Lecture 6
ELE 301: Signals and Systems

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Outline

- LTI System Response
- Filtering

Transfer Function

- Response to LTI system h.

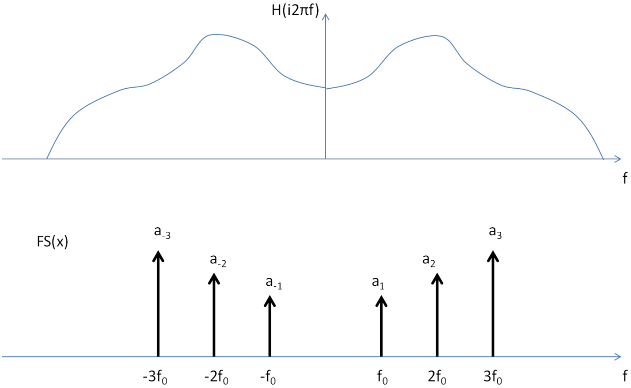
$$\text{Continuous time:} \quad e^{st} \quad \longrightarrow^{h} \quad H_c(s)e^{st},$$
$$\text{Discrete time:} \quad z^{n} \quad \longrightarrow^{h} \quad H_d(z)z^{n}.$$

- We are interested in the cases $s = i2\pi f$ and $z = e^{i2\pi f}$.

$$\text{Continuous time:} \quad y(t) \quad = \quad \sum_{k=-\infty}^{\infty} a_k \; H_c\left(i2\pi f_0 k\right) \; e^{i2\pi f_0 kt},$$
$$\text{Discrete time:} \quad y[n] \quad = \quad \sum_{k=-\infty}^{\infty} a_k \; H_d\left(e^{i2\pi f_0 k}\right) \; e^{i2\pi f_0 kn}.$$

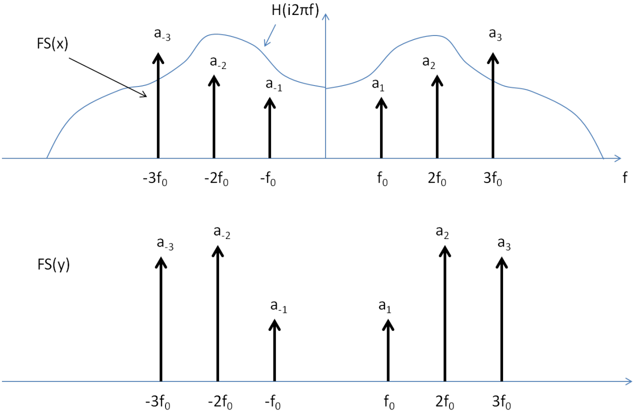
where a_k are the Fourier Series coefficients of the input with period $T = 1/f_0$.

Intuitive Visualization



Note: Plots aren't technically accurate because complex numbers are not one-dimensional.

Intuitive Visualization



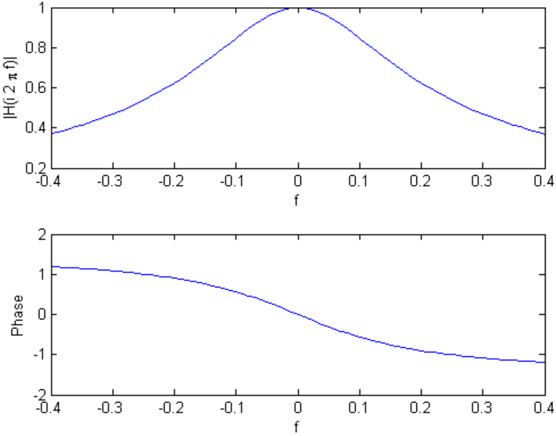
Note: Plots aren't technically accurate because complex numbers are not one-dimensional.

Filtering Example

$$
\begin{aligned}
h(t) &= e^{-t}u(t), \\
H(i2\pi f) &= ?.
\end{aligned}
$$

First-order low-pass filter

$$H(i2\pi f) = \frac{1}{1 + i2\pi f}.$$

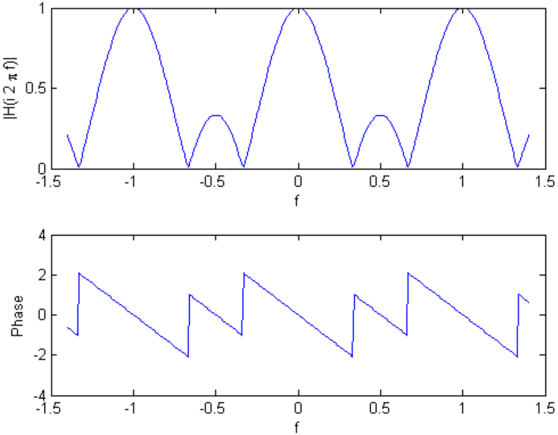


Filtering example - running average

$$
\begin{aligned}
h[n] &= \frac{1}{3}(\delta[n] + \delta[n-1] + \delta[n-2]), \\
H(e^{i2\pi f}) &= ?.
\end{aligned}
$$

Running average

$$H(i2\pi f) \;=\; \frac{1}{3}\left(1 + e^{-i2\pi f} + e^{-i4\pi f}\right).$$



Filtering example - Differentiator

What is the impulse response of a differentiator?

$$h(t) = ?$$

Unit Doublet

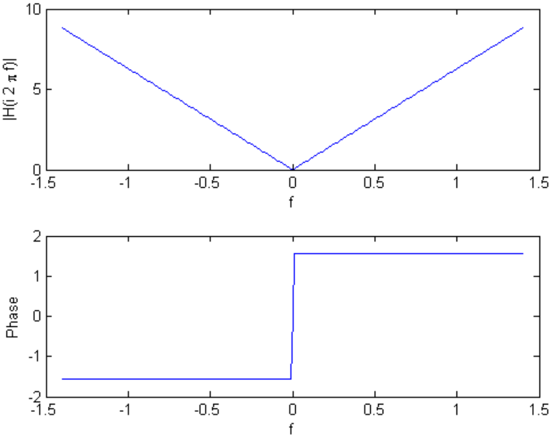
- Another invented pseudo-function
- Conceptually the derivative of the Dirac delta function
- Properties
  - $\delta' * f = f'$
  - $f(t)\delta'(t - t_0) = -f'(t_0)\delta(t - t_0)$
  - $\delta'(-t) = -\delta'(t)$

Differentiator

$$
\begin{aligned}
h(t) &= \delta'(t), \\
H(i2\pi f) &= ?.
\end{aligned}
$$

High-pass filter (Differentiator)

$$H(i2\pi f) = i2\pi f.$$



Filtering example - discrete difference

$$h[n] = \frac{1}{2}(\delta[n] - \delta[n-1]),$$
$$H(e^{i2\pi f}) = ?.$$

Discrete Difference

$$H(i2\pi f) \;=\; i\, e^{-i\pi f} \sin(\pi f).$$