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AN EXPLICIT SOLUTION TO AN OPTIMAL TAX PROBLEM

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Under specific but perhaps not over-restrictive assumptions on social welfare and consumer preferences, an explicit closed-form solution for an optimal linear income tax is derived. Specifically, given linear income supply functions and a rank-order social welfare function, the optimal tax rate and benefit level are characterized by four parameters: I, a measure of pre-tax inequality in the ability (wage) distribution; r, the fraction of potential total income required for (non-redistributed) government revenue; σ, the fraction of potential total income required for consumer subsistence expenditures; and a disincentive parameter, δ, the marginal propensity to spend on leisure or the amount by which earned income is reduced in response to a unit increase in unearned benefit. Defining ϕ, the ratio $I/(1-\sigma-r)$, the optimal tax rate τ is given by:

$$\tau=\frac{1}{1-\delta}-\frac{\delta}{2\phi(1-\delta)^2}\left\{\left(1+\frac{4(1-\delta)\phi}{\delta}\right)^{1/2}-1\right\}.$$

The formula is used to fully characterize τ in terms of the parameters. Results include the following: $\tau=0$ if $I=0$; $\tau=1$ if $\delta=0$; τ is increasing in I, σ and r; τ may be increasing or decreasing in δ depending on the value of ϕ; when disincentive effects are large, τ becomes close to ϕ so that, in such economics, if σ and r are small, the optimal tax rate is equal to the measure of pre-tax inequality. Formulae for the deadweight loss associated with the tax are derived and some observations are offered on the empirical issues associated with the model.

1. Introduction

The theoretical literature on optimal taxation is largely concerned with the derivation of formulae linking taxes, prices and quantities at the (second best) general equilibrium optimum. Such formulae often reveal a great deal about the optimal tax solution, for example whether progressive taxes are desirable or whether it is necessary to have discriminatory as opposed to uniform commodity taxation. However, except in very simple cases, the formulae do not yield explicit closed form solutions which relate tax rates to the ultimate parameters of the problem, that is to preferences, social welfare, distribution,

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and government revenue requirements. Atkinson (1973) [see also Atkinson and Stiglitz (1980, pp. 398–405)] has provided an illustrative model which yields an optimal tax rate equal to the degree of perceived pre-tax inequality; this result depends on government net revenue being zero and on a labor supply function which requires that the sum of earned and transfer income be insensitive to changes in the latter. To obtain results in more general cases, authors such as Mirrlees (1971), Atkinson and Stiglitz (1972), Stern (1976), Deaton (1977) or Bronsard, Salvas-Bronsard and Delisle (1978), have solved numerically the system of non-linear equations generated by the first-order conditions for social welfare maximisation. It seems useful to supplement such calculations by an explicit solution formula, provided the assumptions required encompass a reasonable degree of generality. Such a closed form solution yields a complete description of the optimal tax structure and its relationship to the basic data of the economy. The provision of such a solution is the purpose of this paper. The formula I shall derive is a recognisable generalization of Atkinson's and preserves the close link between the optimal tax rate and the degree of inequality.

The problem I examine is the standard one in which there is a finite population of consumers differing only in the wage rate each faces. The government sets a proportional tax on earnings using the revenue to finance a uniform lump-sum benefit together with some government expenditure. I consider only briefly the possibility that discriminatory commodity taxation may also be used. Government expenditure is not returned to consumers; it can perhaps be thought of as the administrative expense associated with the redistribution. The maximand is the social welfare function

$$W = V(u^1, u^2, \ldots, u^H), \tag{1}$$

where $u^1, u^2, \ldots, u^H$ are the utility levels of the H households (individuals) in the economy. Individual h earns pre-tax wage p_0^h, post-tax wage $w = (1-\tau)p_0^h$ for tax rate τ, pays fixed prices p for a vector of n goods, and receives from the government an untaxed lump-sum benefit μ. In this one-period static framework, all other 'unearned' income can be thought of as having been completely taxed away. Consumer preferences are represented by the full cost function:

$$c(u^h, p_0^h(1-\tau), p) = \mu + p_0^h(1-\tau)T, \tag{2}$$

where T is the one-period endowment of time. The government maximizes social welfare (1) by choosing τ and μ subject to preferences (2) and a revenue constraint:

$$\tau \sum p_0^h(T - q_0^h) = H\mu + R, \tag{3}$$

where q_0^h is hours of leisure, so that $(T-q_0^h)$ is hours worked, and R is the total government revenue requirement. The Lagrangian for the problem may be written:

$$L=V(u^1,u^2,\ldots,u^H)+\sum\lambda^h\{\mu+p_0^h(1-\tau)T-c(u^h,p_0^h(1-\tau),p)\}$$
$$+\xi\left\{\tau\sum_h p_0^h(T-q_0^h)-H\mu-R\right\}, \tag{4}$$

which is maximized with respect to $u^1,\ldots,u^H$, τ and μ. The standard first-order conditions are:

$$\frac{\partial V}{\partial u^h}-\lambda^h\frac{\partial c}{\partial u^h}-\xi\tau p_0^h\frac{\partial q_0^h}{\partial u}=0, \qquad h=1,\ldots,H, \tag{5}$$

$$\sum\lambda^h-H\xi=0, \tag{6}$$

$$-\sum\lambda^h p_0^h(T-q_0^h)+\xi\sum p_0^h(T-q_0^h)+\xi\tau\sum(p_0^h)^2 s_{00}^h=0, \tag{7}$$

where s_{00}^h is the compensated derivative of leisure with respect to the wage.

Manipulation of these first-order conditions in various ways has occupied a good deal of space in the journals. In the next section, I specify forms for social and private preferences which allow an explicit solution.

2. Preferences, social welfare, and a solution

2.1. Consumer preferences

One of the reasons that the first-order conditions are difficult to solve is the presence of a large number of consumers. However, if perfect aggregation assumptions are made, it turns out that the H wage rates p_0^h can be replaced by two: a mean, and a 'socially representative' wage. The essential requirement is that income supply functions be linear in the wage. Preferences necessary for this are derived in Muellbauer (1981) and in Deaton and Muellbauer (1981) and are conveniently represented by the full cost function:

$$c(u,w,p)=\gamma(p)+\gamma_0(p)w+w^{\delta(p)}\{a(p)\}^{1-\delta(p)}u, \tag{8}$$

where, as before, w denotes the post-tax wage $p_0(1-\tau)$, $\gamma(p)$ and $a(p)$ are linearly homogeneous in p, and $\gamma_0(p)$ and $\delta(p)$ are zero degree homogeneous in p. Associated with (8) are:

an indirect utility function

$$u = \frac{\mu + w\{T - \gamma_0(p)\} - \gamma(p)}{w^{\delta(p)}\{a(p)\}^{1-\delta(p)}}; \tag{9}$$

a labor supply function

$$(T - q_0) = (T - \gamma_0)(1 - \delta) - \frac{\delta}{w}(\mu - \gamma); \tag{10}$$

and substitution term

$$s_{00} = -\frac{\delta(1-\delta)}{w^2}\{\mu + w(T - \gamma_0) - \gamma\}. \tag{11}$$

The labor supply function (10) has to be supplemented by inequalities on the net wage w in order to guarantee first that $(T - q_0) \geqq 0$, and secondly that $T - q_0 \leqq T - \gamma_0$. The latter is necessary to ensure that leisure is at least as large as minimum required leisure γ_0, or more fundamentally, to ensure the concavity of the cost function (8). The two inequalities are:

$$w \geqq \left(\frac{\delta}{1-\delta}\right)\frac{(\mu - \gamma)}{(T - \gamma_0)}, \tag{12}$$

$$w \geqq -(\mu - \gamma)/(T - \gamma_0). \tag{13}$$

These inequalities must hold for all individuals if the solutions derived below are to be valid.

The interpretation of these formulae is straightforward. The expression $\gamma(p)$ is subsistence expenditure on goods, while $\gamma_0(p)$ is the subsistence quantity of leisure. The fraction $\delta(p)$, $0 \leqq \delta(p) < 1$, measures preference for leisure and, from (10), is the marginal propensity to spend on leisure or the fraction by which earned income is reduced if unearned income is increased by one unit. The labor supply function (10) gives labor income as a linear function of μ and w and is reasonably general. However, linearity in this context has the restrictive implication that both leisure/unearned income response and substitution response are determined by the single quantity $\delta(p)$. Hence, $\delta(p)$ not only measures the disincentive effects on labor supply of increasing benefits, but $(\delta - 1)$ is the compensated wage elasticity of over-subsistence leisure, $q_0 - \gamma$. As will be seen, a single quantity summarizing all disincentive effects is extremely convenient. It is also the single most restrictive assumption required to derive the results.

The preferences given by (8) contain a number of important special cases. The first occurs when $\delta(p)$ and $\gamma_0(p)$ are constants, δ and γ_0 say, in which case leisure and goods are weakly separable and the conditional demand functions for goods are linear in income $w(T-q_0)+\mu$. These are precisely the conditions shown by Deaton (1979) to be sufficient for the optimality of uniform commodity taxation. Hence, if δ and γ_0 are treated as constants, the linear income tax considered here is optimal in the wider context of possible discriminatory commodity taxes. The cost function (8) also contains as special cases other preferences that have been widely used in the tax literature. For example, with $\gamma=\gamma_0=0$, (8) represents Cobb–Douglas preferences and (10) the associated labor supply function, as used, for example, by Mirrlees (1971). Of greater interest for this paper is the case where $\gamma=0$, $\delta\to 1$ and $(T-\gamma_0)(1-\delta)\to\alpha\neq 0$; this yields the labor supply function:

$$w(T-q_0)=\alpha w-\mu, \tag{14}$$

which is used to obtain an explicit tax solution by Atkinson (1973).

2.2. Social welfare

I work entirely with rank order social welfare functions, i.e. (1) takes the form:

$$W=\sum_h g\{\rho(u^h)\}u^h, \tag{15}$$

where u^h is individual utility as given by (9), and $\rho(u^h)$ is the *rank* of h in the utility distribution (best-off comes first). Such social welfare functions are perhaps relatively unattractive as compared with differentiable additive formulations such as $\sum f(u^h)$, say. However, as the number of consumers becomes large, approaching a continuum, (15) can approximate any additive function of utilities through a suitable choice of the function g. The great convenience of (15) lies in the fact that the ranks $\rho(u^h)$ are *independent of changes in the tax structure* and so can be treated as constants in the analysis. This constancy is a consequence of the fact that consumers differ only in a single characteristic, p_0^h, so that although tax changes can alter welfare levels and the welfare differences between individuals, higher wage individuals will always be at least as well off as lower wage individuals. Note too that, since the ranking never changes, non-differentiabilities in the social welfare function are never encountered.

Respect for equality of utilities requires g to be an increasing function of the ranks. The normalization of utility embodied in (9) corresponds to the usual 'money-metric' form although here, with endogenous labor supply and wages varying over individuals, the marginal (private) utility of money is lower for more able individuals because they face a higher price for leisure. The marginal social utility of money is, of course, a declining function of p_0^h.

There are two special cases of (15) which are worthy of special mention. These are the polar cases where the social welfare function is (i) *utilitarian*, where $g\{\cdot\}=$constant and the social welfare function is simply the sum of individual utilities, and (ii) *Rawlsian*, where $g\{\rho(u^h)\}=0$ if $\rho<H$, and $g(H)=1$, so that society is concerned only with the welfare of the poorest individual.

2.3. *The solution*

This subsection assembles the parts provided by social and private preferences into a solution of the optimal tax first-order conditions. The solution itself is analysed in section 3.

Eq. (6) yields immediately that $\xi=\bar{\lambda}$ so that ξ can be eliminated from both (5) and (7). The former yields:

$$\psi^h=\lambda^h+\bar{\lambda}\tau\,\delta/(1-\tau), \tag{16}$$

where $\psi^h=\partial V/\partial u^h\div\partial c/\partial u^h$ which, by (15) and (8) is given by:

$$\psi^h=g\{\rho(u^h)\}\{p_0^h(1-\tau)\}^{-\delta}a^{\delta-1}, \tag{17}$$

and I have used $\partial q_0^h/\partial u^h\div\partial c/\partial u^h\equiv\partial q_0^h/\partial\mu=\delta/p_0^h(1-\tau)$ by (10). Taking averages of (16) gives:

$$\bar{\psi}=\bar{\lambda}\{1+\tau\delta/(1-\tau)\}. \tag{18}$$

Hence, combining (16), (17) and (18):

$$\frac{\lambda^h}{\sum\lambda^h}=\left(1+\frac{\tau\delta}{1-\tau}\right)\theta^h-\frac{\tau\delta}{H(1-\tau)}, \tag{19}$$

where

$$\theta^h=(p_0^h)^{-\delta}g\{\rho(u^h)\}\Big/\sum_h(p_0^h)^{-\delta}g\{\rho(u^h)\}. \tag{20}$$

Eqs. (19) and (20) give the distributional weights in the form required. Note that θ^h sums over h to unity; otherwise it is the (normalised) marginal social utility of money to individual h. It differs from the utility weights $g\{\rho(u^h)\}$ because of the variation in the cost of leisure with p_0^h. λ^h is the marginal social value of giving an extra unit of currency to h; it differs from θ^h because of the social value of the extra government revenue resulting from the transfer.

Eq. (7), after substitution of $\bar{\lambda}$ for ξ, can be directly evaluated using the labor supply eq. (10), the substitution response (11) and the distributional weights (19). The linearity of $p_0^h(T-q_0^h)$ and $(p_0^h)^2 s_{00}^h$ in p_0^h allows perfect aggregation in terms of a representative consumer with average wage $\bar{p}_0$. Hence, after substitution, (7) becomes:

$$\frac{\tau\delta(1-\delta)}{(1-\tau)^2}\{(\mu-\gamma)+(1-\tau)\bar{p}_0(T-\gamma_0)\}=\left\{1+\frac{\tau\delta}{1-\tau}\right\}(1-\delta)(T-\gamma_0)(\bar{p}_0-\tilde{p}_0), \tag{21}$$

where $\tilde{p}_0$ is the θ^h weighted sum of the p_0^h, i.e.

$$\tilde{p}_0=\sum\theta^h p_0^h. \tag{22}$$

The degree of pre-tax inequality in the distribution of p_0^h can conveniently be measured by the fraction by which $\tilde{p}_0$ falls short of $\bar{p}_0$, i.e.

$$I=(\bar{p}_0-\tilde{p}_0)/\bar{p}_0. \tag{23}$$

Clearly, I depends both on perceptions of inequality, i.e. the weight $g\{\rho(u^h)\}$, and the objective distribution of the p_0^h. As an example, if g is chosen such that the marginal social utility of money is proportional to $2\rho(u^h)-1$ (this requires the introduction of p_0^h into the social welfare function, but otherwise leaves the analysis unaffected), then I is the Gini coefficient for the inequality of the p_0^h. Divide (21) by $\bar{p}_0(T-\gamma_0)$, maximum potential average earnings, to give:

$$\frac{\tau\delta(1-\delta)}{(1-\tau)^2}\{(b-\sigma)+(1-\tau)\}=\left\{1+\frac{\tau\delta}{1-\tau}\right\}(1-\delta)I, \tag{24}$$

where b is the benefit as a fraction of maximum potential earnings, i.e.

$$b=\mu/\bar{p}_0(T-\gamma_0), \tag{25}$$

and σ is subsistence also as a fraction of maximum potential earnings:

$$\sigma=\gamma/\bar{p}_0(T-\gamma_0). \tag{26}$$

Eq. (24) gives one relation between the two instruments b and τ; the other is given by the government budget constraint. Substituting (10) in (3) gives after rearrangement:

$$b-\sigma=\{\tau(1-\delta)-r-\sigma\}/\{1+\delta\tau/(1-\tau)\}, \tag{27}$$

where r is government revenue per head also as a fraction of maximum potential average earnings, i.e.

$$r=R/H\bar{p}_0(T-\gamma_0). \tag{28}$$

Combining (24) and (27) gives:

$$I\{1-(1-\delta)\tau\}^2=\tau\delta(1-r-\sigma), \tag{29}$$

which is quadratic in τ with a single solution between 0 and 1 given by

$$\tau=\frac{1}{1-\delta}-\frac{\delta}{2(1-\delta)^2\phi}\left\{\left(1+\frac{4(1-\delta)\phi}{\delta}\right)^{1/2}-1\right\}, \tag{30}$$

where

$$\phi=I/(1-r-\sigma), \tag{31}$$

and this is the explicit solution for τ. The quantity ϕ is the relevant or 'corrected' inequality measure; only $(1-r-\sigma)$ of GDP is available for redistribution and I must be scaled up to reflect this.

In the foregoing derivation, I have freely divided by $(1-\tau)$ on the implicit assumption that $\tau\neq 1$ at the optimum. In fact, it is easy to show that $\tau=1$ satisfies all the relevant conditions, i.e. first-order conditions and the government budget constraint. However, consideration of (27) shows that $b\to\sigma$ as $\tau\to 1$, in which circumstance each individual u^h equals zero. This can be shown to be less than the utilities arising from τ and the associated b arising from (30) provided $0\leqq\tau<1$. (30) gives $\tau<1$ if $\delta\phi<1$ (provided $\delta\neq 0$) so that given $\delta\phi<1$, this solution dominates $\tau=1$. For $\delta\phi\geqq 1$, $\tau=1$ is optimal. Note finally that for $\delta=1$ (which implies a marginal propensity to consume of zero!) the first-order conditions, i.e. (24), are satisfied by all values of τ and b so that there are multiple solutions corresponding only to the government budget constraint.

It is also necessary to allow for the inequalities (12) and (13) to ensure that the solution derived does not require any individual's hours of work to be either negative or greater than the maximum possible. Clearly, if the inequalities hold for the lowest ability individual, they will hold for everyone. Writing m for the ratio of the minimum p_0^h to be average p_0^h (note that in the

Rawlsian case $I=1-m$), and using (25) and (26), the inequalities become:

$$1-\tau \geqq m(b-\sigma)\delta/(1-\delta) \tag{32}$$

and

$$1-\tau \geqq -m(b-\sigma). \tag{33}$$

Fig. 1 illustrates the circumstances under which (32) and (33) permit a solution of the form (29) or (30). The line AB is the locus of b and τ for which (32) is an equality; along AB, labor supply is zero for the least able individual. Similarly, along CB, the same individual is working at maximum capacity. Hence, it is only (b,τ) combinations within the triangle ABC which permit explicit solutions of the type derived. Note that the base of the triangle AB depends directly on m and will be smaller the less able is the poorest individual; ultimately, if the poorest earns nothing, ABC collapses to GB and the only legitimate solutions are those for which $b=\sigma$. A solution of

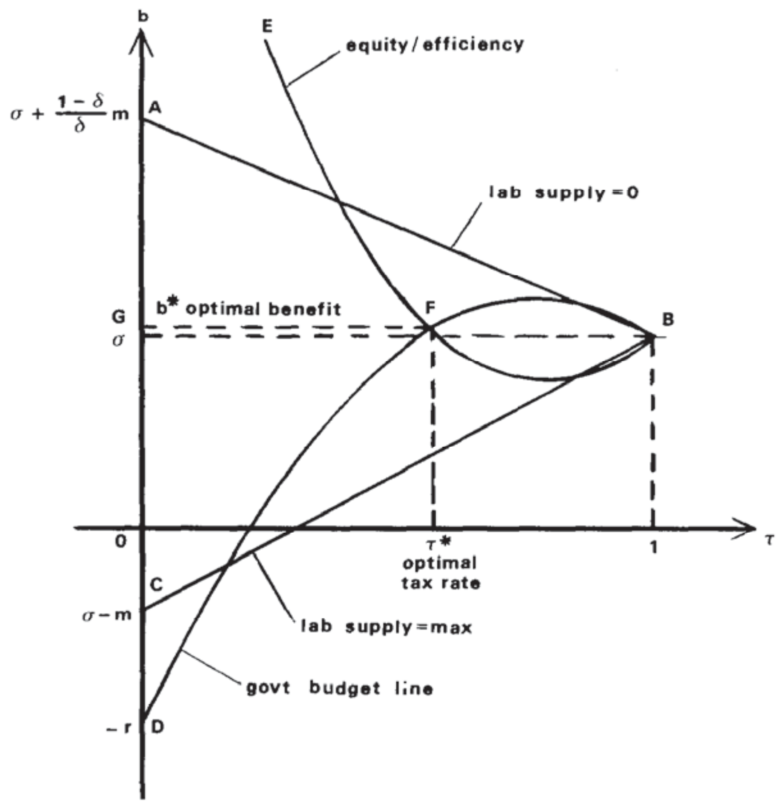

Fig. 1. Determination of optimal tax and benefit.

the normal type is illustrated in the figure. The curve DB is the (b,τ) locus which satisfies the government budget constraint (27). This must be positively sloped for small τ — an increase in the tax rate allows greater benefits — but *may* become negatively sloped before $\tau = 1$. The figure shows such a case and this will always occur if $(1-r-\sigma) > \delta$. The curve EB is the graph of eq. (24); if EB cuts DB within the triangle ABC, the solution is of the type described here. Note that EB rotates clockwise around B as I increases, and since DB is independent of I, this increases τ^*. As illustrated, the point of intersection P yields an optimal benefit greater than subsistence. This is not a general result; b will only be greater than σ if inequality considerations are sufficiently powerful to outweigh disincentive effects, otherwise F may lie below GB; see proposition 6 below.

3. The characterization of the solution

Summarizing the previous section, the optimal tax rate is given by, provided both $\delta\phi < 1$ and the inequalities (32) and (33) are satisfied:

$$\tau = \frac{1}{1-\delta} - \frac{\delta}{2(1-\delta)^2\phi}\left\{\left(1 + \frac{4(1-\delta)\phi}{\delta}\right)^{1/2} - 1\right\}, \tag{34}$$

where $\phi = I/(1-r-\sigma)$. When $\delta\phi \geqq 1$, $\tau = 1$. The implications are summarized in the following set of propositions, which are either obvious or can be derived by elementary manipulation of the formulae.

Proposition 1. When inequality is zero, the optimal tax rate is zero.

Zero inequality gives $I = \phi = 0$ which, from (29), gives $\tau = 0$. Note that, since the pre- and post-tax orderings of welfare are identical, zero post-tax inequality occurs if and only if there is no pre-tax inequality (unless $\delta = 0$, see proposition 2 below). Government revenue is raised as necessary through the benefit level, i.e. $b = -r$.

Proposition 2. When there are no disincentive effects ($\delta = 0$), the optimal tax rate is unity, except when $I = 0$ in which case it is indeterminate. With I positive, all income is taxed and used to pay the same benefit to all. There is no deadweight loss.

Proposition 3. In general, the optimal tax rate is a function of only two parameters, δ and ϕ. Hence, for I, r and σ satisfying $I + k(\sigma + r) = k$, some positive k, only k affects the optimal tax rate. There is thus a linear trade-off between inequality on the one hand and subsistence and/or government revenue *on the other hand. Note that the benefit level b depends on* δ, I, σ *and* r *separately (although* $b-\sigma$ *depends only on* δ, I *and* $1-\sigma-r$).

Proposition 4. *The optimal tax rate is increasing in* I, σ *and* r *independently of the values of the other parameters. For* $0 \leqq \phi \leqq \frac{1}{2}$, *the optimal tax rate is also decreasing in* δ. *For* $\phi > \frac{1}{2}$, $\mathrm{d}\tau/\mathrm{d}\delta$ *is also non-positive unless* $(4\phi-1)^{-1} < \delta < 1$, *in which case* $\mathrm{d}\tau/\mathrm{d}\delta$ *is positive. As* δ *tends to unity, the optimal tax rate tends to* ϕ; *note that* $\phi = I$ *if* $\sigma = r = 0$ *and this is essentially Atkinson's result corresponding to the labor supply function* (14). *The general shape of taxes in response to* σ *and* ϕ *is summarized in fig.* 2.

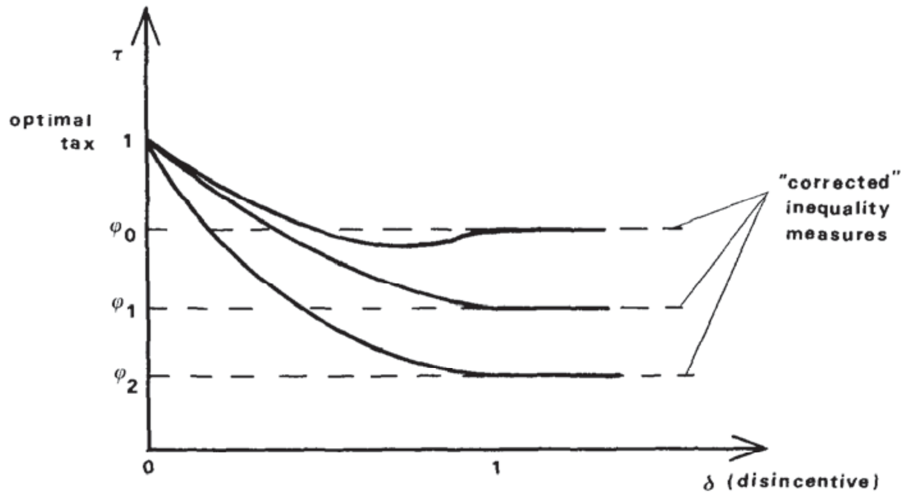

Fig. 2. Tax rates, inequality and disincentives.

Proposition 5. *Holding the benefit level at its optimum value, changes in* τ *from the optimum can either increase or decrease gross government revenue, depending on the values of the parameters. There is no presumption that* τ *should be set so as to maximize revenue, whatever such a statement may mean.*

Proposition 6. *The benefit level b may be greater than or less than the subsidence level* σ. *It is straightforward to show that b is* $>$ *or* $<$ σ *as* $\delta/\{\delta + (1-\delta)I\}$ *is* $<$ *or* $>$ $(1-r-\sigma)$. *Hence, b is likely to be less than* σ *if* δ *is high relative to I or if revenue and subsistence requirements are high, and vice versa.*

Proposition 7. *Finally, it is perhaps worth noting that the model here has assumptions consistent with Broome's* (1975) *'Important theorem on income tax'. In effect Broome assumes* $\delta = \frac{1}{2}$ *and* $\phi = 2-\sqrt{2}$; *application of the formula* (34) *gives* $\tau = 2-\sqrt{2}$, *which is Broome's result.*

4. Measuring deadweight loss

The preferences used in this paper permit straightforward evaluation of the deadweight losses associated with any tax/benefit policy, whether optimal or not. It is perhaps necessary to clarify what is meant by deadweight loss in the context of an optimal tax. As a positive exercise, the individual compensating and equivalent variations can always be evaluated and their sums compared with actual revenue raised. The difference is a measure of the potential product lost because of the inability to levy optimal lump-sum taxes. Although this potential product is not realisable, it can be regarded as a measure of the cost of the egalitarianism embodied in the social welfare function. A utilitarian GDP maximizer would regard the deadweight loss associated with an egalitarian tax scheme as an *actual* loss and even a dedicated egalitarian may be be given pause by the costs of his beliefs in terms of product forgone.

Consider any tax scheme with rate τ and benefit level μ; for the moment there is no presumption of optimality, although the government budget must balance. Pre- and post-tax utilities for individual h, u_0^h and u_1^h, are given by:

$$u_0^h = \{-\gamma + p_0^h(T-\gamma_0)\}/(p_0^h)^\delta a^{1-\delta}, \tag{35}$$

$$u_1^h = \{\mu - \gamma + p_0^h(1-\tau)(T-\gamma_0)\}/(p_0^h(1-\tau))^\delta a^{1-\delta}. \tag{36}$$

The equivalent variation for h, z^h, say, is defined by:

$$\frac{(\mu-\gamma)+p_0^h(1-\tau)(T-\gamma_0)}{(p_0^h)^\delta(1-\tau)^\delta a^{1-\delta}} = \frac{-z^h-\gamma+p_0^h(T-\gamma_0)}{(p_0^h)^\delta a^{1-\delta}}. \tag{37}$$

Note that z^h is the amount which could theoretically be levied by the government if lump-sum taxation were possible and which would yield h the same utility as does the tax. Rearranging (37), averaging and dividing by $\bar{p}_0(T-\gamma)$, gives:

$$z^* - r = (1-\sigma-r) - (1-\tau)^{1-\delta}\left\{\frac{b-\sigma}{1-\tau}+1\right\}, \tag{38}$$

where

$$z^* = \bar{z}/\bar{p}_0(T-\gamma_0). \tag{39}$$

Substituting from the government budget constraint (27) gives finally:

$$z^* - r = (1-\sigma-r)\left\{1 - \frac{(1-\tau)^{1-\delta}}{1-\tau(1-\delta)}\right\}. \tag{40}$$

The left-hand side of (40), the average equivalent variation as a fraction of potential GDP *less* the revenue share, is the share of potential output lost through the inability to levy lump-sum taxes. For an optimal tax–benefit combination, it is the cost in output terms of the egalitarian 'prejudice' which causes τ to differ from zero. (Note that similar calculations can be done using the compensating variation; in this case the formulae are more straightforward using the EV.)

Note that, as must be the case, $(z^* - r)$ is always positive. It is zero when the tax rate is zero or when disincentive effects are absent ($\delta = 0$); it increases with increases in both δ and τ. Its maximum value is $(1 - \sigma - r)$, attained when $\tau = 1$; all non-committed output is lost and consumers are at subsistence. There is nothing to stop this drear situation being optimal if inequality aversion is sufficiently powerful.

5. Empirical considerations

The models discussed above are so abstract that to introduce any element of reality is presumptuous. Nevertheless, labor supply models of the form used here have been estimated both for the United States and for Britain. The two studies by Abbott and Ashenfelter (1976, 1979) and by Phlips (1978) agree on a value of δ at approximately 0.12 for the United States. For Britain, the Family Expenditure System based studies of Atkinson and Stern (1981a, 1981b) and Deaton (1982) give a value for δ of essentially zero; by this token British taxes may well be too low. It should be noted, however, that the results for both countries are subject to very serious doubt, since it is far from clear that the assumptions embodied in the estimation are correct. Even if labor supply functions such as (10) are correct in the long run, labor markets may not clear in the short run and many consumers may not have short-run control over variations in their hours of work. This may be one explanation for the very low estimates of $(T - \gamma_0)$ in all these studies; see also Ashenfelter (1980) for an attempt to explicitly test this possibility.

As for inequality, the Gini is so frequently used that it can serve as a useful base here. At a guess, the coefficient associated with the distribution of abilities (p_0^h) might be close to 0.2. Hence, applying the formulae blindly, and taking $\delta = 0.12$, then if non-redistributed government revenue and consumer Subsidence expenditure together account for 20 percent of potential GDP, $\phi = \frac{1}{4}$ and the optimal tax rate is 0.55 with an associated benefit level (as a proportion of potential per capita GDP) of 0.25 *plus* subsistence. The deadweight loss associated with this scheme is 3.2 percent of potential GDP. To give an idea of sensitivity, the tax rate can be linearized around $\phi = \frac{1}{4}$ to give

$$\tau \simeq 0.55 + 0.765\,(\phi - \tfrac{1}{4}). \tag{41}$$

Hence, if the Gini were 0.3 rather than 0.2, τ is 0.65 rather than 0.55. Deadweight loss may also be approximated in these ranges by

$$(z^* - r) \simeq \tfrac{1}{2}\,\delta(1-\delta)\tau^2(1-\sigma-r). \tag{42}$$

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