Orf 467

Transportation Systems Analysis

Fall 2017/18

Class #6

Part 2: Planning and Analysis Tools of Transportation Demand and Investment

Trip Distribution

Gravity Model:

The gravity Model states that the trips ,(P_i), produced in zone i will be distributed to each other zone j , (T_{ij}), according to the relative intensity of activity of each zone j ,($A_j / \Sigma_{j=1,n} A_j$), in combination with the relative accessibility of each zone j, ($F(t)_{ij}) / \Sigma_{j=1,n} F(t)_{ij}$), to assemble the relative attractiveness of each zone j, : $F(t)_{ij}) K_{ij} A_j$ / $\Sigma_{j=1,n} F(t)_{ij} K_{ij} A_j$ This means that:
Trips between i and j = trips produced at i x {attractiveness characteristics of j} / {attractiveness characteristics of all zones in the area}

$$T_{ij} = \{ P_i\ F(t)_{ij})\ K_{ij}\ A_j \} / \Sigma_{j=1,n} F(t)_{ij}\ K_{ij}\ A_j$$

$$= \{ P_i / \Sigma_{j=1,n} F(t)_{ij}\ K_{ij}\ A_j \}\ F(t)_{ij}\ K_{ij}\ A_j$$

In vector notation: $T = Q\ G$
With: $G = F\ K$; $S = GA$; $R = P / S$; $Q = R\ A^T$
dimensions of P, A, R, S are (nx1), and dimensions of T, G, Q, F, K are (nxn)

Where K_{ij} is a socio-economic adjustment factor (we will take it to be the Identity matrix) and $F(t)_{ij}$ is a disutility (or sometimes called, impedance) measure between i and j which can be different at different times , t. For simplicity, we will take , $F(t)_{ij} = 1/ D^2_{ij}$ where D_{ij} is the distance between i and j. You can take D_{ij} to be the Euclidian distance or the Manhattan distance or other similar measures. You could also multiply these distance times 1.2, a very good estimate of the circuity (the ratio of actual “over-the-road” distance to Euclidean distance between any two points). For intra-zonal computations (i=j), sqrt(zone area) is a good simple distance measure to use. (It is a little longer than what might be the average access distance which can account for some amount of “fixed” disutility associated with any trip) .

If we sum T_{ij} across the rows, then: $\Sigma_{j=1,n} T_{ij} = P_i$ which is good. The sum of the distributed trips cross any row equals the trips produced; however, the same is not guaranteed for the columns. The sum down columns: $\Sigma_{i=1,n} T_{ij}$ <-> A_j is not necessarily equal to the trips attracted to a zone.(<-> reads: “not equal”) Use an *adjustment* factor $\boldsymbol{A}_{jk}$

$$\boldsymbol{A}_{jk} = A_j\ \boldsymbol{A}_{j,(k-1)} / C_{j,(k-1)}$$

Where:

- $\boldsymbol{A}_{jk}$ = adjusted attraction factor for attraction zone (col) j , iteration k
- $\boldsymbol{A}_{j,0} = A_j$ (note: k = 0)
- $C_{j,k}$ = actual computed attraction total for zone j, iteration k
- A_j = desired attractions for zone j
- k = iteration number

iterate until $C_{j,k} = A_j$

Simple 4-zone example

Matrices

Excel can also be used to multiply or invert matrices. For example, suppose you have a 3x3 matirx whose entries are contained in the 9 cells in columns E,F,G and rows 5,6,7. To find the inverse of this matrix, proceed as follows:

Use the mouse to select a 3x3 block of cells into which you want to put inverse matrix. For definiteness, let us suppose that you select the cells in columns J,K,L and rows 9,10 and 11. While the selection is still active (so that all but the top-left cell J9 is dark), write the formula "=MINVERSE(E5:G7)" but **don't press the Enter key**. Instead, press Control-Shift-Enter simultaneously. (This ensures that the formula applies to the entire block of cells from J9 to L11, rather than just the top-left cell J9.) The inverse matrix should then appear in the required block of cells.

Multiplying matrices is similar. For example, suppose you want to multiply the 3x3 matrix X stored in the block of cells E5:G7 by a 3x3 matrix Y stored in the block of cells A5:C7, and put the product XY in the block of cells F1:H3. Use the mouse to select the block F1:H3, and while the selection is still active type the formula "=MMULT(E5:G7, A5:C7)" and then, instead of pressing the Enter key, press Control-Shift-Enter simultaneously.

Now test your skills by multiplying the matrix in the block E5:G7 by its inverse, which is in the block J9:L11. Do you get the identity matrix? If not, why not?

Calibrating Gravity Model:

Take the friction factors to be of the form: $F_{ij} = 1 / W_{ij}^{c_{ij}}$
If we then take the log of booth sides:
$\text{Ln } F_{ij} = -c_{ij} \ln W_{ij}$. This is a linear relationship that we then can use historical data in a regression analysis.

K_{ij} are correction factors that account for special conditions that would substantially change perceived disutility values between zone pairs that aren't properly quantified by the friction factors F_{ij}. For MyCity, you may assume that K_{ij} is an Identity array.

Lincoln NE Trip Distribution used a Gamma Function instead of an inverse power model.
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