ELE 530 March 28, 2011
Theory of Detection and Estimation Handout #8

Homework #4
Due April 4

Notice: Please bring your completed homework with you to your appointment.

1. *Rayleigh Samples.* Suppose Y_i are i.i.d. samples from $p_\theta(y)$, for $i = 1, ..., n$, where $\theta \in \Re$ is unknown, and the family of distributions $p_\theta(y)$ is the Rayleigh family given by

$$p_\theta(y) \quad = \quad \frac{y}{\theta} e^{-\frac{y^2}{2\theta}} u(y),$$

where $u(y)$ is the unit step function.

The distribution of the entire i.i.d. vector of samples $p_\theta(y^n)$ can be inferred from $p_\theta(y)$.

a. Is $p_\theta(y^n)$ an exponential family?

b. Find a complete sufficient statistic for θ.

c. What is the minimum variance unbiased estimator (MVUE) for θ?

d. Can the Cramer-Rao Bound be applied? If so, use it to obtain a lower bound on the variance of unbiased estimators for θ. Is there an efficient estimator? (An unbiased estimator is efficient if it meets the Cramer-Rao bound.)

e. What is the maximum likelihood estimator for θ.

2. *Uniform Samples.* Suppose Y_i are i.i.d. samples $\sim Unif[-\theta, \theta]$, for $i = 1, ..., n$, where $\theta > 0$ is unknown.

The distribution of the entire i.i.d. vector of samples $p_\theta(y^n)$ can be inferred from $p_\theta(y)$.

a. Is $p_\theta(y^n)$ an exponential family?

b. Find a scaler sufficient statistic for θ.

c. What is an unbiased estimator based on the sufficient statistic of part b?

d. Can the Cramer-Rao Bound be applied? If so, use it to obtain a lower bound on the variance of unbiased estimators for θ. Is there an efficient estimator? (An unbiased estimator is efficient if it meets the Cramer-Rao bound.)

e. What is the maximum likelihood estimator for θ.

3. *MMSE vs. MVU and ML.* Consider jointly Gaussian random variables X and Y with zero mean, unit variance, and correlation $\mathbf{E}XY = \rho$.

a. What is the MMSE estimate of X given Y?

b. Now treat X like a parameter (Ignore the marginal distribution on X and just consider the conditional distribution $p(y|x)$.). What is the minimum variance unbiased (MVU) estimate of X given Y, and what is the maximum likelihood (ML) estimate of X given Y?

4. *Efficiency of ML.* Show that if an efficient unbiased estimator exists then it is the maximum likelihood estimator. Consider the necessary and sufficient condition for an efficient estimator to exist, stated in Theorem 3.1 of Kay Vol. 1.