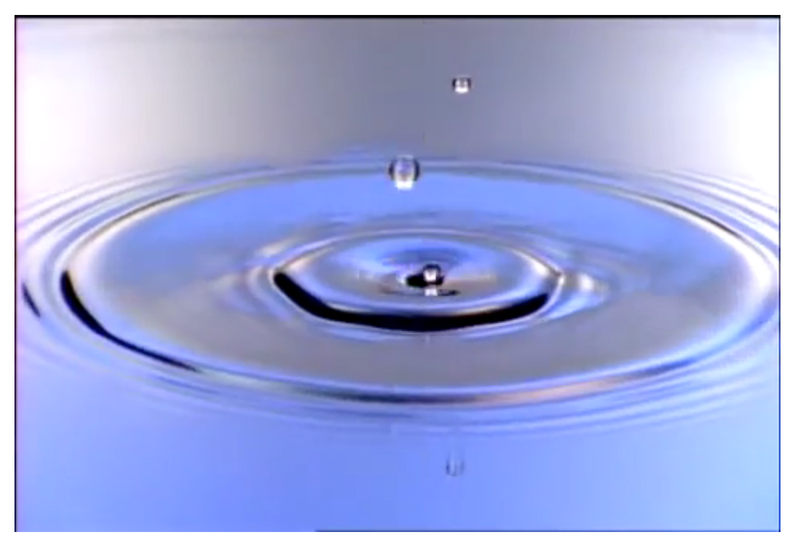
$$c = \sqrt{\frac{F}{\rho A}}$$

F **tensile force**

ρ **mass density**

A **cross-section area**

waves on liquid surfaces



shallow water

$$c = \sqrt{gh}$$

deep water

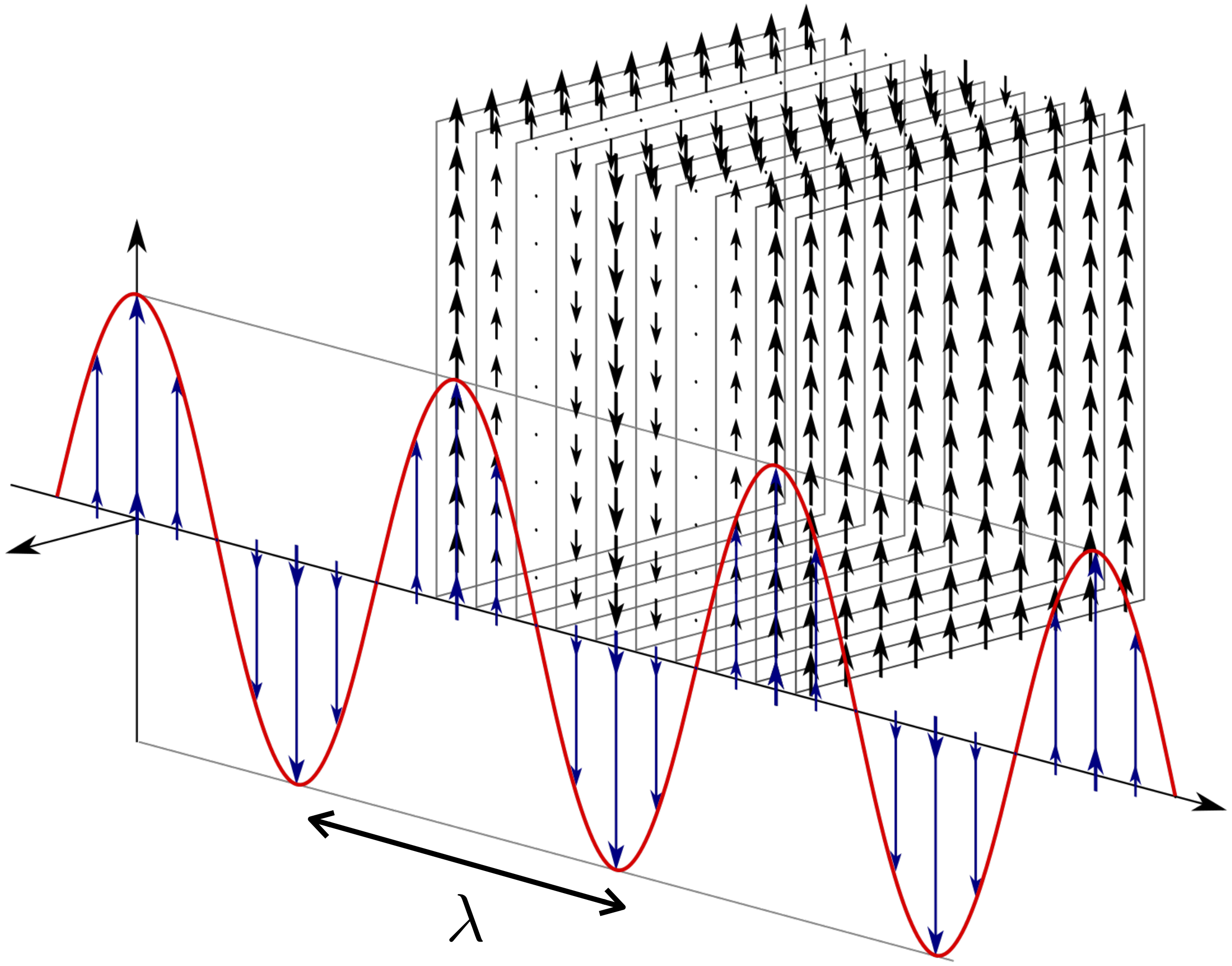
$$c = \sqrt{\frac{g\lambda}{2\pi}}$$

g **gravitational const.**

h **water depth**

λ **wavelength**

Plane waves



Planes of constant phases:

$$\vec{k} \cdot \vec{r} = \text{const}$$

Solutions of wave equation can be described as a linear superposition of plane waves:

$$u(x,t) = \sum_{\vec{k}} A_{\vec{k}} e^{i(\vec{k}\cdot\vec{r} - \omega t)}$$

$k = \frac{2\pi}{\lambda}$ **wavevector**

$\omega = 2\pi\nu$ **angular frequency**

Plane waves travel in direction of $\vec{k}$ with velocity:

$$c = \frac{\omega}{k} = \lambda\nu$$

Note: velocity of plane waves may depend on the wavevector $c(\vec{k})$!

Interference

constructive interference

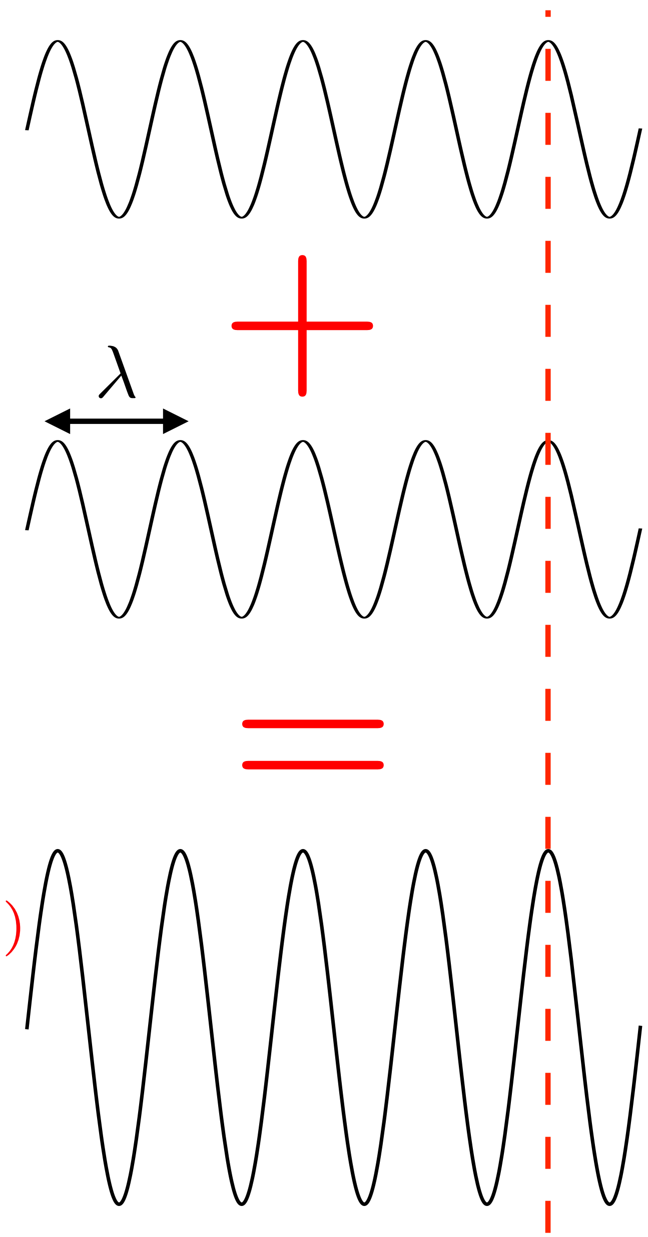
$|A|e^{i(\vec{k}\cdot\vec{r}-\omega t)}$

+

$|B|e^{i(\vec{k}\cdot\vec{r}-\omega t)}$

=

$(|A|+|B|)\,e^{i(\vec{k}\cdot\vec{r}-\omega t)}$



Constructive interference occurs when the two waves are in phase: waves offset by $m\lambda$,

$$m = 0, \pm 1, \pm 2, \ldots$$

$$e^{ikm\lambda} = e^{i2\pi m} = +1$$

destructive interference

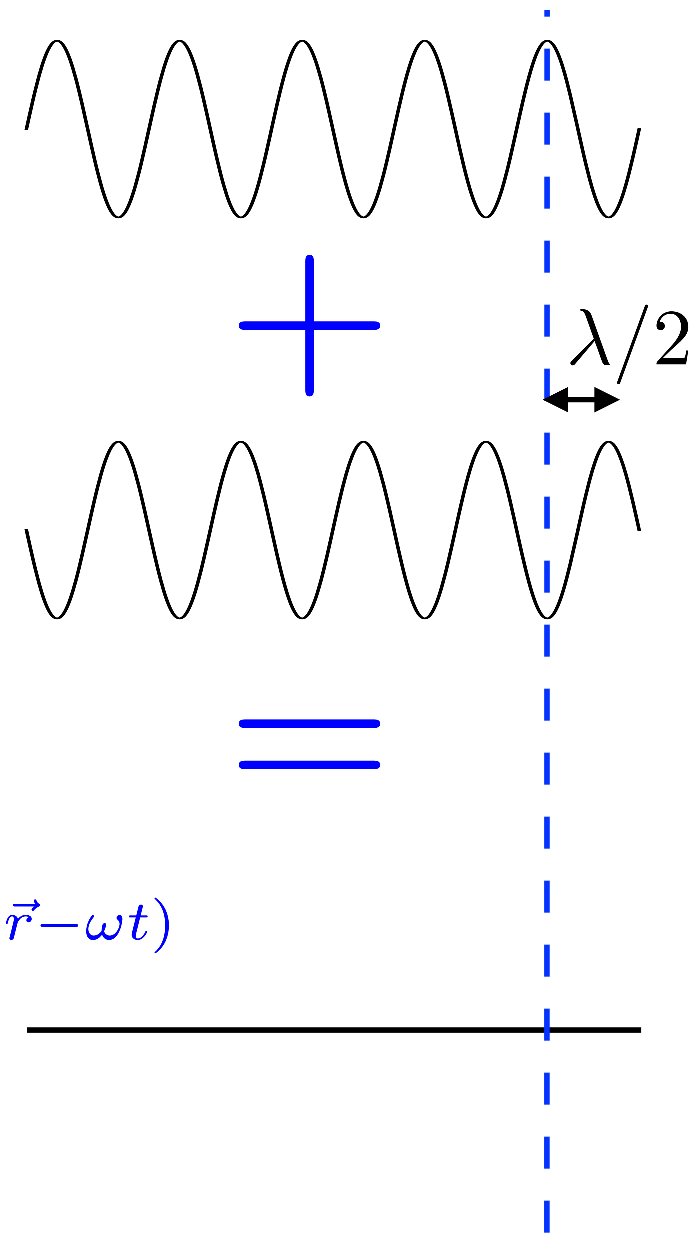
$|A|e^{i(\vec{k}\cdot\vec{r}-\omega t)}$

+

$|B|e^{i(\vec{k}\cdot\vec{r}-\omega t+\pi)}$

=

$(|A|-|B|)\,e^{i(\vec{k}\cdot\vec{r}-\omega t)}$

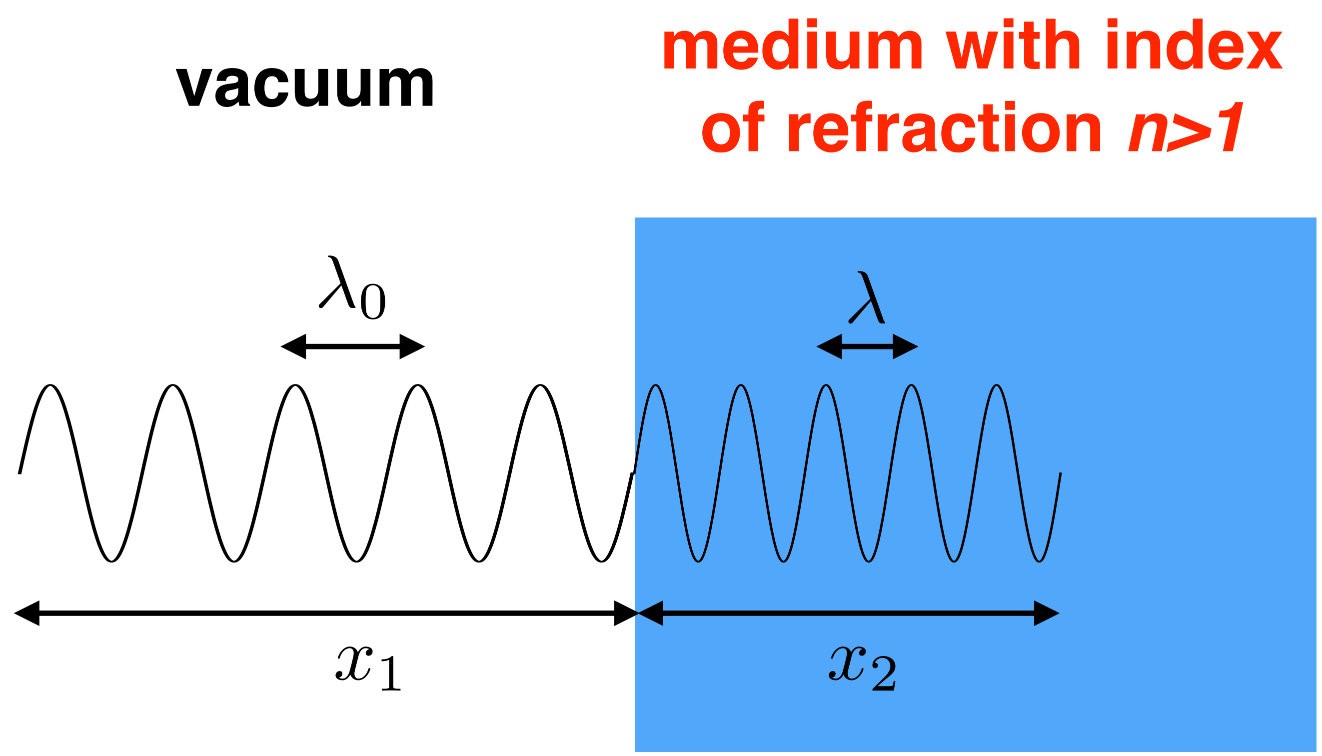


Destructive interference occurs when the two waves are out of phase: waves offset by $(m+1/2)\lambda$,

$$m = 0, \pm 1, \pm 2, \ldots$$

$$e^{ik(m+1/2)\lambda} = e^{i(2\pi m+\pi)} = -1$$

Propagation of light in medium



	vacuum	medium with index of refraction $n>1$
speed of light	$c_0 = 3 \times 10^8 \mathrm{m/s}$	$c = c_0/n$
frequency	ν_0	$\nu = \nu_0$
wavelength	λ_0	$\lambda = \lambda_0/n$
	$c_0 = \nu_0 \lambda_0$	$c = \nu\lambda$

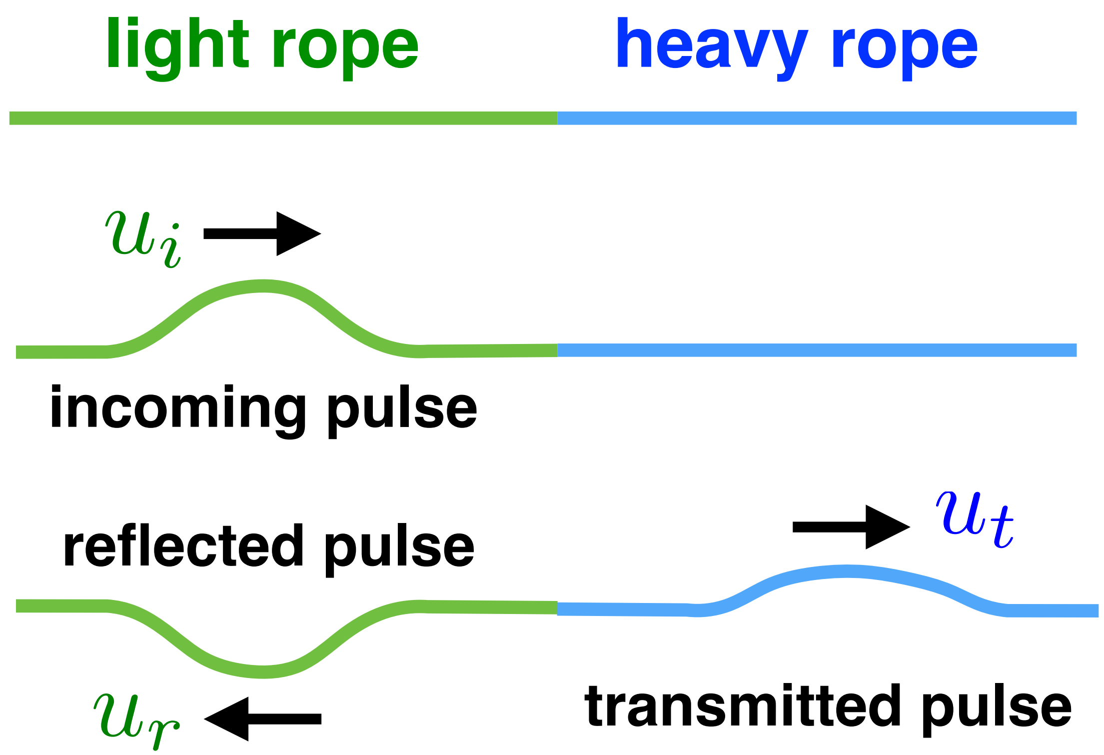
---

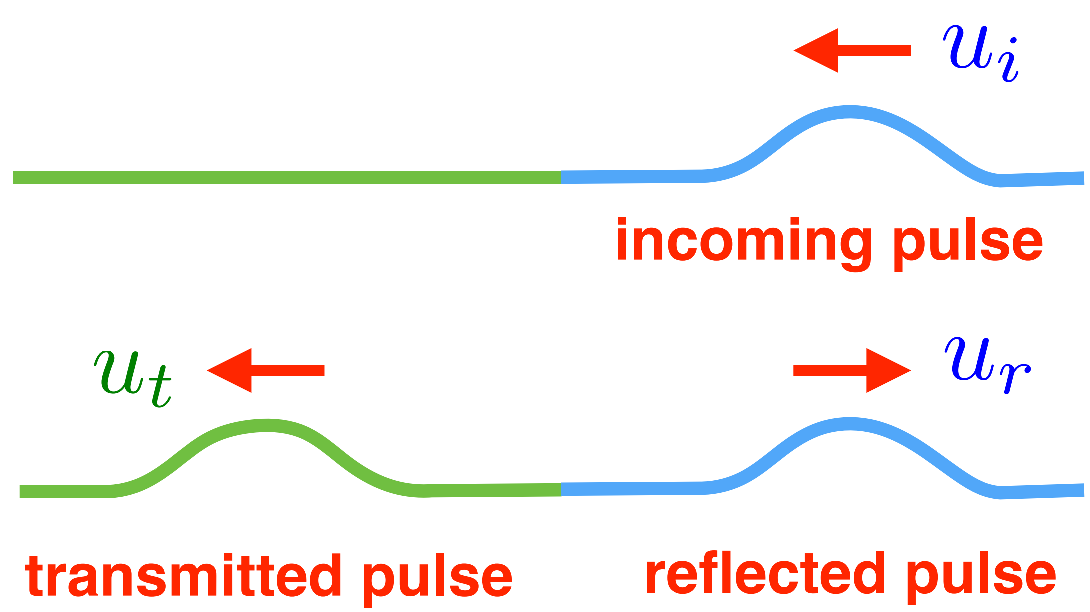
total number of cycles

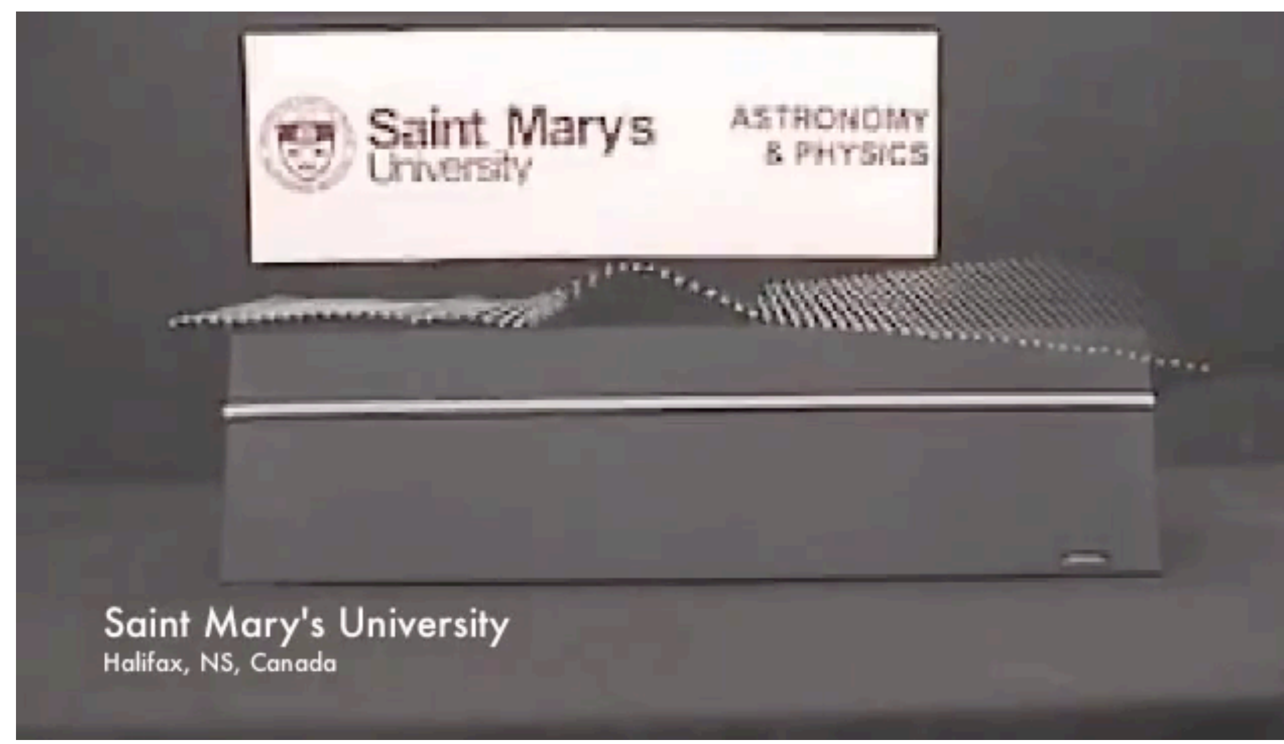
$$\frac{x_1}{\lambda_0} + \frac{x_2}{\lambda} = \frac{x_1 + n x_2}{\lambda_0}$$

Optical path length is geometric distance multiplied by the index of refraction!

Reflection of waves







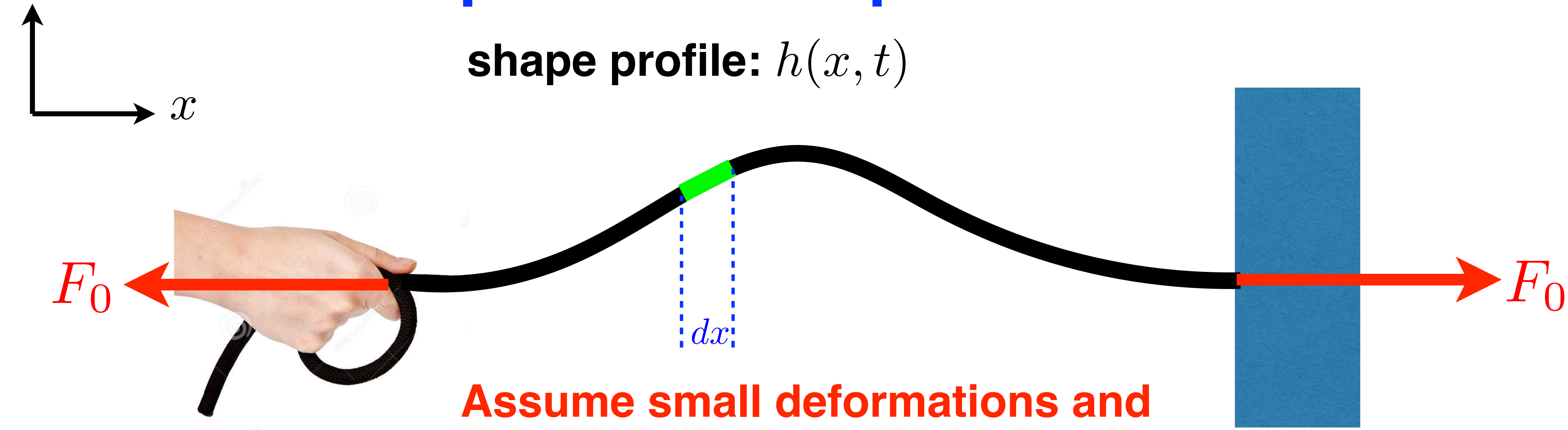
amplitude of reflected pulse

$$\frac{u_r}{u_i} = \frac{c_2 - c_1}{c_1 + c_2}$$

amplitude of transmitted pulse

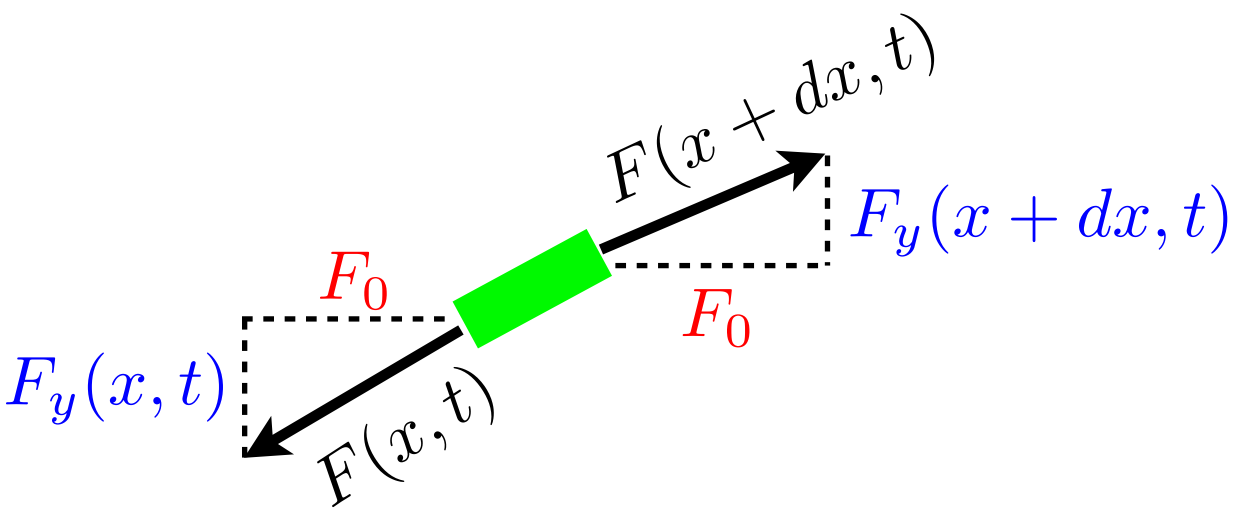
$$\frac{u_t}{u_i} = \frac{2c_2}{c_1 + c_2}$$

Wave equation for rope under tension



Assume small deformations and ignore movement in *x* direction!

Forces acting on a small rope element:



Forces act only in direction of the rope:

$$\frac{F_y(x,t)}{F_0} = \frac{\partial h(x,t)}{\partial x}$$

Second Newton's law for a small rope element:

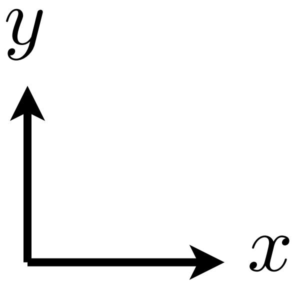
$$\rho A dx \frac{\partial^2 h}{\partial t^2} = F_y(x+dx,t) - F_y(x,t)$$

$$\rho A \frac{\partial^2 h}{\partial t^2} = \frac{\partial F_y}{\partial x} = F_0 \frac{\partial^2 h}{\partial x^2}$$

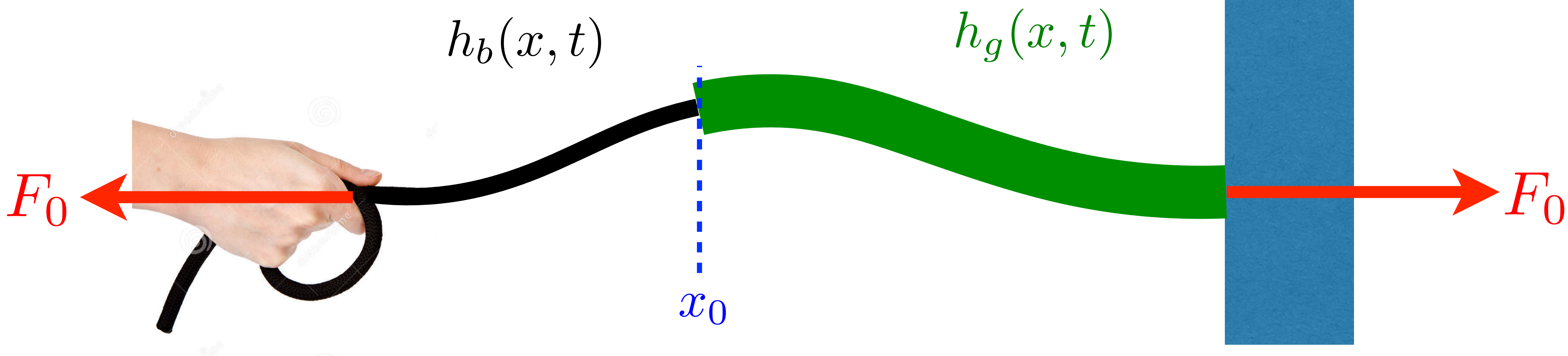
Wave equation:

$$\frac{\partial^2 h}{\partial t^2} = \frac{F_0}{\rho A}\frac{\partial^2 h}{\partial x^2} \equiv c^2 \frac{\partial^2 h}{\partial x^2}$$

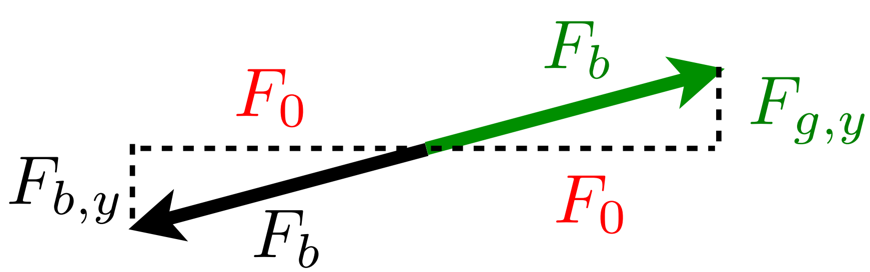
Boundary conditions between connected ropes



shape profiles:



Forces acting on the massless point, where ropes are connected:



Newton's law for this massless point:

$$F_{g,y} - F_{b,y} = ma = 0$$

Continuity: ropes are connected

$$h_b(x_0,t) = h_g(x_0,t)$$

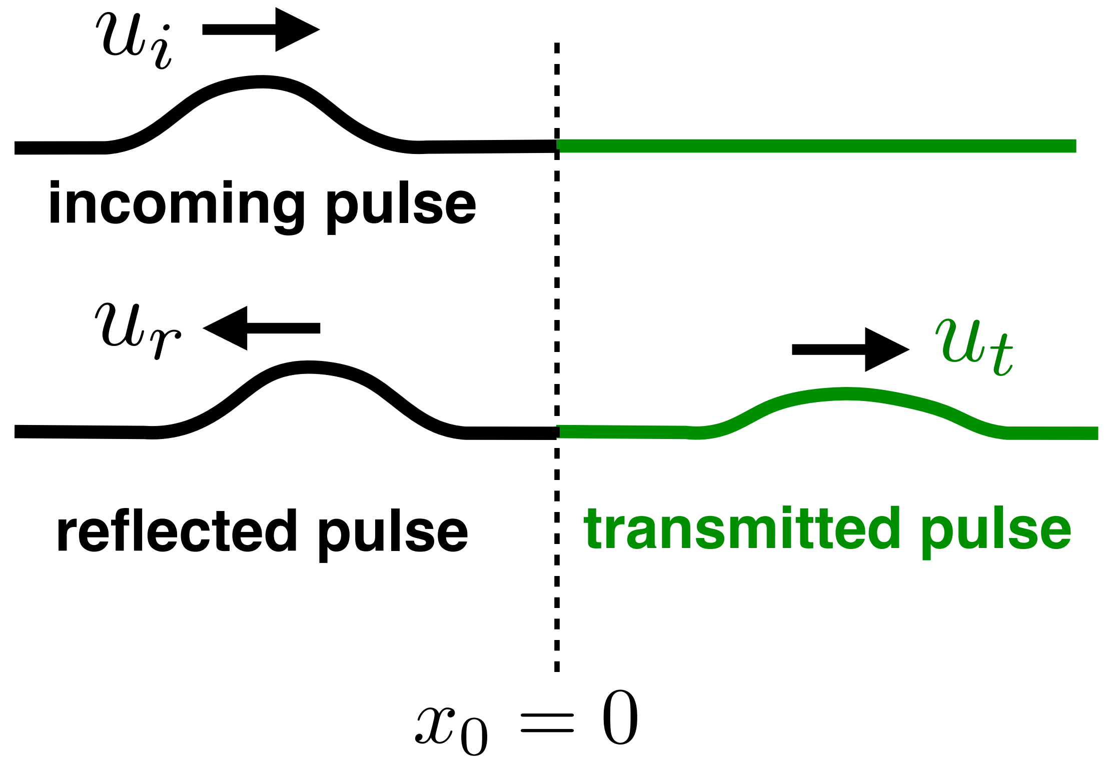
Force balance:

$$\frac{\partial h_b}{\partial x}(x_0,t) = \frac{\partial h_g}{\partial x}(x_0,t)$$

Reflection of waves on ropes

wave speed in black rope

$$c_1 = \frac{\omega}{k_1}$$



wave speed in green rope

$$c_2 = \frac{\omega}{k_2}$$

Solutions of wave equations can be expanded in Fourier series:

incoming pulse — reflected pulse

$$u_b(x,t) = \sum_\omega \left(A_\omega e^{i(k_1 x - \omega t)} + B_\omega e^{i(-k_1 x - \omega t)} \right)$$

transmitted pulse

$$u_g(x,t) = \sum_\omega \left(C_\omega e^{i(k_2 x - \omega t)} \right)$$

boundary conditions:

$$u_b(0,t) = u_g(0,t)$$

$$\frac{\partial u_b}{\partial x}(0,t) = \frac{\partial u_g}{\partial x}(0,t)$$

→

$$A_\omega + B_\omega = C_\omega$$

$$ik_1(A_\omega - B_\omega) = ik_2 C_\omega$$

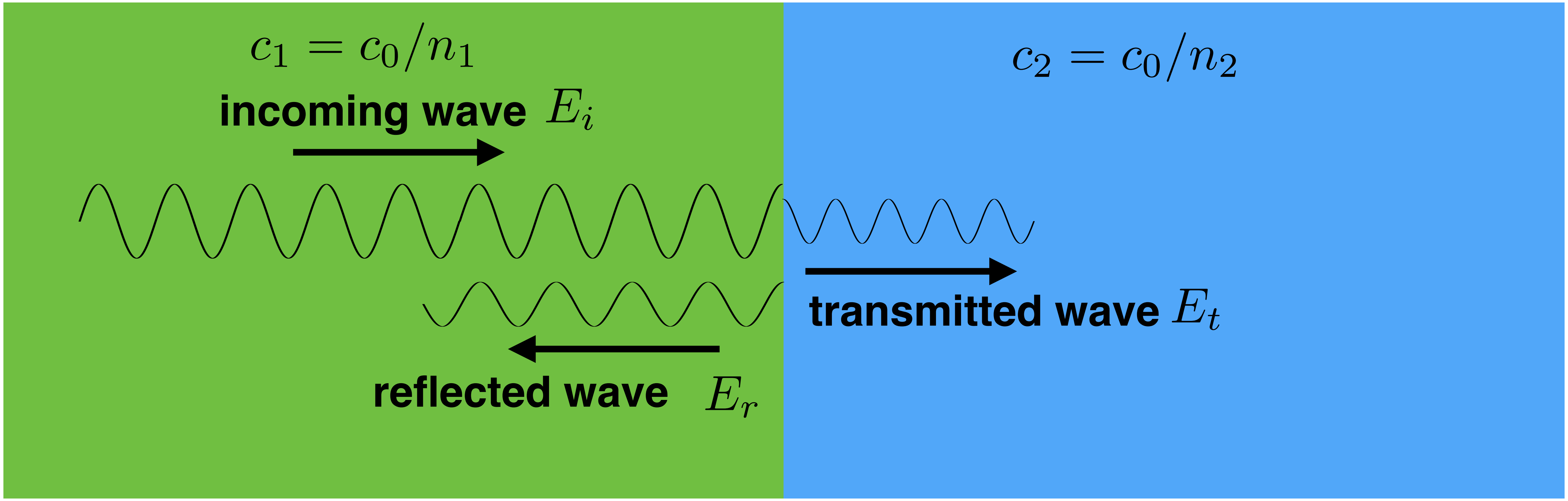
→

amplitudes of reflected and transmitted waves:

$$B_\omega = A_\omega \frac{(c_2 - c_1)}{(c_1 + c_2)}$$

$$C_\omega = A_\omega \frac{2c_2}{(c_1 + c_2)}$$

Reflection of light at the interface between two media



boundary conditions for incident waves normal to the interface:

$$E_1 = E_2 \qquad H_1 = H_2 \rightarrow \frac{\partial E_1}{\partial x} = \frac{\partial E_2}{\partial x}$$

amplitude of reflected electric field

$$r \equiv \frac{E_r}{E_i} = \frac{n_1 - n_2}{n_1 + n_2}$$

amplitude of transmitted electric field

$$t \equiv \frac{E_t}{E_i} = \frac{2n_1}{n_1 + n_2}$$

energy density of electromagnetic waves

$$\propto n|E|^2$$

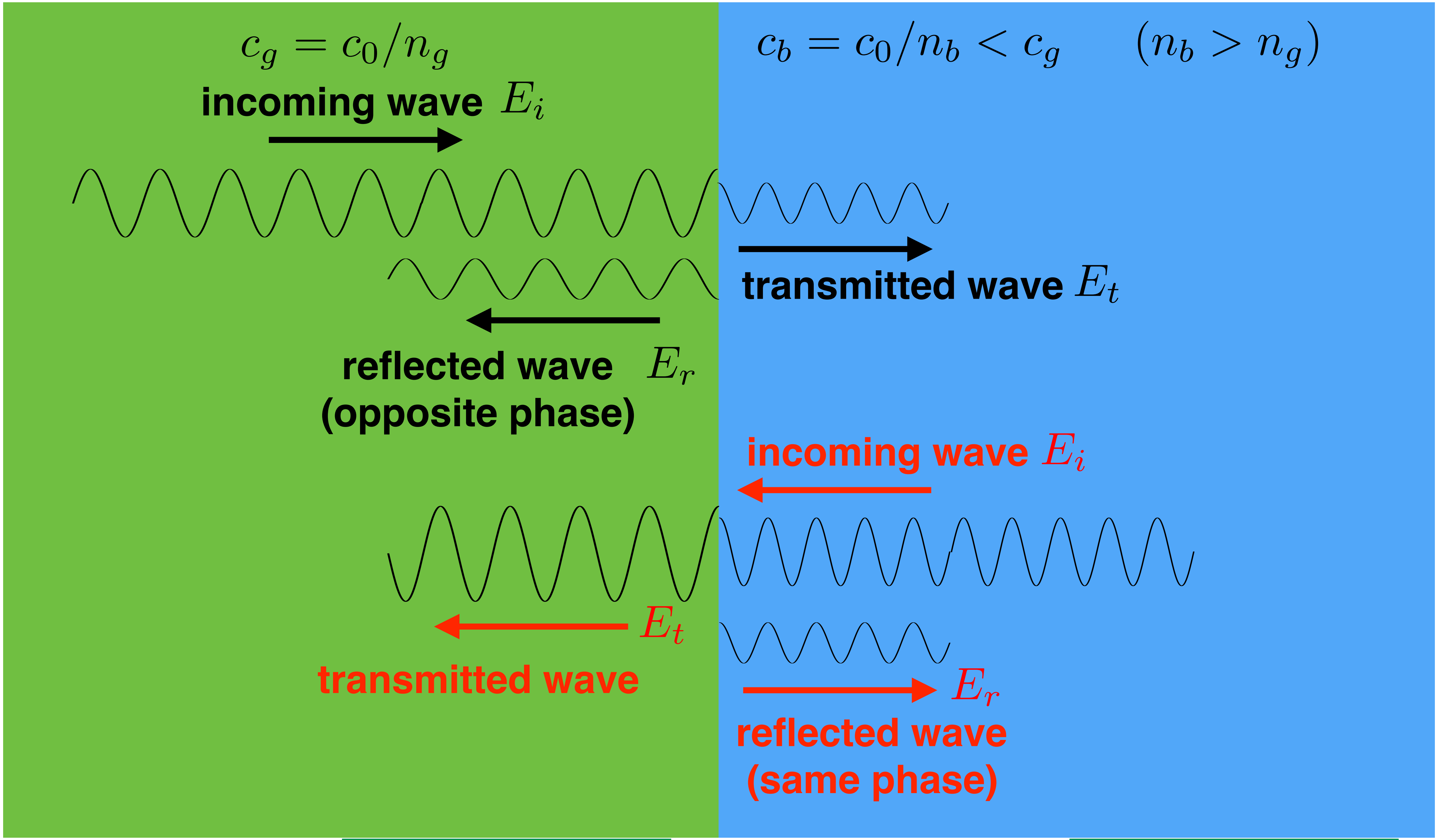
reflectance

$$R \equiv \frac{n_1|E_r|^2}{n_1|E_i|^2} = |r|^2$$

transmittance

$$T \equiv \frac{n_2|E_t|^2}{n_1|E_i|^2} = |t|^2\frac{n_2}{n_1} = 1 - R$$

Reflection of light at the interface between two media



amplitude of reflected electric field

$$\frac{E_r}{E_i} = \frac{n_1 - n_2}{n_1 + n_2}$$

amplitude of transmitted electric field

$$\frac{E_t}{E_i} = \frac{2n_1}{n_1 + n_2}$$