Lecture 12

Tuesday, October 25, 2011
1:30 PM

Midterm on Thursday

DTFS:

$x[n]$ has period $T = \frac{1}{f}$

$$a_k = \frac{1}{T} \sum_{\langle T \rangle} x[n] e^{-i2\pi k f n}$$

$$x[n] = \sum_{\langle T \rangle} a_k e^{i2\pi k f n}$$

Possible Topics:

Convolution

Differences between DT and CT

- FS (DT CT)
- FT (DT CT) — DTFT CTFT
- DFT

↳ x has finite duration.

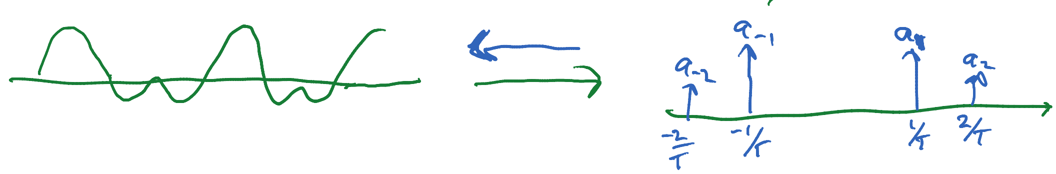
DFT(x) is the sampled Fourier transform

Alternatively

↳ Take FS of periodic extension of x.

FS is periodic. Return only one period.

Almost exactly the same as DFT. ($\frac{1}{N}$ in diff. spot).



With the use of delta functions,
everything is a special case of CTFT

$x(t)$	$X(f)$	
Periodic	Discrete (only δ functions)	FS
Discrete	Periodic	DTFT
Disc. + Periodic	Disc. + Per.	DTFS (DFT)

Properties:

DT:

Time-scaling

Multiplication: Convolution in freq. only over one period

$$x[n] \cdot y[n] \longrightarrow \int X(\tau) Y(f-\tau) \, d\tau$$

$$\frac{1}{2\pi} \int X(e^{j\tau}) Y(e^{j(\omega-\tau)}) \, d\tau$$

$$\frac{1}{2\pi}\int_{2\pi} X(e^{j\tau})\,Y(e^{j(w-\tau)})\,d\tau$$

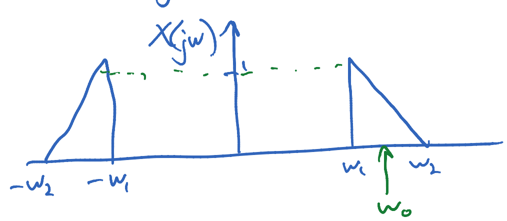
Properties of Systems:

- Linear
- Time-invariant
- Memoryless
- BIBO Stable $\qquad \int_{-\infty}^{\infty} |h(\tau)|\,d\tau < \infty$
- Causal
- Invertible

Practice Problem: 7.27:

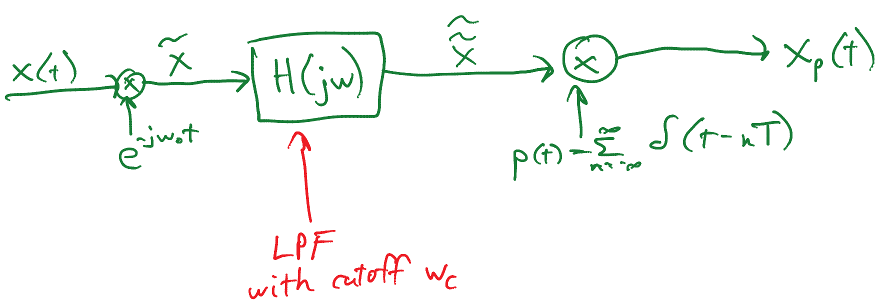
Suppose $x(t)$ is real:

and $X(jw) = 0 \quad \forall |w| < w_1$ and $\forall |w| > w_2$

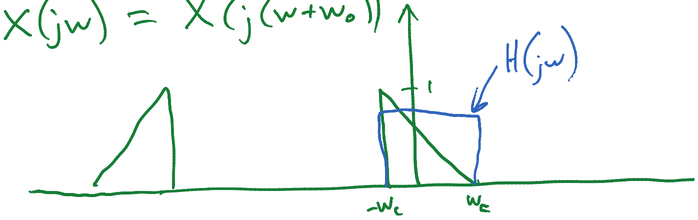


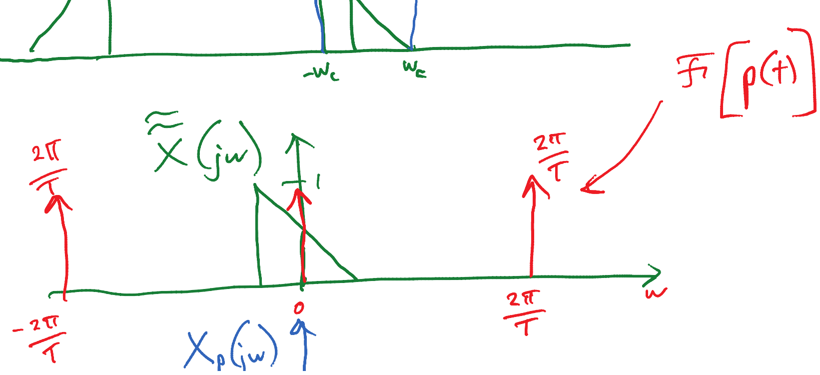
Define: $w_0 = \frac{1}{2}(w_1 + w_2)$

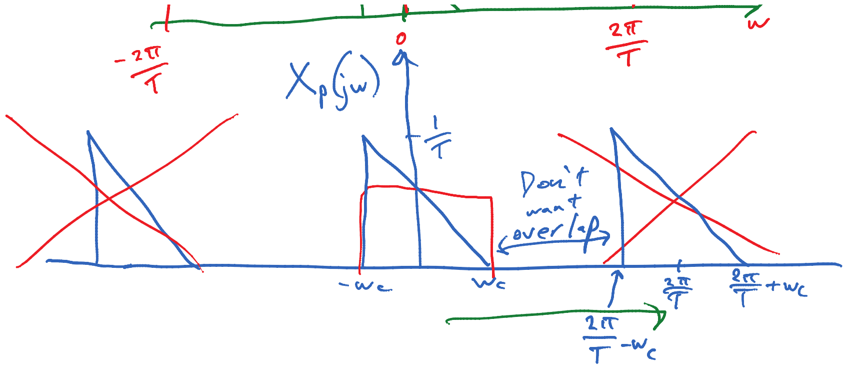
$w_c = \frac{1}{2}(w_2 - w_1)$



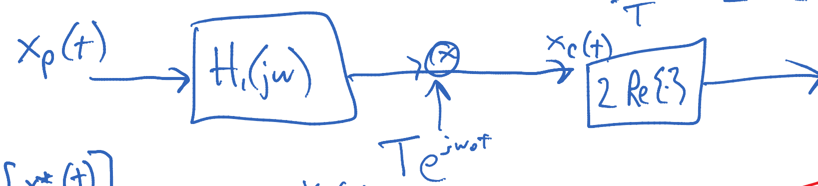
$\tilde{X}(jw) = X(j(w + w_0))$

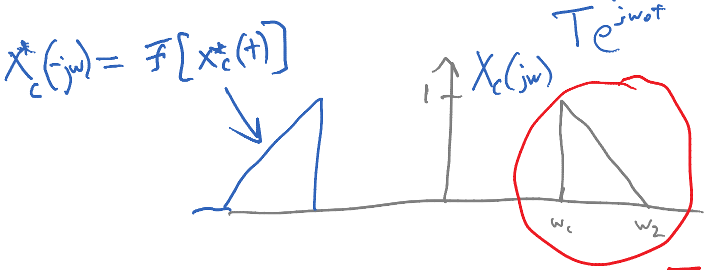






$$\frac{2\pi}{T} > 2w_c \Rightarrow T < \frac{\pi}{w_c}$$





$X_c(jw) + X_c^*(-jw) = X_c(jw)$ ~~doesn't~~ $\forall\, w > 0$.

If $x(t)$ is real $\Rightarrow$ $X(jw) = X^*(-jw)$

$\boxed{5.42:}$ Frequency shift Property is a special case of Mult. Prop.

$$\underline{x[n]\,e^{jw_0 n}} \longrightarrow X(e^{j(w-w_0)})$$

$e^{jw_0 n} = g[n]$

$$G(jw) = \mathcal{F}[e^{jw_0 n}] = 2\pi \sum_{l=-\infty}^{\infty} \delta(w - w_0 - 2\pi l)$$

$$\mathcal{F}[x[n]\,g[n]] = \frac{1}{2\pi}\int X(e^{j(\theta)})\,G(e^{j(w-\theta)})\,d\theta$$

$$= \int_0^{2\pi} X(e^{j\theta}) \sum_{l=-\infty}^{\infty} \delta(w - \theta - w_0 - 2\pi l)\,d\theta$$

Find l^* s.t. $w - w_0 - 2\pi l \in [0, 2\pi]$

$$= \int^{2\pi}$$

s.t. $w - w_0 - 2\pi l \in [0, 2\pi]$

$$= \int_0^{2\pi} X(e^{j\theta})\, \delta(w - \theta - w_0 - 2\pi l^*)\, d\theta$$

$$= X\left(e^{j(w - w_0 - 2\pi l^*)}\right)$$

$$= X\left(e^{j(w - w_0)}\right)$$