

$$= n I(M, Y_T | Y^{T-1}, Z_{T+1}^1, T) - n I(M, Z_T | Y^{T-1}, Z_{T-1}^1, T)$$

$$= n I(U; Y|V) - n I(U; Z|V)$$

$$\leq n \max_v I(U; Y|V=v) - I(U; Z|V=v)$$

Consider $P_{U \times Y|Z|V=v}$ satisfies the statement

Check: cond. on V

$U-X-YZ$

$P_{Y|Z|X}$ matches channel.

Project: (Presentation and Write-up: 2-10 pages)

10/27/2016
Thursday

Look ISIT 2016 Technical Program / Trans. on Inf. Theory (recent)

1) New inquiry

OR

2) Explore the history of analysis

3) Try their ideas on other problems

→ Lecture starts with slides on Soft Covering: Check the slides.

$$\text{We want } \frac{P_{v^n}}{Q_{v^n}} \approx 1 \Rightarrow \frac{1}{2^{nR}} \sum_m \frac{Q_{v^n|u^n}(v^n|U^n(m))}{\underbrace{Q_{v^n}(v^n)}_{Z_m}} \approx 1$$

$\uparrow$ iid average $\sim \pi Q_u$

We check the expected value:

$$\mathbb{E} Z_1 = \mathbb{E}_c \left[\frac{Q_{v^n|u^n}(v^n|U^n(1))}{Q_{v^n}(v^n)} \right] = \sum_{u^n} Q_{u^n}(u^n) \frac{Q_{v^n|u^n}(v^n|u^n)}{Q_{v^n}} = 1.$$

Focus on $(u^n, v^n) \in \mathcal{T}_\varepsilon^{(n)}$



$$\frac{Q_{v^n|u^n}(v^n|u^n)}{Q_{v^n}(v^n)} \in \left[2^{n(I(u;v) - \epsilon)}, \underbrace{2^{n(I(u;v) + \epsilon)}}_{\text{what we care about.}} \right]$$

Roughly speaking: Z_m is binary $\{0, 2^{nI}\}$

$$\text{Variance}(Z_m) = \frac{1}{2^{nI}} \left(1 - \frac{1}{2^{nI}} \right) \cdot 2^{n2I} = 2^{nI} - 1$$

$$\text{Variance}\left(\frac{1}{2^{nR}} \sum Z_m\right) \approx 2^{-n(R-I)}$$

$$\begin{aligned} P_1(v^n) &= \frac{1}{2^{nR}} \sum_m Q_{v^n|u^n}(v^n|U^n(m)) \mathbb{1}\{(U^n(m), v^n) \in \mathcal{T}_\epsilon^{(n)}\} \\ P_2(v^n) &= \frac{1}{2^{nR}} \text{ " " " " " } \notin \mathcal{T}_\epsilon^{(n)} \end{aligned} \quad \left| \quad \mathbb{E}_Q P_{v^n} = Q_{v^n} \right.$$

$$P_1 + P_2 = P_{v^n}$$

Total Variation Distance:

$$\|P_{v^n} - Q_{v^n}\|_{TV} = \frac{1}{2} \sum |P_{v^n}(v^n) - Q_{v^n}(v^n)|$$

$$\|P_{v^n} - Q_{v^n}\|_{TV} = \|P_1 + P_2 - \mathbb{E}P_1 - \mathbb{E}P_2\|_{TV}$$

$\uparrow$
 $\mathbb{E}P_{v^n}$

$$\leq \|P_1 - \mathbb{E}P_1\|_{TV} + \|P_2 - \mathbb{E}P_2\|_{TV}$$

$$= \underbrace{\frac{1}{2} \sum_{v^n \in \mathcal{T}_\epsilon^{(n)}} |P_1(v^n) - \mathbb{E}P_1(v^n)|}_{(*)} + \frac{1}{2} \sum_{v^n \notin \mathcal{T}_\epsilon^{(n)}} |P_2(v^n) - \mathbb{E}P_2(v^n)|$$

$$\forall v^n \in \mathcal{T}_\epsilon^{(n)}$$

$$\mathbb{E} |P_i(v^n) - \mathbb{E}[P_i(v^n)]| \leq \sqrt{\mathbb{E}[(P_i(v^n) - \mathbb{E}[P_i(v^n)])^2]} \quad \leftarrow \text{Jensen}$$

(what is random?
randomness is inside $P_i(v^n)$
it depends on the random codebook)

$$= \sqrt{\text{Var}(P_i(v^n))}$$

$$= \sqrt{2^{-nR} \text{Var}[Q_{v^n|u^n}(v^n|u^n(1)) \mathbb{1}\{(u^n(1), v^n) \in \mathcal{T}_\epsilon^{(n)}\}]}$$

$$\leq \sqrt{2^{-nR} \mathbb{E}[(Q_{v^n|u^n}(v^n|u^n(1)))^2 \mathbb{1}\{(u^n(1), v^n) \in \mathcal{T}_\epsilon^{(n)}\}]}$$

$$\leq \sqrt{2^{-nR} \sum_{u^n} Q_{u^n}(u^n) (Q_{v^n|u^n}(v^n|u^n))^2 \mathbb{1}\{(u^n(1), v^n) \in \mathcal{T}_\epsilon^{(n)}\}}$$

$$= \sqrt{2^{-nR} \sum_{(u^n, v^n) \in \mathcal{T}_\epsilon^{(n)}} Q_{u^n}(u^n) (Q_{v^n|u^n}(v^n|u^n))^2}$$

$$\leq \sqrt{2^{-nR} 2^{-n(H(v|u)-\epsilon)} \sum_{u^n} Q_{u^n v^n}(u^n, v^n)}$$

$$\leq \sqrt{2^{-n(R+H(v|u)-\epsilon)} Q_{v^n}(v^n)}$$

$$\leq \sqrt{2^{-n(R+H(v|u)+H(v)-2\epsilon)}}$$

u^n is typical (joint typ $\Rightarrow$ typ.)

since v^n is typical

* in prev. page satisfies:

$$(*) \leq \frac{1}{2} \cdot 2^{n(H(v)+\epsilon)} \cdot 2^{-\frac{1}{2}n(R+H(u|v)+H(v)-2\epsilon)}$$

$$= \frac{1}{2} \cdot 2^{-\frac{1}{2}n(R+H(u|v)-H(v)-4\epsilon)}$$

$$= \frac{1}{2} 2^{-\frac{1}{2}n(R-I-4\epsilon)}$$

$$\mathbb{E} \|P_2 - \mathbb{E} P_2\|_{TV} \leq \mathbb{E} \|P_2 - 0\|_{TV} + \mathbb{E} \|\mathbb{E} P_2 - 0\|$$

$$= 2 \cdot \frac{1}{2} \left(\sum_{v^n} \mathbb{E} P_2(v^n) \right) \mathbb{P}[(u^n, v^n) \notin \mathcal{T}_\epsilon^{(n)}]$$

Distances:

$D(P||Q) \leftarrow$ not symmetric, no triangle inequality

Total Variation

$$\|P-Q\|_{TV} = \max_A [P(A) - Q(A)] \quad \left(\begin{array}{l} \text{True distance metric} \\ \text{TV is } L_1 \text{ distance} \end{array} \right)$$

$$\begin{aligned} \text{Let } P \text{ and } Q \text{ be discrete: } \|P-Q\|_{TV} &= \frac{1}{2} \sum_x |P(x) - Q(x)| \\ &= \frac{1}{2} \|P-Q\|_1 \end{aligned}$$

Let P and Q be cont.

$$\|P-Q\|_{TV} = \frac{1}{2} \int |p(x) - q(x)| dx$$

$$\text{Note: } L_q \text{ distance } \|P-Q\|_q = \left(\sum_x |P(x) - Q(x)|^q \right)^{1/q}$$

L_1 is the most relevant for probabilities.