0RF 467

Transportation Systems Analysis

Fall2007/8

Class #10

Part 2: Planning and Analysis Tools of Transportation Demand and Investment

Traffic Assignment:

- Constant link costs (linear networks):
  - "All-or-nothing" assignments
  - Multiple path assignments:
    - K-shortest paths
    - "Essentially-equal" shortest path
- Volume-dependent link costs
  - $\$_Q = \$_0 \{1 + a (Q / Q_{max})^b\}$

    where

    $\$_Q$ = link cost at traffic flow q

    $\$_0$ = "zero flow" link cost

    Q = traffic flow (veh/hr.)

    Q_{max} = practical capacity

    a , b are parameters

  - System-optimum assignments
  - User-optimum assignments
    - Wardropt's first principle : There exists no path that has a lower cost.

Analytical Representations of Spatial Elements:

- Points (Nodes)
  - Location in "n"- dimensional space
  - Often 2-D; ex. Latitude, Longitude in some defined coordinate system such as WGS-84 ellipsoid Overview Detailed equations Transformations to other datum National Geodetic Survey (2cm accuracy objective)
  - Other attributes of nodes: name, type, number
- Lines (Arcs (Links))
  - Defined by two (2) end points ("A"-node, "B"-node)
  - Attributes:
    - Distance: UTM Coord System (using a "ruler")
    - Name, speed limit, travel time, travel time distribution, …
- Polygons (Surface areas (often planar))
  - Defined by ordered closed sequence of directed arcs ("Closed Directed Walk")
  - Attributes: type, name, value,…
  - GeoTIFF (geographic referencing of TIFF file types)
- Volumes (Closed ensemble of surface areas)
  -

Networks:

Definitions and notation:

Directed graphs and Networks: A *directed graph,* G = (N, A) consists of a set of N Nodes and a set of A arcs whose elements are ordered pairs of distinct nodes. A *directed network* is a directed graph whose nodes and/or arcs have associated numerical values.

Undirected graphs and networks: A directed graph with arcs having unordered pairs of distinct nodes.
Tails and Heads: A directed arc (i,j) has two end points i and j. We refer to i ("A" node) as the tail node and j ("B" node) as the head node. Arc (i,j) *emenates* from i and *terminates* at j. Arc (i,j) is incident to nodes i and j. It is *outgoing* from i and *incoming* to j. Whenever an arc (i,j) $\in$ A, then node j is adjacent to i.
Degrees: The *indegree* of a node is the number on incoming arcs to that node and the *outdegree* is the number of its outgoing arcs. The *degree* of a node is the sum of the in- and out- degrees.
Adjacency list: The *arc adjacency list*, A(i) of a node i is the set of arcs emenating from that node. A(i) = { (i,j) $\in$ A : j$\in$ N }. The *node adjacency list*, N(i) is the set of nodes adjacent to that node; N(i) = { j$\in$ N: (i,j) $\in$ A }
Multiarcrs and Loops: *Multiarcs* are two or more arcs having the same tail and head nodes. A *loop* is an arc whose tail node is the same as its head node.
Subgraph: A graph G' = (N', A') is a *subgraph* of G = (N,A) if N' $\subseteq$ N and A' $\subseteq$ A. A graph G' = (N', A') is a *spanning subgraph* of G = (N,A) if N' = N and A' $\subseteq$ A.
Walk: A *walk* in a directed graph is a subgraph of G containing a connected sequence of nodes (without any mention of arcs) or a connected sequence of arcs (without any mention of nodes).
Directed Walk: A *directed* walk is an "oriented" version of a walk in that any two consecutive nodes on the walk must be a member of the set of arcs (i_k, i_{k+1}) $\in$ A .
Path: A path is a walk without any repetition of nodes. We can partition the arcs of a path into two groups : forward arcs and backward arcs. An arc (i,j) in the path is a *forward arc* if the path visits node i prior to visiting node j , and is a *backward arc* otherwise.
Directed Path: A *directed path* is a directed walk without any repetition of nodes. We can store a path easily by defining a *predecessor index*, *pred (j)* for every node j in the path.
Cycles: A *cycle* is a path $i_1 - i_2 - i_3 - \dots i_r - i_1$. Cycles can be directed.
Acyclic Graph: A graph is *acyclic* if it contains no cycles.
Connectivity: We will say that two nodes, i and j , are *connected* if the graph contains at least one path from node i to node j. A graph is *connected* if every pair of nodes is connected; otherwise it is *disconnected*.
Strong connectivity: A connected graph is strongly connected if it contains at least one *directed* path from every node to every other node.
Cut: A cut is a partition of the node set N into two parts, S and $\underline{S}$ = N – S. Each cut defines a set of arcs consisting of those arcs that have one endpoint in S and the other in $\underline{S}$.
s – t Cut: This cut has node s $\in$ S ant t $\in$ $\underline{S}$.
Tree: A *tree* is a connected graph that contains no cycles.

A tree of n nodes contains exactly n – 1 arcs.
A tree has at least two *leaf* nodes (nodes with degree 1)
Every two nodes of a tree are connected by a unique path.

Forest: A graph that contains no cycles is a *forest*. Alternatively, a forest is a collection of trees.
Subtree: A connected subgraph of a tree is a *subtree*.
Rooted trees: A trooted tree is a tree with a specially designated node called a *root*. Rooted trees are viewed as hanging from the root.
Directed-out-tree: every path from node s is a directed path