Rotary Motion Servo Plant: SRV02

Rotary Experiment #01: Modeling

SRV02 Modeling using QUARC

Instructor Manual

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1. Introduction

The objective of this experiment is to find a transfer function that describes the rotary motions of the SRV02 load shaft. The dynamic model is derived analytically from classical mechanic principles and using experimental methods.

The following topics are covered in this laboratory:

- From first-principles, derive the dynamical equation representing the position of the rotary servo plant.
- Obtain the transfer function that describes the load shaft position with respect to the motor input voltage from the dynamical equation previously found.
- Obtain SRV02 transfer function using a frequency response experiment.
- Compare the obtained transfer function with the actual response to validate the model.

> ***Regarding Gray Boxes***:
>
> Gray boxes present in the **instructor manual** are not intended for the students as they provide solutions to the pre-lab assignments and contain typical experimental results from the laboratory procedure.

2. Prerequisites

In order to successfully carry out this laboratory, the user should be familiar with the following:

- Data acquisition card (e.g. QPID), the power amplifier (e.g. VoltPAQ), and the main components of the SRV02 (e.g. actuator, sensors), as described in References [1], [4], and [5], respectively.
- Wiring and operating procedure of the SRV02 plant with the amplifier and DAQ device, as discussed in Reference [5].
- Transfer function fundamentals, e.g. obtaining a transfer function from a differential equation.
- Laboratory described in Reference [6] in order to be familiar using QUARC with the SRV02.

3. Overview of Files

Table 1 below lists and describes the various files supplied with the SRV02 Modeling laboratory.

File Name	*Description*
01 – SRV02 Modeling – Student Manual.pdf	This laboratory guide contains pre-lab and in-lab exercises demonstrating how to model the Quanser SRV02 rotary plant. The in-lab exercises are explained using the QUARC software.
setup_srv02_exp01_mdl.m	The main Matlab script that sets the SRV02 motor and sensor parameters. **Run this file only to setup the laboratory.**
config_srv02.m	Returns the configuration-based SRV02 model specifications *Rm*, *kt*, *km*, *Kg*, *eta_g*, *Beq*, *Jeq*, and *eta_m*, the sensor calibration constants K_POT, K_ENC, and K_TACH, and the amplifier limits VMAX_AMP and IMAX_AMP.
calc_conversion_constants.m	Returns various conversions factors.
q_srv02_mdl.mdl	Simulink file that implements the open-loop controller for the SRV02 system using QUARC.
01 – SRV02 Modeling – Instructor Manual.pdf	Same as the student version except the gray boxes are no longer shaded to reveal the solution to the pre-lab and in-lab exercises.
srv02_exp01_modeling.mws	Maple worksheet used to derive the transfer function model involved in the experiment. Waterloo Maple 9, or a later release, is required to open, modify, and execute this file.
srv02_exp01_modeling.html	HTML presentation of the Maple Worksheet. It allows users to view the content of the Maple file without having Maple 9 installed. No modifications to the equations can be performed when in this format.
d_model_param.m	Calculates the SRV02 model parameters *K* and *tau* based on the device specifications *Rm*, *kt*, *km*, *Kg*, *eta_g*, *Beq*, *Jeq*, and *eta_m*.
sample_freq_rsp.m	Generates bode plot and finds model parameters based on measured data.
sample_bumptest.m	Finds the model parameters given an input step voltage and the corresponding measured load speed response. Users can use the saved responses contained in the MAT files *sample_mdl_data_wl.mat* and *sample_mdl_data_vm.mat*, or perform the experiment again and use the response currently saved in the Matlab workspace..
sample_model_validation.m	Generates a Matlab figure that plots the response from the nominal, frequency response, and bumptest models using the MAT files described below.

File Name	Description
data_mdl_val_nom.mat	Sample step load speed response using nominal model.
data_mdl_val_freqrsp.mat	Sample step load speed response using frequency response model.
data_mdl_val_bumptest.mat	Sample step load speed response using bumptest model.
data_bumptest_wl.mat	Bumptest data file: sample load speed step response used by sample_bumptest.m script..
data_bumptest_Vm.mat	Bumptest data file: sample input voltage used by sample_bumptest.m script..

Table 1: Files supplied with the SRV02 Modeling experiment.

4. Pre-Lab Assignments

The angular rate of the SRV02 load shaft with respect to the input motor voltage can be described by the following first-order transfer function

$$\frac{\Omega_l(s)}{V_m(s)} = \frac{K}{\tau\, s + 1} \qquad [1]$$

where the $\Omega_l(s)$ is the Laplace transform of the load shaft rate $\omega_l(t)$, $V_m(s)$ is the Laplace transform of motor input voltage $v_m(t)$, K is the steady-state gain, τ is the time constant, and s is the Laplace operator.

The SRV02 transfer function model is derived analytically in Section 4.1 and its K and τ parameters are evaluated. These are known as the *nominal* model parameter values. The model parameters can also be found experimentally. Section 4.2 and 4.3 describe how to use the frequency response and bump-test methods to find K and τ. These methods are useful when the dynamics of a system are not known, for example in a more complex system. After the in-lab exercises, the experimental model parameters are compared with the nominal values.

4.1. Modeling from First-Principles

This section contains exercises that teaches the student to obtain a differential equation that describes the rotary motions of the load shaft with respect to the motor input voltage. The motor circuit equation is derived in Section 4.1.1 and the load shaft motion equation with respect to an applied motor torque is found in Section 4.1.2. These equations are combined in Section 4.1.3 and then transformed into a transfer function in Section 4.1.4. Thereafter, in Section 4.1.5 the model parameters are evaluated using the SRV02 motor and gear train specifications.

4.1.1. Electrical Equation

The DC motor armature circuit schematic and gear train is illustrated in Figure 1. Recall, as specified in Reference [5], that the R_m is the motor resistance, L_m is the inductance, and k_m is the back-emf constant.

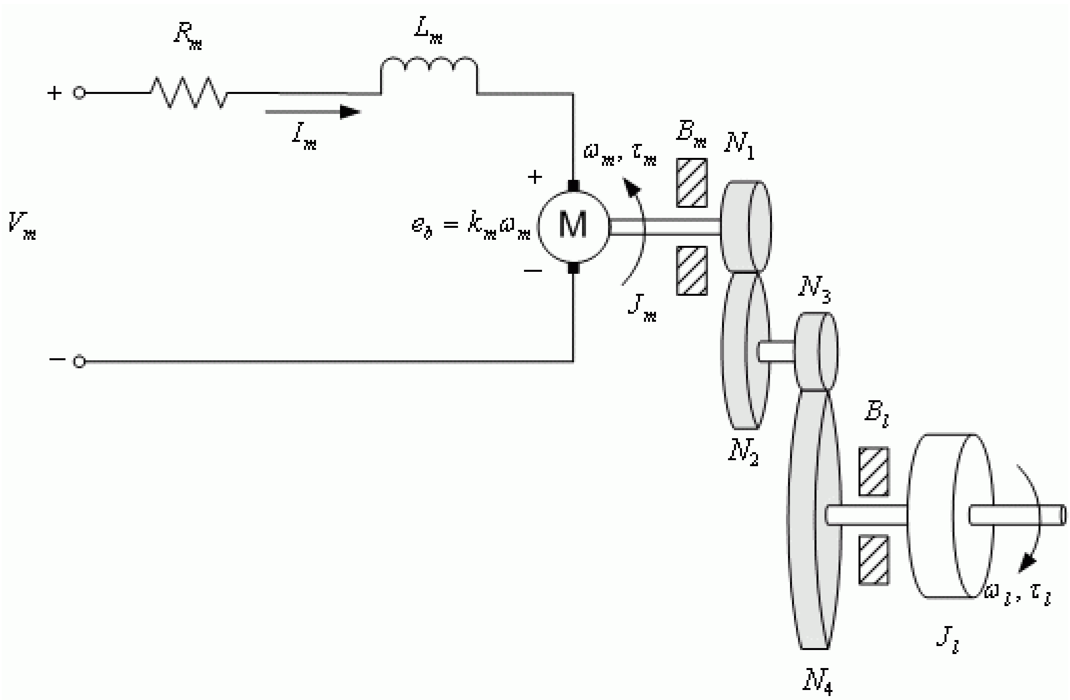

Figure 1: SRV02 DC motor armature circuit and gear train.

1. Using Kirchoff's Voltage Law, find the electrical equation that describes the amount of current that runs through the motor leads, I_m, when a motor voltage of V_m is applied and a back emf voltage of

$$e_b(t) = k_m \omega_m(t) \quad [2]$$

is present. The back emf (electromotive) voltage depends on the speed of the motor shaft ω_m and the back-emf constant of the motor, k_m. It opposes the current flow. The SRV02 motor has a very low inductance and since its much lower then the motor resistance, i.e. $L_m << R_m$, the circuit equation can be simplified by setting $L_m = 0$.

Solution:
The raw equation found using KVL is

$$V_m(t) - R_m I_m(t) - L_m \left(\frac{d}{dt} I_m(t) \right) - K_m \omega_m(t) = 0 \qquad [s1]$$

Since the motor inductance is much less than the resistance, the equation becomes

$$V_m(t) - R_m I_m(t) - K_m \omega_m(t) = 0 \qquad [s2]$$

Solving for $I_m(t)$, the motor current equation equals

$$I_m(t) = -\frac{-V_m(t) + K_m \omega_m(t)}{R_m} \qquad [s3]$$

0	1	2

4.1.2. Mechanical Equation

For a one-dimensional rotary system, Newton's Second Law of Motion can be written

$$J\alpha = \tau \qquad [3]$$

where J is the moment of inertia of the body (about its center of mass), α is the angular acceleration of the system, and τ is the sum of the torques being applied to the body. In this section the equation of motion describing the speed of the load shaft, ω_l, with respect to the applied motor torque, τ_m, is developed. As illustrated in Figure 1, the SRV02 gear train along with the viscous friction acting on the motor shaft, B_m, and the load shaft B_l are considered. The load equation of motion is

$$J_l \left(\frac{d}{dt} \omega_l(t) \right) + B_l \omega_l(t) = \tau_l(t) \qquad [4]$$

where J_l is the moment of inertia of the load and τ_l is the total torque applied on the load. The load inertia includes the inertia from the gear train and from any external loads attached, e.g. disc or bar. The motor shaft equation is expressed

$$J_m \left(\frac{d}{dt} \omega_m(t) \right) + B_m \omega_m(t) + \tau_{ml}(t) = \tau_m(t) \qquad [5]$$

where J_m is the motor shaft moment of inertia and τ_{ml} is the resulting torque acting on the motor shaft from the load torque. The torque at the load from an applied motor torque is described

$$\tau_l(t) = \eta_g K_g \tau_m(t) \qquad [6]$$

where K_g is the gear ratio and η_g is the gearbox efficiency. The planetary gearbox that is directly mounted on the SRV02 motor (see Reference [5] for more details) is represented by the N_1 and N_2 gears in Figure 1 and has a gear ratio of

$$K_{gi} = \frac{N_2}{N_1} \qquad [7]$$

This is the internal gear box ratio. The motor gear, N_3, and the load gear, N_4, are directly meshed together and visible from the outside. These gears comprise the external gear box and it has an associated gear ratio of

$$K_{ge} = \frac{N_4}{N_3} \quad [8]$$

The gear ratio of the SRV02 gear train is

$$K_g = K_{ge} K_{gi} \quad [9]$$

Thus the torque seen at the motor shaft through the gears is expressed

$$\tau_{ml}(t) = \frac{\tau_l(t)}{\eta_g K_g} \quad [10]$$

1. Find the relationship between the angular rate of the motor shaft, ω_m, and the angular rate of the load shaft, ω_l.

   Solution:
   The position relationship between the load and motor shafts is

   $$\theta_l(t) = \frac{\theta_m(t)}{K_g} \quad [s4]$$

   Intuitively, the motor shaft must rotate K_g times for the output shaft to rotate one revolution. Taking the time derivatives and solving for the motor shaft speed gives the equation

   $$\omega_m(t) = K_g \omega_l(t) \quad [s5]$$

0	1	2

2. Using the motor-load rate relationship found above, the load equation given in [4], the motor equation in [5], and the torque equation in [10], find the differential equation that describes the motion of the load shaft with respect to an applied motor torque. Express the equation found in terms of the equivalent moment of inertia, J_{eq}, and the equivalent viscous damping B_{eq}, defined as shown below

   $$J_{eq}\left(\frac{d}{dt}\omega_l(t)\right) + B_{eq}\omega_l(t) = \eta_g K_g \tau_m(t) \quad [11]$$

   Ensure both the equation derivation is shown and the J_{eq} and B_{eq} parameters are found.

Solution:
Substitute equations [10] and [s5] followed by the load equation [4] into the motor equation [5] to get the following equation

$$J_m K_g \left(\frac{d}{dt}\omega_l(t)\right) + B_m K_g \omega_l(t) + \frac{J_l\left(\frac{d}{dt}\omega_l(t)\right) + B_l \omega_l(t)}{\eta_g K_g} = \tau_m(t) \quad \text{[s6]}$$

Collecting the coefficients in terms of the load shaft velocity and acceleration gives

$$(\eta_g K_g^2 J_m + J_l)\left(\frac{d}{dt}\omega_l(t)\right) + (\eta_g K_g^2 B_m + B_l)\omega_l(t) = \eta_g K_g \tau_m(t) \quad \text{[s7]}$$

From the above equation, the equivalent moment of inertia and viscous damping parameters are

$$J_{eq} = \eta_g K_g^2 J_m + J_l \quad \text{[s8]}$$

and

$$B_{eq} = \eta_g K_g^2 B_m + B_l \quad \text{[s9]}$$

0	1	2

4.1.3. Adding Actuator Dynamics

In this section the electrical equation derived in Section 4.1.1 and the mechanical equation found in Section 4.1.2 are brought together to get an expression that represents the motions of the load shaft speed in terms of the applied motor voltage.

1. The amount of motor torque is proportional to the voltage applied and is described

$$\tau_m(t) = \eta_m k_t I_m(t), \quad \text{[12]}$$

where k_t the current-torque constant (N.m/A), η_m is the motor efficiency, and I_m is the armature current. See Reference [5] for the details on the SRV02 motor specifications. Express the motor torque-current relationship in [12] in terms of input voltage $V_m(t)$ and load shaft rate $\omega_l(t)$.

Solution:
Substitute the motor armature current equation [s3], found in Section 4.1.1, into the current-torque relationship given in [12] to get

$$\tau_m(t) = -\frac{\eta_m k_t (-V_m(t) + k_m \omega_m(t))}{R_m} \quad \text{[s10]}$$

This equation describes the motor torque with respect to the input voltage V_m, and the motor shaft speed, ω_m. To express this in terms of V_m and ω_l, insert the motor-load shaft rate relationship, Equation [s5], into [s10] and get

$$\tau_m(t) = -\frac{\eta_m k_t (-V_m(t) + k_m K_g \omega_l(t))}{R_m} \quad \text{[s11]}$$

0	1	2

2. Using the equation of motion in [11] and the voltage-to-motor torque relationship found above, find the voltage to load shaft equation of motion

$$\left(\frac{d}{dt}\omega_l(t)\right) J_{eq} + B_{eq,v}\,\omega_l(t) = A_m V_m(t) \quad \text{[13]}$$

Recall that J_{eq} is the equivalent moment of inertia previously found in Section 4.1.2. The viscous damping parameter $B_{eq,v}$ includes the back-emf influence and A_m is the actuator gain parameter. Make sure the equation derivation is shown and the parameters $B_{eq,v}$ and A_m are expressed symbolically.

Solution:
Substitute this expression into the load shaft and motor torque equation of motion in [11] to obtain

$$J_{eq}\left(\frac{d}{dt}\omega_l(t)\right) + B_{eq}\,\omega_l(t) = -\frac{\eta_g K_g \eta_m k_t(-V_m(t) + k_m K_g \omega_l(t))}{R_m} \quad [s12]$$

After collecting the terms, the equation becomes

$$\left(\frac{d}{dt}\omega_l(t)\right) J_{eq} + \left(\frac{k_m \eta_g K_g^2 \eta_m k_t}{R_m} + B_{eq}\right)\omega_l(t) = \frac{V_m(t)\,\eta_g K_g \eta_m k_t}{R_m} \quad [s13]$$

From this, the new equivalent damping term is

$$B_{eq,v} = \frac{\eta_g K_g^2 \eta_m k_t k_m + B_{eq} R_m}{R_m} \quad [s14]$$

and the actuator gain equals

$$A_m = \frac{\eta_g K_g \eta_m k_t}{R_m} \quad [s15]$$

0	1	2

4.1.4. Obtaining the Transfer Function

1. Find the transfer function $\Omega_l(s)/V_m(s)$ that represents the load shaft rate with respect to an applied motor voltage. Assume the motor speed is initially zero, i.e. $\omega_l(0^-) = 0$.

Solution:
Taking the Laplace transform of Equation [13] and assuming $\omega_l(0^-) = 0$ gives

$$J_{eq}\, s\, \Omega_l(s) + B_{eq,v}\, \Omega_l(s) = A_m V_m(s) \quad [s16]$$

Solving for $\Omega_l(s)/V_m(s)$ gives the plant transfer function

$$\frac{\Omega_l(s)}{V_m(s)} = \frac{A_m}{J_{eq}\, s + B_{eq,v}} \quad [s17]$$

0	1	2

2. Express the steady-state gain, K, and the time constant, τ, of the process model in [1] in terms of the J_{eq}, $B_{eq,v}$, and A_m parameters.

Solution:
The time constant parameter is

$$\tau = \frac{J_{eq}}{B_{eq,v}} \qquad [s18]$$

and the steady-state gain is

$$K = \frac{A_m}{B_{eq,v}} \qquad [s19]$$
.

0	1	2

4.1.5. Evaluating the Model Parameters

The model parameters will now be evaluated numerically using the SRV02 specifications given in Reference [5]. The parameters are to be calculated based on an SRV02-T (has a tachometer) in the low-gear configuration.

1. Calculate the viscous damping parameter $B_{eq,v}$ and the actuator gain A_m.

Solution:
The $B_{eq,v}$ viscous damping expression is given in Equation [s14] above. All the parameters are defined in Reference [5] including the experimentally determined equivalent viscous damping parameter $B_{eq} = 1.5\times10^{-3}$ N.m.s/rad (in the high-gear configuration). Substituting all the specifications into [s14] gives

$$B_{eq,v} = 0.0844 \left[\frac{N\,m\,s}{rad} \right] \qquad [s20]$$
.

Evaluating the actuator gain expression in [s15] with the SRV02 parameters gives

$$A_m = 0.129 \left[\frac{N\,m}{V} \right] \qquad [s21]$$
.

0	1	2

2. Evaluate the moment of inertia that is acting about the motor shaft, J_m. Recall that the SRV02-T system is being used and therefore the inertia from the tachometer must be considered.

Solution:
The moment of inertia about the motor shaft equals

$$J_m = J_{tach} + J_{m,\,rotor} \quad [s22]$$

where J_{tach} and $J_{m,rotor}$ is the moment of inertia of the specified for the tachometer and the SRV02 DC motor rotor. Evaluating the above expression with the parameters outlined in Reference [5] gives

$$J_m = 0.4606251061\ 10^{-6} \quad [s23]$$

0	1	2

3. The load attached to the motor shaft includes a 24-tooth gear, two 72-tooth gears, and a single 120-tooth gear along with any other external load that is attached to the load shaft. Thus for the gear moment of inertia J_g and the external load moment of inertia $J_{l,ext}$, the load inertia is

$$J_l = J_g + J_{l,\,ext} \quad [14]$$

Find the total moment of inertia from the gears J_g.

Solution:
The formula to calculate the moment of inertia of a disc is

$$J_{disc} = \frac{m\,r^2}{2}, \quad [s24]$$

where *m* is the mass and *r* is the radius. Assuming the gears are discs and using the parameters given in Reference [5], the moment of inertia of a 24-tooth, 72-tooth, and 120-tooth gears are

$$J_{24} = 0.101\ 10^{-6}\ [kg\ m^2], \quad [s25]$$

$$J_{72} = 0.544\ 10^{-5}\ [kg\ m^2] \quad [s26]$$

and

$$J_{120} = 0.0000418\,[kg\ m^2]. \quad [s27]$$

The total moment of inertia from the gears is

$$J_g = J_{24} + 2\,J_{72} + J_{120} \quad [s28]$$

which equals

$$J_g = 0.0000528\,[kg\ m^2] \quad [s29]$$

when evaluated.

0	1	2

4. Assuming the disc load is attached at one of its ends to the load shaft, calculate the total load

moment of inertia, J_l, using Equation [14].

Solution:

Using the formula in [s24] with the m_b and r_b disc load parameters found in Reference [5], the external load moment of inertia equals

$$J_{l,\,ext} = 0.0000500\ [kg\ m^2] \quad \text{[s30]}$$

Using Equation [14], the total load moment of inertia is

$$J_l = 0.0000663\ [kg\ m^2] \quad \text{[s31]}$$

0	1	2

5. Evaluate the equivalent moment of inertia J_{eq}.

Solution:

Using Equation [s8] with the gear train and motor specifications listed in Reference [5] and the load inertia found in [s15], the equivalent moment of inertia acting on the SRV02 motor shaft is

$$J_{eq} = 0.00213\ [kg\ m^2] \quad \text{[s32]}$$

0	1	2

6. Give the steady-state model gain and time constant values. These will be known as the nominal model parameters and will be used to compare with parameters that are found experimentally.

Solution:

Using equations [s18] and [s19] with the $B_{eq,v}$, A_m, and J_{eq} parameters found in equations [s20], [s21], and [s32], the steady-state gain is

$$K = 1.53 \left[\frac{rad}{s\ V} \right] \quad \text{[s33]}$$

and the model time constant is

$$\tau = 0.0253\ [s] \quad \text{[s34]}$$

0	1	2

4.2. Modeling Experimentally

A linear model of a system can also be determined purely experimentally. Basically, the idea is to observe how a system reacts to different inputs and change structure and parameters of a model until a reasonable fit is obtained. The inputs can be chosen in many different ways and there is a large variety of methods. In sections 4.2.1 and 4.2.2, two methods of modeling the SRV02 are outlined: frequency response and bumptest.

4.2.1. Frequency Response

In Figure 2, the response of a typical first-order time-invariant system when subject to a sine wave is shown. Thus the input signal, *u*, is a sine wave with a set amplitude and frequency and the output, *y*, is a sinusoid with the same frequency but with a different amplitude. By varying the frequency of the sine wave and observing the resulting outputs, a bode plot of the system similarly as shown in Figure 3 can be obtained.

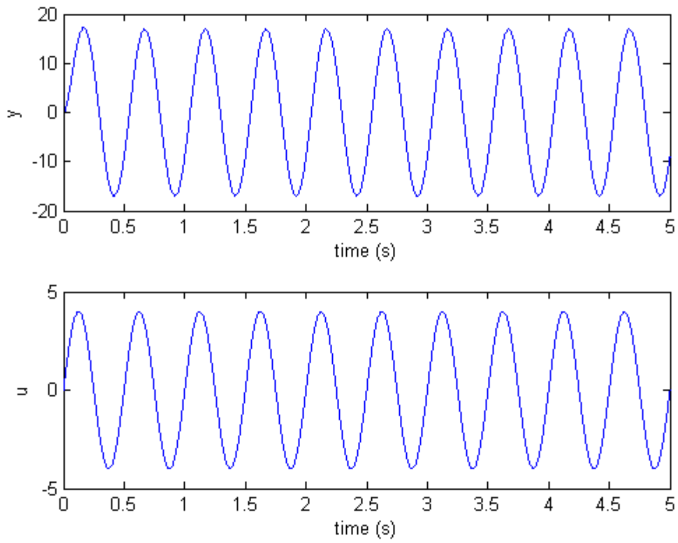

Figure 2: Typical frequency response.

The bode plot can then be used to find the steady-state gain, i.e. the DC gain, and the time constant of the system. As shown in Figure 3, the cutoff frequency, ω_c, is defined as the frequency where the gain is 3 dB less then the maximum gain (i.e. the DC gain). When working in the linear non-decibel range, the 3 dB frequency is defined as the frequency where the gain is $1/2^{1/2}$, or about 0.707, of the maximum gain. Remark that in this case, the cutoff frequency is the the bandwidth of the system and it represents how fast the system responds to a given input.

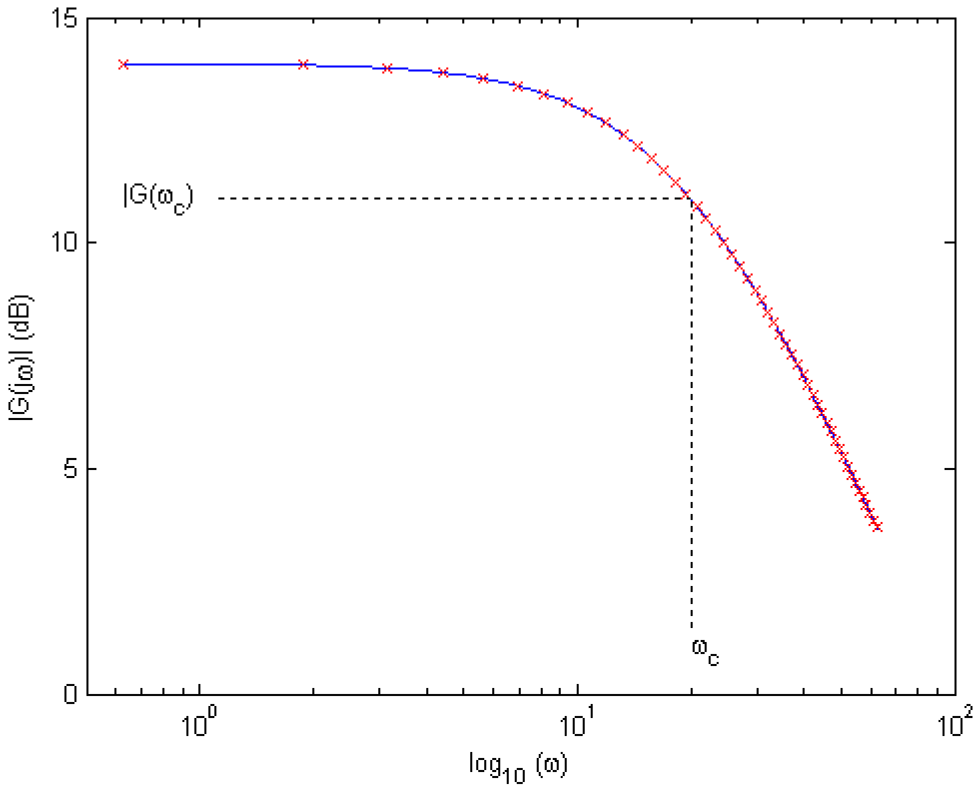

Figure 3: Magnitude bode plot.

1. Find the magnitude of the frequency response of the SRV02 plant transfer function [1],

$$\left| G_{wl,v}(\omega) \right| = \left| \frac{\Omega_l(\omega j)}{V_m(\omega j)} \right|, \qquad [15]$$

where ω is the frequency of the motor input voltage signal V_m.

Solution:

Substituting s= jω in Equation [1] gives the SRV02 frequency response

$$\frac{\Omega_l(\omega j)}{V_m(\omega j)} = \frac{K}{\tau \omega j + 1} \qquad [s35]$$

and its magnitude equals

$$\left| G_{wl,v}(\omega) \right| = \frac{K}{\sqrt{1 + \tau^2 \omega^2}}. \qquad [s36]$$

0	1	2

2. Call the frequency response model parameters $K_{e,f}$ and $\tau_{e,f}$ in order to differentiate them from the

nominal model parameters, K and τ, calculated previously in Section 4.1. The steady-state gain is the DC gain (i.e. gain at zero frequency) of the model, thus

$$K_{e,f} = \left| G_{wl,v}(0) \right| \qquad [16]$$

3. Find the time constant of the model, $\tau_{e,f}$, using the 3 dB frequency ω_c.

Solution:

The 3 dB frequency is defined

$$\left| G_{wl,v}(\omega_c) \right| = \frac{1}{2} \left| G_{wl,v}(0) \right| \sqrt{2} \qquad [s37]$$

Thus its the frequency when the decibel magnitude is -3 dB, or 1/2^(1/2) of the DC gain. Applying this to the SRV02 frequency response magnitude in [s36] above resulting in the equation

$$\frac{1}{2} \left| G_{wl,v}(0) \right| \sqrt{2} = \frac{\left| G_{wl,v}(0) \right|}{\sqrt{1 + \tau_{e,f}^2 \omega_c^2}} \qquad [s38]$$

Solving for the time constant gives

$$\tau_{e,f} = \frac{1}{\left| \omega_c \right|} \qquad [s39]$$

0	1	2

4.2.2. Bumptest

The bumptest is a simple test based on a step response for a stable system. It is carried out in the following way. A constant input is chosen. A stable system will then reach an equilibrium. The input is then changed rapidly to a new level and the output is recorded.

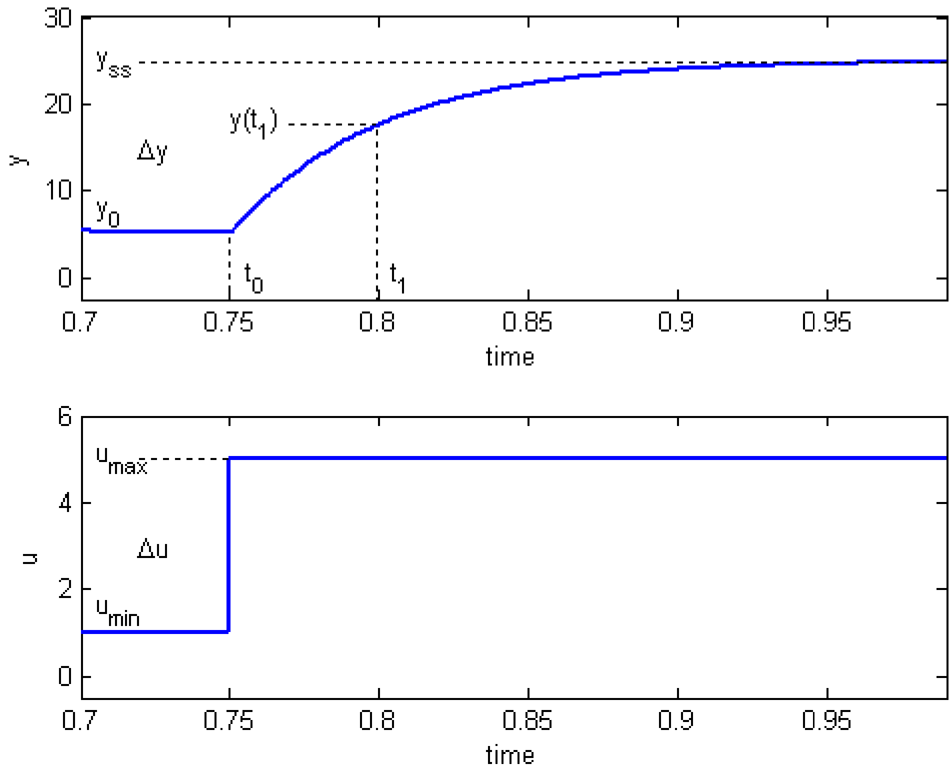

Figure 4: Input and output signal used in the bumptest method.

The step response shown in Figure 4 is generated using the transfer function

$$\frac{Y(s)}{U(s)} = \frac{K}{\tau\, s + 1} \quad [15]$$

with the parameters

$$K = 5.0 \left[\frac{rad}{s\, V} \right] \quad [16]$$

and

$$\tau = 0.05\ [s] \,. \quad [17]$$

The input signal, *u*, is a step that begins at time t_0. The input signal has a minimum value of u_{min} and a maximum value of u_{max}. The resulting output signal is initially at y_0. Once the step is engaged, the output eventually settles to its steady-state value y_{ss}. From the output and input signals, the steady-state gain is

$$K = \frac{\Delta y}{\Delta u} \quad [18]$$

where

$$\Delta y = y_{ss} - y_0 \,. \quad [19]$$

and

$$\Delta u = u_{max} - u_{min} . \qquad [20]$$

In order to find the model time constant, τ, the output signal at $y(t_1)$ must be measured. It is defined

$$y(t_1) = 0.632\, y_{ss} + y_0 \qquad [21]$$

and the time is

$$t_1 = t_0 + \tau \qquad [22]$$

From this, the model time constant is

$$\tau = t_1 - t_0 . \qquad [23]$$

1. Find the the SRV02 load speed transfer function, $\Omega_l(s)$, with the step input

$$V_m(s) = \frac{A_v\, \mathbf{e}^{(-s\, t_0)}}{s} . \qquad [24]$$

where A_v is the amplitude of the step and t_0 is the step time (i.e. the delay).

Solution:
Substitute the step given in Equation [24] into the SRV02 process model in [1] above to get

$$\Omega_l(s) = \frac{K A_v\, \mathbf{e}^{(-s\, t_0)}}{(\tau s + 1)\, s} \qquad [s40]$$

0	1	2

2. Find the SRV02 load speed step response, $\omega_l(t)$, by taking the inverse Laplace of the above system. Careful with the time delay t_0 and note that the initial condition is $\omega_l(0^-) = \omega_l(t_0)$.

Solution:
Take the inverse Laplace transform of Equation [s40]

$$\omega_l(t) = K A_v \left(1 - \mathbf{e}^{\left(-\frac{t - t_0}{\tau}\right)}\right) + \omega_l(t_0) \quad [s41]$$

0 | 1 | 2

3. Find the steady-state gain of the step response and compare it with Equation [18].

Solution:
Let the steady-state value of the load shaft rate be defined

$$\omega_{l,ss} = \lim_{t \to \infty} \omega_l(t) \quad [s42]$$

The limit of the servo step response [s41] is

$$\omega_{l,ss} = K A_v + \omega_l(t_0) \quad [s43]$$

and the steady-state gain is therefore

$$K = -\frac{-\omega_{l,ss} + \omega_l(t_0)}{A_v} \quad [s44]$$

This is consistent with the $\Delta y / \Delta u$ relationship in Equation [18].

0 | 1 | 2

4. Evaluate the step response when $t = t_0 + \tau$ (called t_1 above) and compare it with Equation [21].

Solution:
Substituting $t = t_0 + \tau$ in Equation [s41] gives the load shaft rate

$$\omega_l(t_0 + \tau) = K A_v (1 - \mathbf{e}^{(-1)}) + \omega_l(t_0) \quad [s45]$$

This is consistent with the $y(t_1)$ expression in Equation [18].

0 | 1 | 2

5. In-Lab Procedures

The *q_srv02_mdl* Simulink diagram shown in Figure 5 is used to perform the modeling exercises in this laboratory. The *SRV02-ET* subsystem contains QUARC blocks that interface with the DC motor and sensors of the SRV02 system, as discussed in Reference [6]. The *SRV02 Model* uses a Transfer Fcn block from the Simulink library to simulate the SRV02 system. Thus both the measured and simulated load shaft speed can be monitored simultaneously given the set open-loop input voltage.

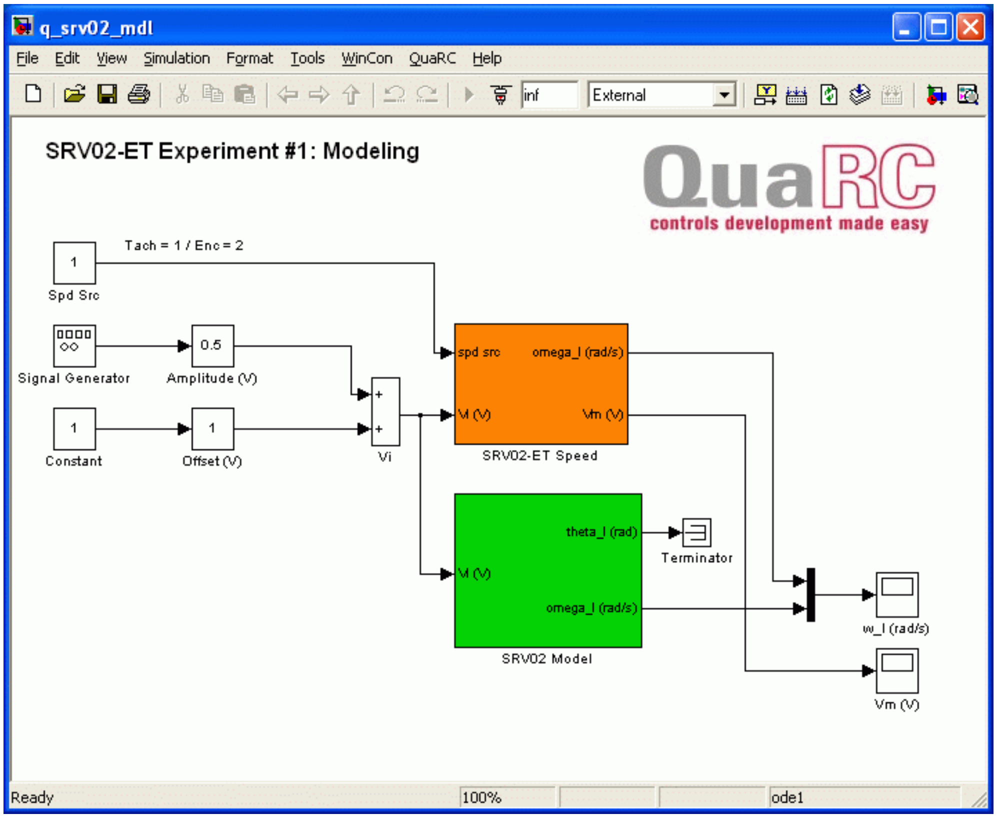

Figure 5: q_srv02_mdl Simulink diagram used to model SRV02.

Students are asked to model the SRV02 using frequency response in Section 5.2 and then using the bumptest technique in Section 5.3. The nominal value as well as the results from both experimental methods are then validated in Section 5.4. Before going through these experiments, go through Section 5.1 to configure the lab files according to your SRV02 setup.

5.1. Configuring the SRV02 and the Lab Files

Before beginning the in-lab exercises the SRV02 device, the q_srv02_mdl Simulink diagram, and the setup_srv02_exp02.m script must be configured.

Follow these steps to get the system ready for this lab:

1. Setup the SRV02 in the high-gear configuration and with the disc load as described in Reference [5].
2. Load the Matlab software.
3. Browse through the *Current Directory* window in Matlab and find the folder that contains the SRV02 modeling files, e.g. *q_srv02_mdl.mdl*.
4. Double-click on the *q_srv02_mdl.mdl* file to open the Simulink diagram shown in Figure 5.
5. **Configure DAQ**: Double-click on the HIL Initialize block in the Simulink diagram and ensure it is configured for the DAQ device that is installed in your system. For instance, the block shown in Figure 5 is setup for the Quanser Q8 hardware-in-the-loop board. See Reference [6] for more information on configuring the HIL Initialize block.
6. **Configure Sensor**: The speed of the load shaft can be measured using various sensors. Set the *Spd Src* Source block in q_srv02_mdl, as shown in Figure 5, as follows:
   - 1 to use tachometer
   - 2 to use to the encoder

   It is recommended that the tachometer sensor be used to perform this laboratory. However, for users who do not have a tachometer with their servo, e.g. SRV02 or SRV02-E options, then they may choose to use the encoder with a high-pass filter to get a velocity measurement.
7. Go to the *Current Directory* window and double-click on the *setup_srv02_exp01_mdl.m* file to open the setup script for the q_srv02_mdl Simulink model.
8. **Configure setup script**: The beginning of the setup script is shown Text 1. Ensure the script is setup to match the configuration of your actual SRV02 device. For example, the script given in Text 1 is setup for an SRV02-ET plant in the high-gear configuration mounted with a disc load and it is actuated using the Quanser VoltPAQ device with a motor cable gain of 1. See Reference [5] for more information on SRV02 plant options and corresponding accessories. Next, ensure the MODELING_TYPE is set to 'MANUAL'.

```
%% SRV02 Configuration
% External Gear Configuration: set to 'HIGH' or 'LOW'
EXT_GEAR_CONFIG = 'HIGH';
% Encoder Type: set to 'E' or 'EHR'
ENCODER_TYPE = 'E';
% Is SRV02 equipped with Tachometer? (i.e. option T): set to 'YES' or 'NO'
TACH_OPTION = 'YES';
% Type of Load: set to 'NONE', 'DISC', or 'BAR'
LOAD_TYPE = 'DISC';
% Amplifier Gain: set VoltPAQ amplifier gain to 1
K_AMP = 1;
% Power Amplifier Type: set to 'VoltPAQ', 'UPM_1503', 'UPM_2405', or 'Q3'
AMP_TYPE = 'VoltPAQ';
% Digital-to-Analog Maximum Voltage (V)
VMAX_DAC = 10;
%
%% Lab Configuration
% Type of Controller: set it to 'AUTO', 'MANUAL'
MODELING_TYPE = 'AUTO';
% MODELING_TYPE = 'MANUAL';
```

Text 1: Configuration sections in the setup_srv02_exp01_mdl.m file.

9. Run the script by selecting the Debug | Run item from the menu bar or clicking on the *Run* button in the tool bar. The messages shown in Text 2, below, should be generated in the Matlab Command Window. Theses are default model parameters and do not accurately represent the SRV02 system.

```
Calculated SRV02 model parameters:
   K = 1 rad/s/V
   tau = 0.1 s
```

Text 2: Display message shown in Matlab Command Window after running setup_srv02_exp01_mdl.m.

5.2. Frequency Response

In this laboratory, a sine wave of varying frequency is fed to the DC motor and the resulting speed is recorded. The bode plot of the system is constructed using the collected data and then, as discussed in Section 4.2.1, the model parameters are found.

Follow the steps below:

1. Enter the nominal model parameter values, K and τ, found in pre-lab exercise #6 in Section 4.1.5 in Table 3.
2. In the Simulink diagram, double-click on the *Signal Generator* block and ensure the following parameters are set:
   Wave form: sine
   Amplitude: 0.0
   Frequency: 1.0
   Units: Hertz
3. Set the *Amplitude (V)* slider gain to 0.0 V.
4. Set the *Offset (V)* block to 2.0 V.
5. Open the load shaft speed scope, *w_l (rad/s)*, and the motor input voltage scope, *Vm (V)*.
6. Click on QUARC | Build to compile the Simulink diagram.
7. Select QUARC | Start to begin running the controller. The SRV02 unit should begin rotating in one constant direction. The scopes should be reading something similar to as shown in figures 6 and 7. Note that in the *w_l (rad/s)* scope, the yellow trace is the measured speed while the purple trace is the simulated speed (generated by the *SRV02 Model* block).

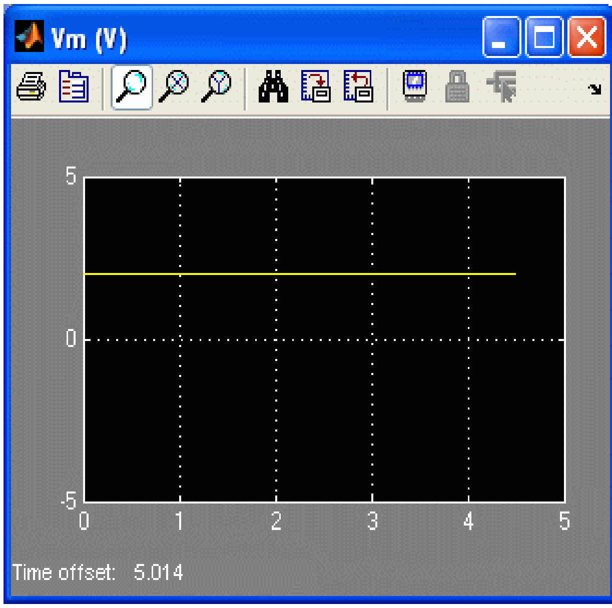

Figure 6: Constant input motor voltage.

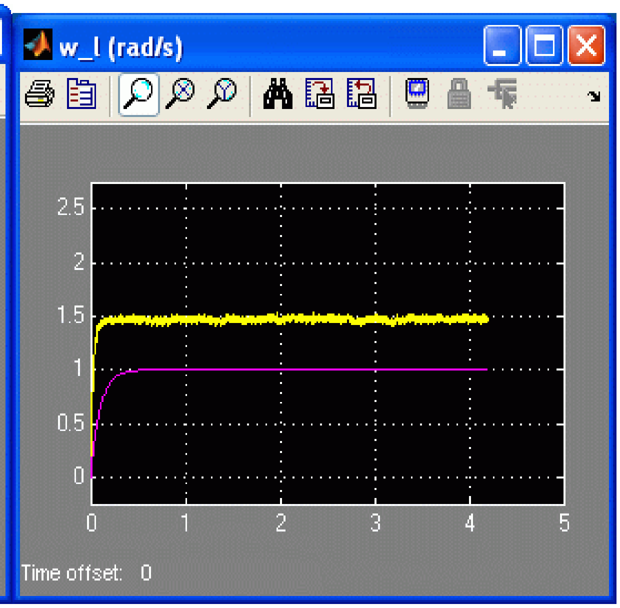

Figure 7: Load shaft speed response to a constant input.

8. Measure the speed of the load shaft. The measurement can be done directly from the scope. Alternatively, users can use Matlab commands to find the maximum load speed using the saved *w_l* variable. When the controller is stopped, the *w_l (rad/s)* scope saves the last 5 seconds of response data to the Matlab workspace in the *w_l* parameter. It has the following structure: *w_l(:,1)* is the time vector, *w_l(:,2)* is the measured speed, and *w_l(:,3)* is the simulated speed. In either method, enter the speed measurement in Table 2 under the f = 0 Hz row.

f (Hz)	*Amplitude (V)*	*Maximum Load Speed (rad/s)*	*Gain: $\|G(\omega)\|$ (rad/s/V)*	*Gain: $\|G(\omega)\|$ (rad/s/V, dB)*
0.0	2.0	3.31	1.66	4.37
1.0	2.0	3.25	1.62	4.22
2.0	2.0	3.14	1.57	3.91
3.0	2.0	2.96	1.48	3.40
4.0	2.0	2.77	1.39	2.83
5.0	2.0	2.59	1.29	2.24
6.0	2.0	2.45	1.22	1.75
7.0	2.0	2.34	1.17	1.38
8.0	2.0	2.22	1.11	0.89

0	1	2

Table 2: Collected frequency response data.

9. As instructed in Section 4.2.1, calculate the gain when the input signal is a constant signal and express the values in both the linear and decibel (dB) units. Show your calculations below and

enter the resulting numerical value in the f = 0 Hz row of Table 2. Remark that this is the steady-state gain of the system. Enter its non-decibel result in Table 3 below.

Solution:

The frequency response magnitude is found from the collected data using Equation [15]. In terms of the frequency in Hertz, i.e. $\omega = 0$, the relationship is presented

$$\left| G_{wl,\,v}(0) \right| = \left| \frac{\Omega_l(0)}{V_m(0)} \right| . \qquad [s46]$$

As shown in Table 2, at f = 0.0 Hz the maximum load speed measured is $\Omega_l(2\pi) = 1.48$ rad/s and the voltage is $V_m(0) = 2.0$ V. The gain is therefore

$$\left| G_{wl,\,v}(0) \right| = 1.66 \left[\frac{rad}{s\ V} \right] . \qquad [s47]$$

Using the expression

$$\left| G_{wl,\,v}(0) \right|_{dB} = 20\ \mathrm{Log10}(1.66) \qquad [s48]$$

the gain in dB is

$$\left| G_{wl,\,v}(0) \right|_{dB} = 4.40\ [dB] . \qquad [s49]$$

This is the steady-state gain of the system

0	1	2

10. Set the *Offset (V)* block to 0 V.
11. Set the *Amplitude (V)* slider gain to 2.0 V.
12. The SRV02 unit should begin rotating smoothly back and forth and the scopes should be reading a response similar to as shown in figures 8 and 9.

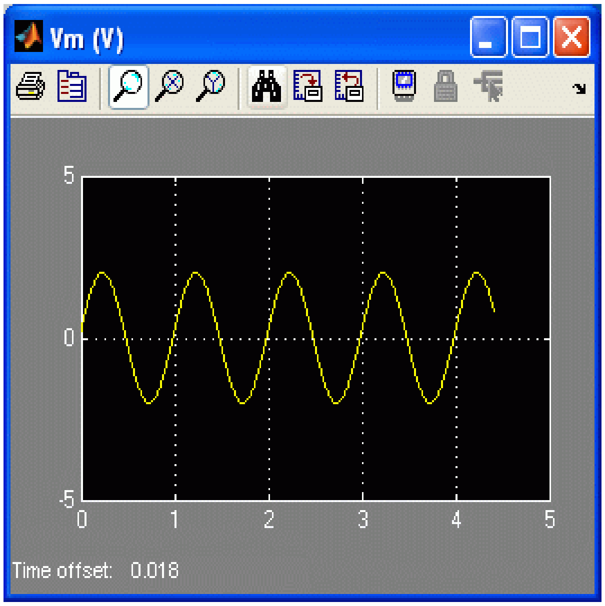

Figure 8: Input motor voltage scope.

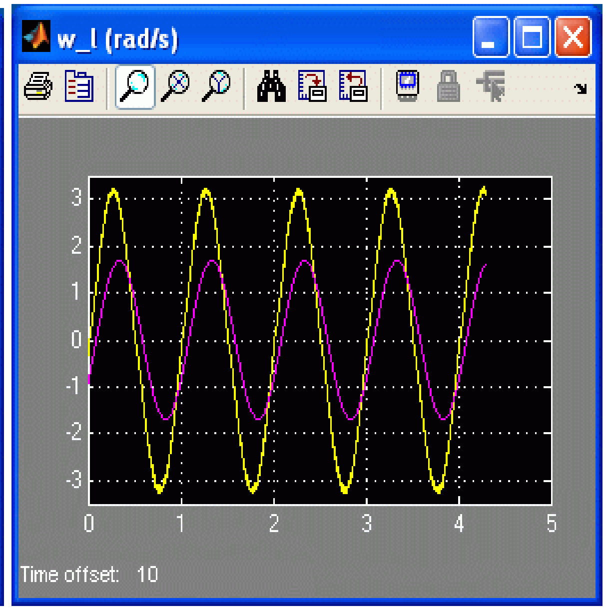

Figure 9: Load shaft speed sine wave response.

13. Measure the maximum positive speed of the load shaft. As before, this measurement can be done directly from the scope or, preferably, users can use Matlab commands to find the maximum load speed using the saved *w_l* variable. Enter the maximum load speed when f = 1.0 Hz in Table 2 above.
14. Calculate the gain of the system (in both linear and dB) and enter the results in Table 2.
15. By adjusting the frequency parameter in the *Signal Generator* block, measure the maximum load speed and calculate the gain for all frequencies listed in Table 2.
16. Generate a magnitude bode plot and attach it to the report. Make sure the amplitude and frequency scales are in decibels. When plotting the bode, ignore the f = 0 Hz entry as the logarithm of 0 is not defined.

Solution:
The sample_freq_rsp.m script contains the collected data in Table 2 and generates the corresponding magnitude bode plot shown in Figure 10.

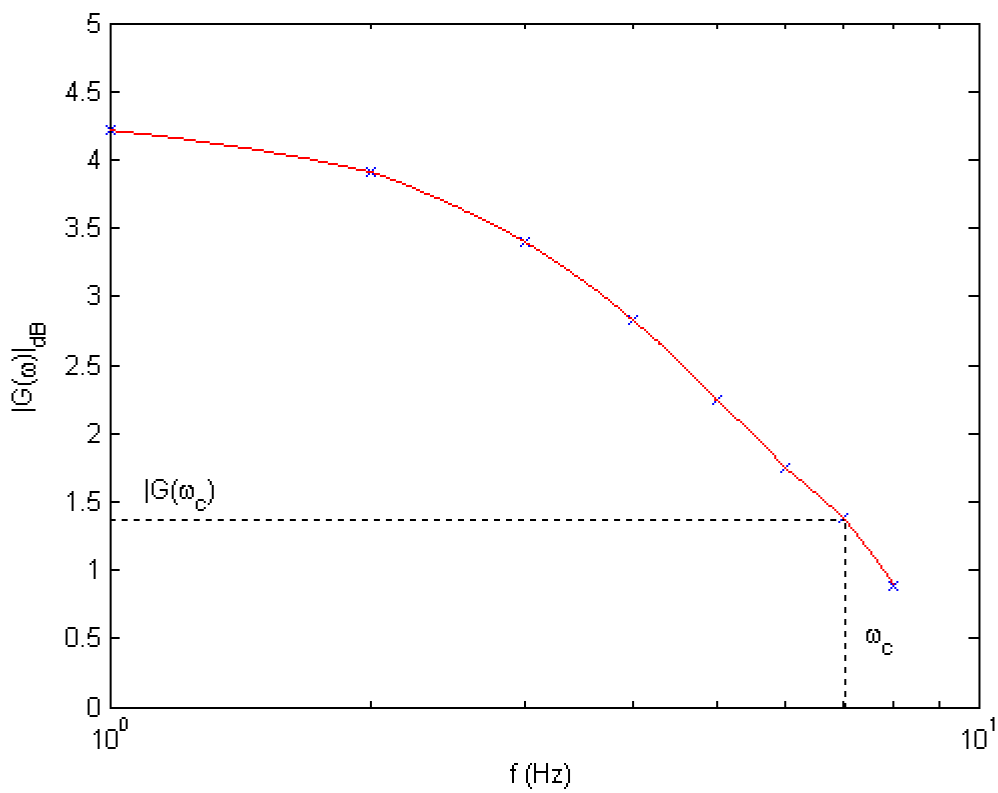

Figure 10: Sample Bode plot after performing frequency response laboratory.

0	1	2

17. Calculate the time constant derived using the frequency response method, $\tau_{e,f}$, using the obtained bode plot by finding the cutoff frequency. Label the bode plot with the -3 dB gain and the cutoff frequency and enter the resulting time constant in Table 3. *Hint*: Use the *ginput* command to obtain values from a Matlab figure.

Solution:
As illustrated in Figure 10, the -3 dB gain is

$$\left| G_{wl,v}(\omega_c) \right|_{dB} = 1.36\,[dB] \qquad [s50]$$

and the corresponding cutoff frequency is

$$f_c = 7.04\,[Hz] \qquad [s51]$$

or

$$\omega_c = 44.3 \left[\frac{rad}{s} \right] \qquad [s52]$$

Using Equation [11], the time constant is

$$\tau_{e,f} = 0.0226\,[s] \qquad [s53]$$

0	1	2

18. Click on the *Stop* button on the Simulink diagram tool bar (or select QUARC | Stop from the menu) to stop running the code.
19. Shut off the power of the amplifier if no more experiments will be performed on the SRV02 in this session.

5.3. Bumptest

In this laboratory a step voltage is given to the SRV02 and the corresponding load shaft response is recorded. Using the saved response, the model parameters are found as discussed in Section 4.2.2.

Follow this procedure to model the SRV02 using the bumptest technique:

1. Double-click on the *Signal Generator* block and ensure the following parameters are set:
   Wave form: square
   Amplitude: 1.0
   Frequency: 0.4
   Units: Hertz
2. Set the *Amplitude (V)* slider gain to 1.5 V.
3. Set the *Offset (V)* block to 2.0 V.
4. Open the load shaft speed scope, *w_l (rad/s)*, and the motor input voltage scope, *Vm (V)*.
5. Click on QUARC | Build to compile the Simulink diagram.
6. Select QUARC | Start to begin running the controller. The gears on the SRV02 should be rotating in the same direction and alternating between low and high speeds. The response in the scopes should similar to the readings shown in figures 11 and 12.

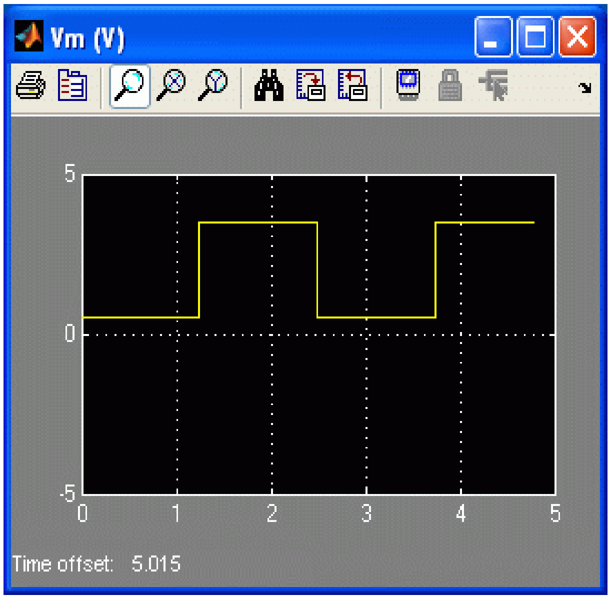

Figure 11: Square input motor voltage.

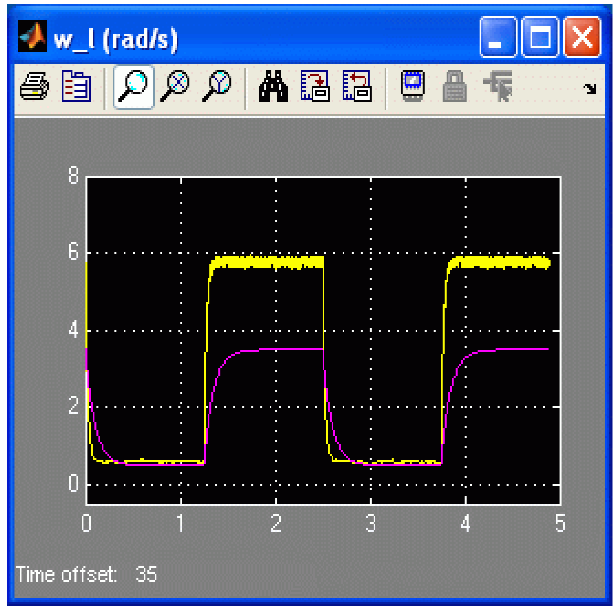

Figure 12: Load shaft speed step response.

7. Plot the response in a Matlab figure and attach it to the report. It is reminded that the maximum load speed is saved in the Matlab workspace under the *w_l* variable.

Solution:
See the sample_bumptest.m Matlab script. It loads sample measured data and plots the response shown in

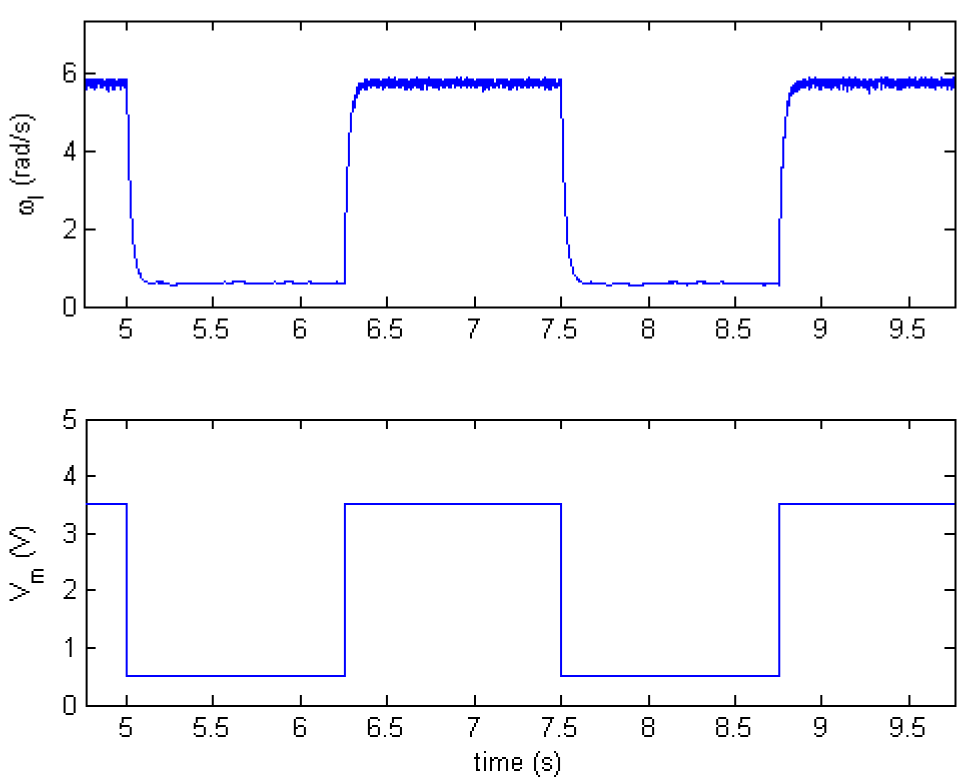

Figure 13: Sample SRV02 step response.

0	1	2

8. Find the steady-state gain using the measured step response. Give the calculation below and enter the resulting gain in Table 3 below.

Solution:
See the sample_bumptest.m Matlab script for more details on calculating the steady-state gain automatically.

From Figure 13, the measured initial and steady-state load shaft speeds are

$$\omega_l(t_0) = 0.530 \left[\frac{rad}{s}\right] \quad [s54]$$

and

$$\omega_{l,\,ss} = 5.92 \left[\frac{rad}{s}\right] \quad [s55]$$

and the input voltage amplitude is

$$A_v = 3.0\,[V]. \quad [s56]$$

Using Equation [s44] with the collected results above, the resulting steady-state gain is

$$K = 1.79 \left[\frac{rad}{s}\right]. \quad [s57]$$

0	1	2

9. Find the time constant from the obtained response. Show the calculations in the box below and enter the result in Table 3.

Solution:
See the sample_bumptest.m Matlab script for more details on the method used to calculate the time constant from a saved response.

In order to find time of first decay $t_1 = t_0 + \tau$, the corresponding speed measurement defined in Equation [s45] must be found. From Figure 13, the time at the shaft speed

$$\omega_l(t_0 + \tau) = 3.92 \left[\frac{rad}{s}\right] \quad [s58]$$

is

$$t_1 = 6.273\,[s]. \quad [s59]$$

The step start time is

$$t_0 = 6.250\,[s]. \quad [s60]$$

Given the step start time t_0 and decay time t_1 with Equation [23], the time constant

$$\tau = 0.023\,[s]. \quad [s61]$$

0	1	2

10. Click on the *Stop* button on the Simulink diagram tool bar (or select QUARC | Stop from the menu) to stop running the code.
11. Shut off the power of the amplifier if no more experiments will be performed on the SRV02 in this session.

5.4. Model Validation

As previously discussed, the q_srv02_mdl Simulink diagram includes a subsystem that interfaces with the actual SRV02 plant and a subsystem that simulates the plant using the transfer function. In this lab, students are asked to tune the transfer function parameters and do some basic model fitting. Then, the model parameters developed in Section are entered and the accuracy of the developed model is assessed.

Follow this procedure:

1. Double-click on the *Signal Generator* block and ensure the following parameters are set:
   Wave form: square
   Amplitude: 1.0
   Frequency: 0.4
   Units: Hertz
2. Set the *Amplitude (V)* slider gain to 1.0 V.
3. Set the *Offset (V)* block to 1.5 V.
4. Open the load shaft speed scope, *w_l (rad/s)*, and the motor input voltage scope, *Vm (V)*.
5. Click on QUARC | Build to compile the Simulink diagram.
6. Select QUARC | Start to begin running the controller. The gears on the SRV02 should be rotating in the same direction and alternating between low and high speeds and the scopes should be as shown in figures 14 and 15. Recall that the yellow trace is the measured load shaft rate and the purple trace is the simulated trace. By default, the steady-state gain and time constant values set in the Matlab workspace and use by the transfer function used for simulation is set to: K = 1 rad/s/V and τ = 0.1 s. These model parameters do not accurately represent the system.

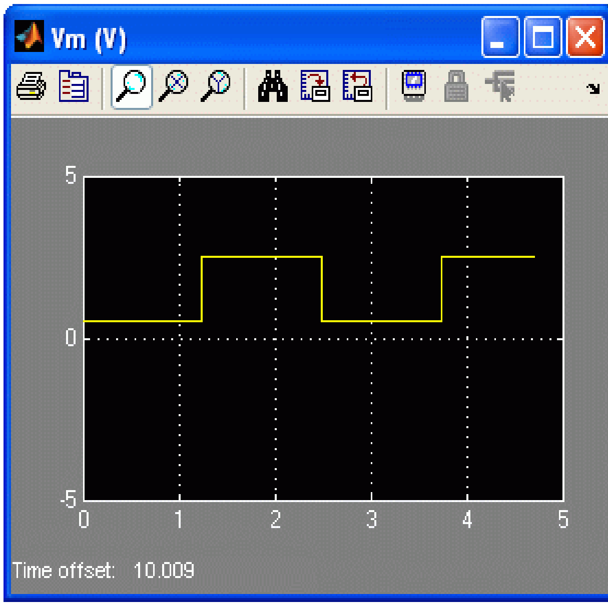

Figure 14: Input square voltage.

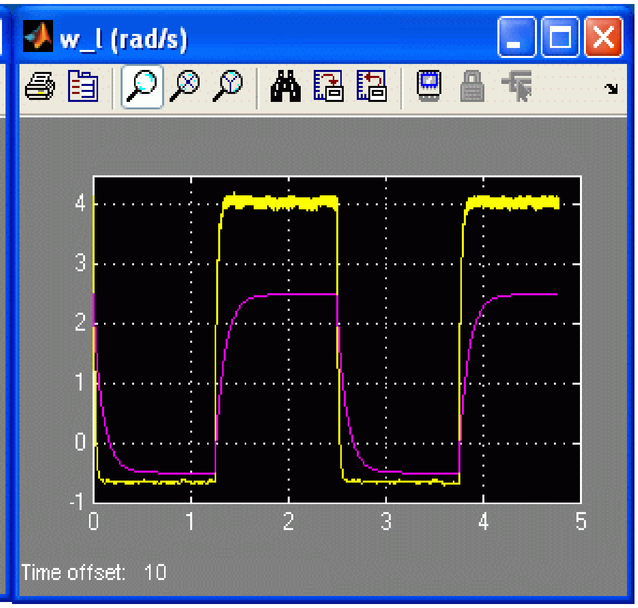

Figure 15: Speed step response. Simulation done with default model parameters: K =1 and τ = 0.1.

7. Enter the command "K = 1.25" in the Matlab Command Window.
8. Update the simulation by selecting the Edit | Update Diagram item in the q_srv02_mdl Simulink diagram and examine how the simulation changes. This updates the parameters used by the Transfer Function block contained in the *SRV02 Model* subsystem.
9. Enter the command "tau = 0.2" in the Matlab Command Window.
10. Update the simulation by selecting the Edit | Update Diagram and examine how the simulation changes.
11. Vary the gain and time constant model parameters and summarize the effects of changing them in the box below.

Solution:
The steady-state of the simulated speed increases as K is increased. When the time constant, τ, is increased, the peak time of the response increases. That is, it takes longer for the speed to reach its steady-state from the point when the step is engaged.

0	**1**	**2**

12. Enter the nominal values, K and τ, that were found in Section 4.1.5 in the Matlab Command Window. Update the parameters and examine how well the simulated response matches the measured one.
13. If the calculations were done properly, then the model should represent the actual system quite well. However, there are always some differences between each servo unit and, as a result, the

model can always be tuned to match the system better. Try varying the model parameters until the simulated trace matches the measured response better. Enter these tuned values under the Model Validation section of Table 3.

14. Name two reasons why the nominal model does not represent the SRV02 with better accuracy?

Solution:

Here are a few reasons:

- Inductance not taken into account. Using a second-order model to represent the SRV02 would be more accurate.
- Because the SRV02 specifications vary, e.g. back-emf has a rated variance of 12%, the model parameters that are calculated from these specifications will have an inherit variance.
- Equivalent viscous damping parameter was derived experimentally for one SRV02 unit. The viscous friction in each SRV02 is slightly different.
- Coulomb friction different in each SRV02.

0	1	2

15. Evaluate how well the nominal model, the frequency response model, and the bumptest model represent the SRV02 system. Thus enter the nominal values, K and tau, in the Matlab Command Window, update the parameters, and examine the response. Repeat for the frequency response parameters $K_{e,f}$ and $\tau_{e,f}$ along with the bumptest variables $K_{e,b}$ and $\tau_{e,b}$. Create a Matlab figure that shows the measured and simulated response of each method and attach it to the report.

Solution:
In the setup_srv02_exp01_mdl.m file, set MODELING_TYPE = 'AUTO' and run the script in order to load the nominal model parameters. The response is shown in Figure 16.

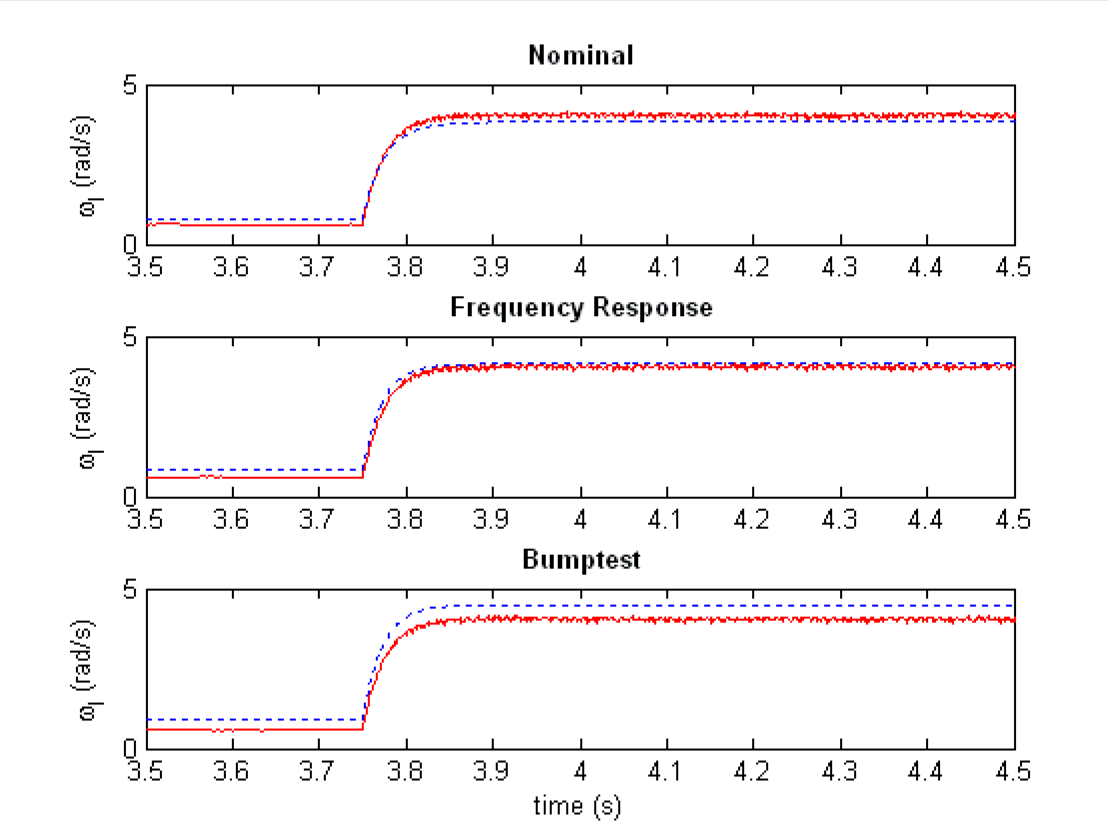

Figure 16: Model comparison

The nominal and frequency response model parameters both represent the SRV02 well. The frequency response model represents the steady-state system slightly better while the transient is represented more accurately with the nominal method. The parameters derived using the bumptest method do not represent the SRV02 as well as the other models. As shown in the bottom plot of Figure 16, the simulated steady-state value is higher then the measured speed.

0	1	2

16. Click on the *Stop* button on the Simulink diagram tool bar (or select QUARC | Stop from the menu) to stop running the code.
17. Shut off the power of the amplifier if no more experiments will be performed on the SRV02 in this session.

5.5. Results Summary

Fill out Table 3, below, with the pre-lab and in-lab results obtained above. For instance, enter the nominal model parameters that were found in the Pre-Lab Exercise #6 in Section 4.1.5 along with the model parameters that were found experimentally using the frequency response method, the bumptest method, and by model tuning, as dictated in sections 5.2, 5.3, and 5.4, respectively.

Section	*Description*	*Symbol*	*Value*	*Unit*
4.1.5.	***Nominal Values***			
6.	Open-Loop Steady-State Gain	K	1.53	rad/(V.s)
6.	Open-Loop Time Constant	τ	0.0254	s
5.2.	***In-Lab: Frequency Response Modeling***			
9.	Open-Loop Steady-State Gain	$K_{e,f}$	1.65	rad/(V.s)
17.	Open-Loop Time Constant	$\tau_{e,f}$	0.0226	s
5.3.	***In-Lab: Bumptest Modeling***			
8.	Open-Loop Steady-State Gain	$K_{e,b}$	1.80	rad/(V.s)
9.	Open-Loop Time Constant	$\tau_{e,b}$	0.023	s
5.4.	***In-Lab: Model Validation***			
13.	Open-Loop Steady-State Gain	$K_{e,v}$	1.60	rad/(V.s)
13.	Open-Loop Time Constant	$\tau_{e,v}$	0.0254	s

Table 3: SRV02 Experiment #1: Modeling results summary.

6. References

[1] DAQ User Manual
[2] QUARC User Manual (type `doc quarc` in Matlab to access)
[3] QUARC Installation Manual
[4] Amplifier User Manual
[5] SRV02 User Manual
[6] SRV02 QUARC Integration - Instructor Manual