Cascade Multiterminal Source Coding

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Stanford University

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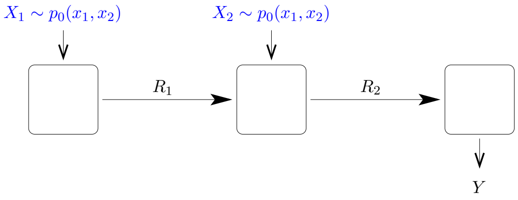
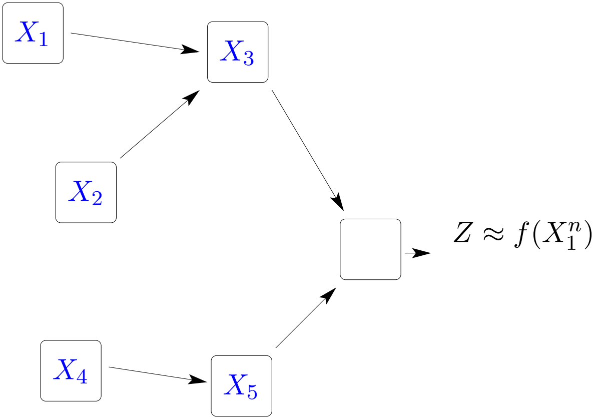


Distributed Source Coding

Aggregate information in a network:



Small sample of the large variety of related work:

- Sensor Networks [Giridhar, Kumar 05-06]
- Function Computation [Ayaso, Shah, & Dahleh 08]

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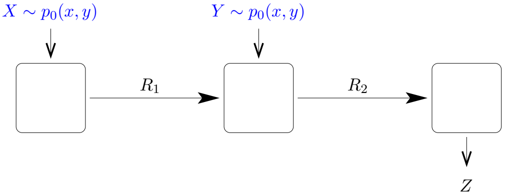

Related Investigations:

- [Vasudevan, Tian, Diggavi 06]
- [Gu, Effros 06]
- [Bakshi, Effros, Gu, Koetter 07]

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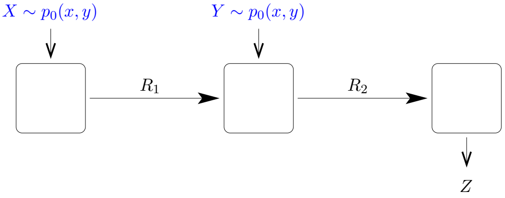

Encoding/Decoding:

$$
\begin{aligned}
i &: \mathcal{X}^n \longrightarrow \left[2^{nR_1}\right], \\
j &: \left[2^{nR_1}\right] \times \mathcal{Y}^n \longrightarrow \left[2^{nR_2}\right], \\
g &: \left[2^{nR_2}\right] \longrightarrow \mathcal{Z}^n.
\end{aligned}
$$

$$Z^n = g(j(i(X^n), Y^n)).$$

Objective:

$$\frac{1}{n}\sum_{i=1}^{n} d(X_i, Y_i, Z_i) \leq D.$$

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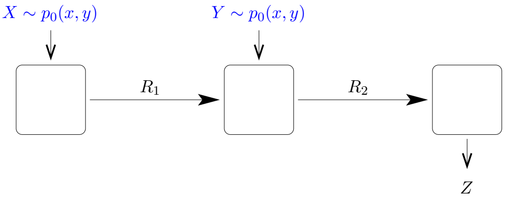
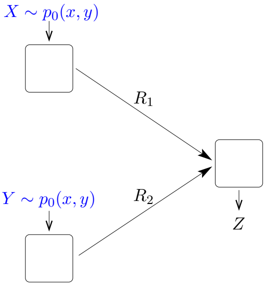
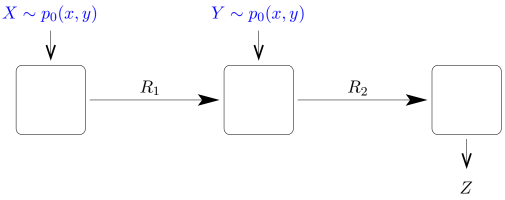


Multiterminal Source Coding:

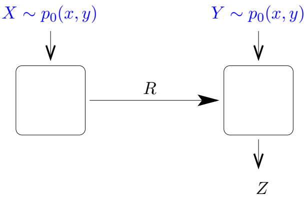


- Slepian-Wolf 73
- Berger-Tung 78
- Korner-Marton 79
- Wanger-Tavildar-Viswanath 08
- Krithivasan-Pradhan 08

Cascade Multiterminal Source Coding



Wyner-Ziv Source Coding:

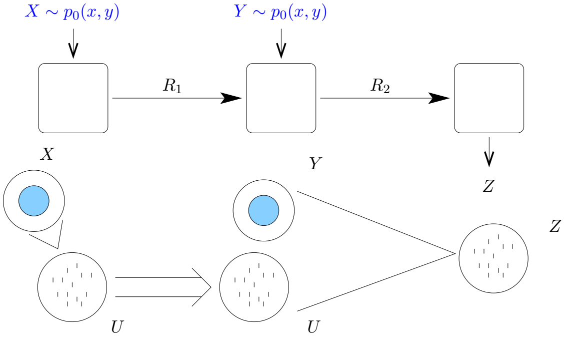


- Wyner-Ziv 76

$$\begin{aligned} U &- X - Y, \\ Z &= f(U,Y), \\ R &\geq I(X;U|Y). \end{aligned}$$

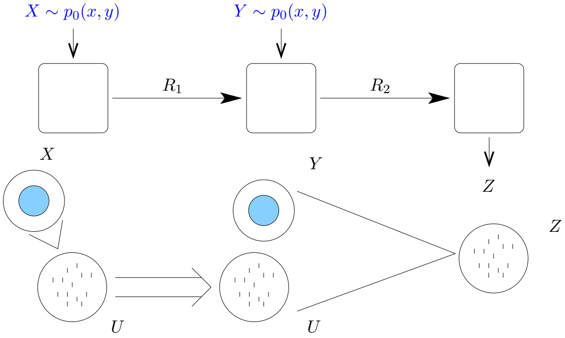
- Yamamoto 82
- Orlitsky-Roche 01

Idea 1: "Recompress"



$$
\begin{aligned}
U &- X - Y, \\
X &- (U,Y) - Z, \\
R_1 &\geq I(X;U|Y), \\
R_2 &\geq I(Y,U;Z).
\end{aligned}
$$

Idea 1: "Recompress"



- Tight when $Z = f(X,Y)$.

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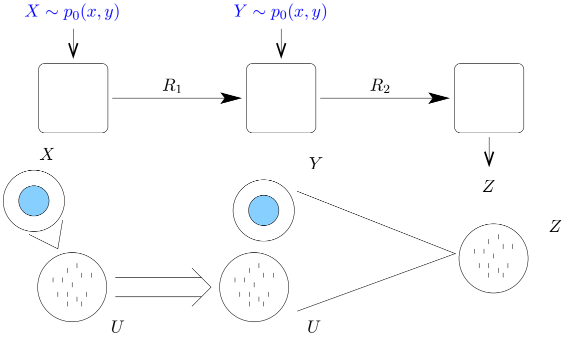

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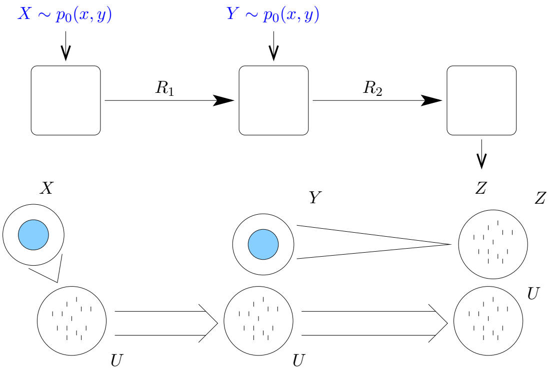
R_2 &\geq I(Y,U;Z).
\end{aligned}
$$

- Tight when $Z = f(X,Y)$.
- Loose when $Y = \emptyset$.

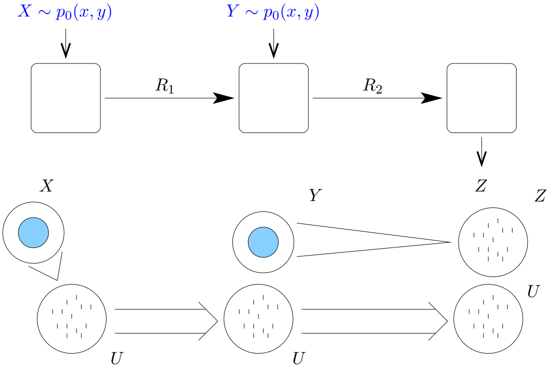
$$
\begin{aligned}
X &- U - Z, \\
R_1 &\geq I(X;U), \\
R_2 &\geq I(U;Z).
\end{aligned}
$$

Idea 2: "Forward"



$$
\begin{aligned}
U &- X - Y, \\
X &- (U,Y) - Z, \\
R_1 &\geq I(X;U|Y), \\
R_2 &\geq I(X;U) + I(Y;Z|U).
\end{aligned}
$$

Idea 2: "Forward"



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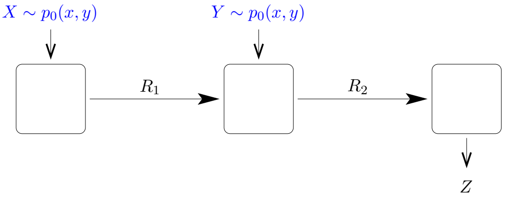
\end{aligned}
$$

- Tight when $Y = f(X)$. (Gu, Effros 06)

Theorem: Inner Bound

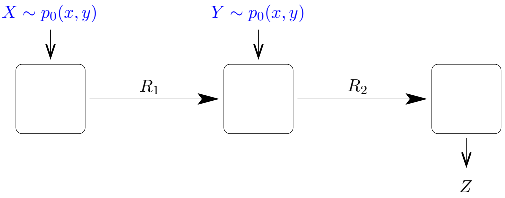


Theorem: Inner Bound

The triple (R_1, R_2, D) are achievable if there exist U, V, and Z such that,

$$
\begin{aligned}
Y &- X - (U,V), \\
X &- (U,V,Y) - Z, \\
D &> E(d(X,Y,Z)), \\
R_1 &> I(X;U,V|Y), \\
R_2 &> I(X;U) + I(Y,V;Z|U).
\end{aligned}
$$

Theorem: Outer Bound

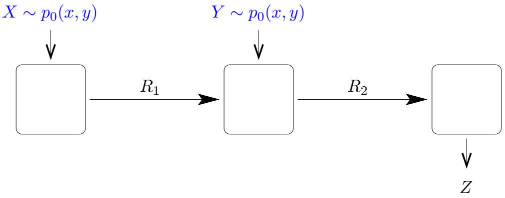


Theorem: Outer Bound

For any achievable triple (R_1, R_2, D) there must exist U, and Z such that,

$$
\begin{aligned}
Y \; &- \; X \; - \; U, \\
X \; &- \; (U,Y) \; - \; Z, \\
D \; &\geq \; E(d(X,Y,Z)), \\
R_1 \; &\geq \; I(X;U|Y), \\
R_2 \; &\geq \; I(X,Y;Z).
\end{aligned}
$$

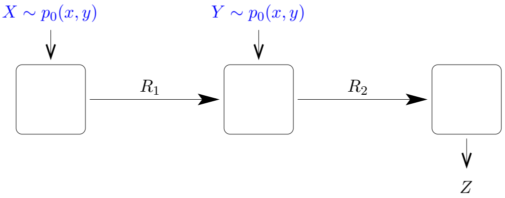
Sum of Jointly Gaussian Sources



Jointly Gaussian Sources:

$$
\begin{aligned}
X &\sim N(0, P_X), \\
Y &\sim N(0, P_Y), \\
E(XY) &= \rho\sqrt{P_X P_Y}, \\
d(x,y,z) &= ((x+y)-z)^2.
\end{aligned}
$$

Sum of Jointly Gaussian Sources



Jointly Gaussian Sources:

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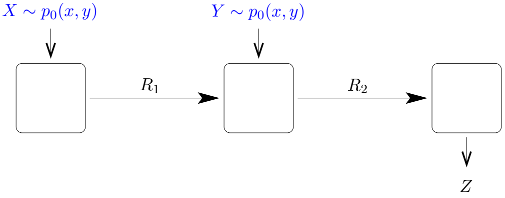
Recompress = compute average at intermediate node.
Forward = compute average at final destination.

Sum of Jointly Gaussian Sources



First Observation:

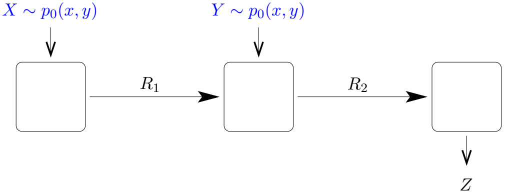
Forward if and only if

$$\frac{1}{2}\log_2 \frac{P_X}{P_Y} \quad \geq \quad R_1,$$

regardless of ρ.

(Notice that if $P_X \leq P_Y$ local computation is always better.)

Sum of Jointly Gaussian Sources



First Observation:

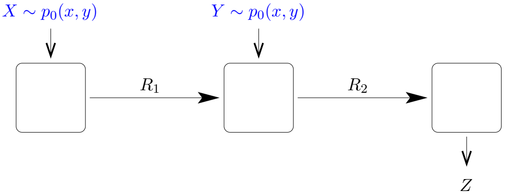
Forward if and only if

$$2^{-nR_1} P_X \quad \geq \quad P_Y,$$

regardless of ρ.

(Notice that if $P_X \leq P_Y$ local computation is always better.)

Sum of Jointly Gaussian Sources



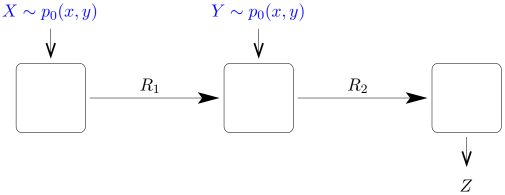
Second Observation:

Let $P_X = P_Y$,
let $R(D)$ be the required sum rate $R_1 + R_2$ for distortion D.

Low Distortion Regime:

$$R(D) \geq \frac{1}{2}\log_2\left(\frac{P_{X+Y}}{D}\right) + \frac{1}{2}\log_2\left(\frac{(1-\rho^2)P_X}{D}\right)$$

Sum of Jointly Gaussian Sources



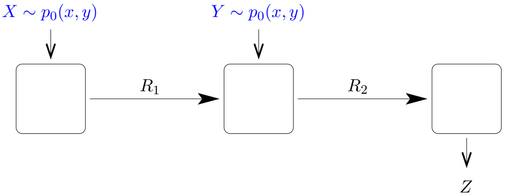
Second Observation:

Let $P_X = P_Y$,
let $R(D)$ be the required sum rate $R_1 + R_2$ for distortion D.

Low Distortion Regime:

$$R(D) \leq \frac{1}{2}\log_2\left(\frac{P_{X+Y}}{D}\right) + \frac{1}{2}\log_2\left(\frac{(1-\rho^2)P_X}{D}\right) + \log_2\left(1+\sqrt{1-\frac{D}{P_{X+Y}}}\right).$$

Sum of Jointly Gaussian Sources



Second Observation:

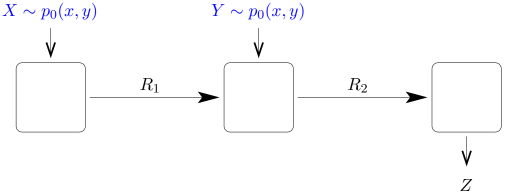
Let $P_X = P_Y$,
let $R(D)$ be the required sum rate $R_1 + R_2$ for distortion D.

High Distortion Regime:

$$R(D) \leq \frac{1}{2}\log_2\left(\frac{P_{X+Y} - (1-\rho^2)P_X}{D - (1-\rho^2)P_X}\right).$$

$$R(D) \geq \frac{1}{2}\log_2\left(\frac{P_{X+Y}}{D}\right).$$

Remarks



- Inner Bound: Two auxiliary variables.
  1. Forward U.
  2. Recompress V.
- Outer Bound derived from two cuts in network.
- Bounds are tight in some interesting cases
  - Tight when Y is function of X (for all distortion functions).
  - Tight when computing a function $Z = f(X,Y)$.
- Sum of Gaussian: Gap between bounds is less than 1 bit.