BULLETIN *of the* OXFORD UNIVERSITY INSTITUTE *of* ECONOMICS *and* STATISTICS

Volume 33 *May 1971* *No. 2*

ECONOMETRIC MODELS FOR THE PERSONAL SECTOR*

A. S. DEATON AND K. J. WIGLEY

1. *Introduction*

This paper discusses the characteristics of three alternative econometric systems designed to explain, in terms of prices and incomes alone, saving and the components of consumers' expenditure. A number of fundamental issues are raised by this process, and it is these we wish to discuss.

For some time improvements have been sought in the expenditure system employed in the Cambridge Growth Model; most experience to date has been with the linear expenditure system and this forms a starting point for this paper. More recently one of the authors found this system unsatisfactory for use in a fuel sub-model and subsequently employed a non-linear or 'direct addilog' model as an alternative. Finally, attempts to include savings behaviour within this type of framework led to the extension of the linear expenditure system to incorporate the concept of permanent income. Thus each of the systems presented has been considered in response to problems concerning different aspects of consumers' behaviour; yet they have much in common. Each can be derived from the constrained maximization of a directly additive utility function; this alone implies that certain restrictions among the price and income elasticities are common to all three models [4]. Even given these common factors, the differences between the systems are as notable as their similarities; alternative models imply very different responses to the same economic stimuli, yet by many tests these alternative systems are equally satisfactory.

Here, then, are the essential problems faced in this paper: given several sets of data on consumers' expenditure, income and prices, to what extent is it possible to deduce information about (*a*) consumers' preferences, if any, (*b*) consumers' responses to changes in prices, relative prices, and income, and (*c*) consumers' behaviour faced with some heretofore unseen configuration of income and prices. The three models have been fitted to alternative sets of data in an attempt to throw some light on these three fundamental issues.

*An earlier version of this paper was presented to the Second World Congress of the Econometric Society in Cambridge, September 8th–14th, 1970.

2. *Notation*

The following symbols are used throughout the paper:

x_i = real consumption of good i (i=1, ... n)

p_i = price of good i (price index) (i=1, ... n)

e_i = $p_i x_i$ = expenditure on good i (i=1, ... n)

y = consumers' money disposable income

p = overall consumers price index = implicit price deflator of consumers' expenditure (excluding durable good purchases)
= $\Sigma p_i x_i / \Sigma p_i^o x_i$, where p_i^o are base period prices

z = real permanent income

e = $\Sigma p_i x_i$ = total consumers expenditure

$$n_i = \frac{\partial x_i}{\partial e}\frac{e}{x_i} \text{ or } \frac{\partial x_i}{\partial y}\frac{y}{x_i} = \text{total expenditure or income elasticity of good i}$$

$$n_{ij} = \frac{\partial x_i}{\partial p_j}\frac{p_j}{x_i} = \text{uncompensated price elasticity of good i with respect to price of good j}$$

$$n^*_{ij} = \left(\frac{\partial x_i}{\partial p_j}\frac{p_j}{x_i}\right)_{u=\text{constant}} = n_{ij} + \frac{e_j}{e} n_i = \text{compensated elasticity as above}$$

E^{-1} and Δ^* = backward shift operator and backward first difference operators respectively.

The time suffix has been omitted for simplicity of presentation and should be understood wherever appropriate.

3. *The Linear Expenditure System (LES)*

There are two ways of setting up a system of consumer demand equations. We may state a utility function and derive from it explicit or implicit demand functions. Alternatively we may postulate a system of demand equations which are a priori reasonable and proceed directly from there. Because we are dealing with *aggregate* and not *individual* demand functions, the two cannot in general be shown to be equivalent. It follows that we cannot necessarily reject a set of empirically determined demand functions because they imply an unsatisfactory formulation of utility. In this paper we shall in each case make explicit the utility function 'underlying' the demand equations; we shall state the convexity conditions and explain their implications. Nevertheless, these convexity requirements must not be rigidly imposed in the aggregate and we shall see that greater flexibility can be allowed in the demand equations if we do not always do so. These points will be elaborated and illustrated as we proceed.

Our first system has been fully described elsewhere (e.g. [11] and [12]) and only a summary of the model and its properties are given here. The demand functions may be written

$$e_i = p_i c_i + b_i (e - \Sigma p_k c_k),\ \Sigma b_i = 1 \tag{1}$$

where c_i and b_i are constants. In [11], Stone has shown that the LES is the most general linear expenditure system (i.e. each expenditure is a linear function of income and all the prices) compatible with the conditions of additivity, homogeneity, and symmetry of the substitution matrix.

Alternatively we may derive the LES by maximizing the utility function

$$U = \Sigma\, b_k \log (x_k - c_k) \text{ subject to } \Sigma\, p_k x_k = e \tag{2}$$

Equation (1) states that there are two components to the expenditure on each commodity, the first independent of the level of total expenditure, the second proportional to the excess of total expenditure over the sum of the first components. In a more restrictive interpretation of the system the first components have been termed 'committed' expenditures and the second components proportional to 'supernumerary' expenditure. The utility function (2) represents a convex preference ordering if and only if $b_i > 0$ for all i; even so, the demand equations (1) seem reasonable even with some negative b's provided that expenditures on individual commodities remain positive.[1] It is easy to confirm that

$$n_i = \frac{b_i e}{e_i} \qquad\qquad n_{ij} = \delta_{ij}\left(\frac{c_i}{x_i} - 1\right) - \frac{b_i}{e_i} c_j p_j \tag{3}$$

and

$$n^*_{ij} = (b_i - \delta_{ij}) \frac{e_j}{e_i}\left(1 - \frac{c_j}{x_j}\right) \qquad \text{where } \delta_{ij} \text{ is the Kronecker delta.}$$

If convexity is satisfied, all income elasticities are positive; furthermore, if each committed level of consumption is more than met, the own and cross price elasticities are always negative, whereas the compensated elasticities are negative for the own price terms and positive otherwise. We can also see that the uncompensated own price elasticity always lies between zero and minus one, implying inelasticity of the demand curves. This inelasticity and the negativity of *all* the uncompensated elasticities may seem unsatisfactory from a theoretical point of view; nevertheless, we shall see that, even if we do not relax the convexity restrictions, this is not an empirically serious drawback in most of the cases we consider.

Estimation and Results

One of the several available non-linear estimation techniques must be used to estimate the parameters of the LES. The iterative algorithm used is due to Marquardt [6] and is a relatively simple extension of the Gauss-Newton method for non-linear least squares minimization. Using this algorithm the parameters of the LES have been estimated successfully over several sets of data; annually and quarterly over six broad commodity groupings, annually over forty-three commodities and again annually with six commodities, four of which were fuels. These together provide a fairly thorough test of what is basically a very simple hypothesis; nevertheless the system works remarkably well in all these contexts.

A colleague, Dr. L. J. Slater, fitted the annual forty-three commodity system, and the results may be summarized as follows; all the c parameters were positive, only one, running costs of vehicles—a rapidly expanding item—being not

[1] If real expenditure is steadily increasing a negative b_i is likely to imply negative expenditure on that commodity eventually. This would pose obvious difficulties for long-term forecasting.

significantly different from zero; of the b's, fifteen out of the forty-three were negative, but of these only nine were absolutely larger than their standard errors. These were bread and cereals, sugar and preserves, coal and coke, bus, coach and tram fares, railway travel, cigarettes, cinema, domestic services, and expenditure of income in kind. All of these could be categorised as inferior or complementary goods, neither of which can be accommodated within any linear system. Aside from these goods, the fit was excellent; thus a simple linear relationship between expenditure, prices and income is capable of explaining adequately the post-war variations in demand for thirty-six out of forty-three commodity groups.

In the fuel sub-model, we used only six commodities, four of which, solid fuel, oil, gas and electricity accounted for only five per cent of the total. This required some adjustment to the fitting technique given that a priori we expected that the variances of the commodity error terms would differ widely. To overcome this problem we constructed a weighted residual sum of squares and varied the weights by trial and error in an attempt to equalise the R^2 statistics for each commodity. This technique is similar to the maximum likelihood method for dealing with heteroscedastic errors suggested by Fisher [3]. This was reasonably successful in that the R^2 statistics were much improved and were brought above .90 for each category. Even so, the negative b for coal and coke in the forty-three commodity system persisted here for solid fuels.

In its simple unweighted form, the LES worked well (apart from considerable positive serial correlation among the residuals) for six more equal commodity groups both annually and quarterly. All b's and c's were positive and highly significant; each R^2 was greater than .96 with most over .99.

One may be tempted to conclude that linearity in spite of all its attendant restrictions, is a largely adequate description of reality. Before this position would be tenable, however, it would be necessary to test the LES against a set of data with much greater price variation than at present exists for the UK. We shall return to this point at the end of the next section.

4. *A Non-Linear Expenditure System (NLES)*

This system, introduced in the literature by Houthakker [4] as the 'direct addilog system', has not attracted so much attention as the LES, consequently we shall give rather more space to the discussion of its properties. Like Houthakker, we have not found it possible to derive explicit demand functions; instead we have a linear double logarithmic relationship between the real consumption of pairs of categories and relative prices. This takes the form.

$$\log \frac{e_i}{e_i} = \log \frac{\alpha_i}{\alpha_i} + \beta_i \log x_i - \beta_j \log x_j \tag{4}$$

Alternatively, we may prefer the form

$$x_i^{\ 1-\beta_i} = \frac{\alpha_i \ e}{p_i \Sigma \alpha_k \ x_k^{\beta_k}} \tag{5}$$

This lacks the intuitive appeal of the LES demand equations, nevertheless we shall see that the implications of (4) and (5) are not unreasonable. Alternatively, we may derive the NLES by maximizing the utility function

$$U = \Sigma \frac{\alpha_k}{\beta_k} x_k^{\beta_k} \quad \text{subject to } \Sigma p_k x_k = e. \tag{6}$$

By differentiation of (5) we can derive the following relationships

$$(1-\beta_i)\, n_{ij} = -\Sigma \beta_k \frac{e_k}{e} n_{kj} - \delta_{ij} \tag{7}$$

$$(1-\beta_i)\, n_i = -\Sigma \beta_k \frac{e_k}{e} n_k + 1$$

The elasticities may be calculated from (7) by matrix inversion. As the matrix involved is very simple, we may perform the operation algebraically. The elasticities and their signs turn out as follows:

$$n_{ii} = \frac{\beta_i e_i - (1-\beta_i)\sigma}{(1-\beta_i)^2\sigma} \qquad <0 \text{ for } \beta_k<1 \text{ for all k} \tag{8 (i)}$$

$$n_{ij} = \frac{\beta_j\, e_j}{(1-\beta_i)\,(1-\beta_j)} \cdot \frac{1}{\sigma} \quad \begin{matrix} >0 \text{ for } 0<\beta_k<1 \text{ for all k} \\ <0 \text{ for } \beta_j<0 \text{ and } \beta_k<1 \text{ for all k} \end{matrix} \tag{8 (ii)}$$

$$n_i = \frac{e}{1-\beta_i} \cdot \frac{1}{\sigma} \qquad >0 \text{ for } \beta_k<1 \text{ for all k} \tag{8 (iii)}$$

$$n^*_{ii} = \frac{e_i - \sigma(1-\beta_i)}{(1-\beta_i)^2\sigma} \qquad <0 \text{ for } \beta_k<1 \text{ for all k} \tag{8 (iv)}$$

$$n^*_{ij} = \frac{e_j}{(1-\beta_i)(1-\beta_j)} \cdot \frac{1}{\sigma} \quad > 0 \text{ for } \beta_k<1 \text{ for all k} \tag{8 (v)}$$

$$\text{Where } \sigma = \Sigma \frac{e_k}{1-\beta_k} \tag{8 (vi)}$$

In order that (6) should represent a convex preference ordering β_i must be less than unity for all i. Accepting this condition, equations (8) give us the signs of the various elasticities. Note in particular that (8) (i) rules out inferior goods and (8) (v) rules out complementary goods, a limitation shared with the LES. In fact, Houthakker has shown that for a system derived from a directly additive utility, the substitution effects are proportional to both the income derivatives concerned. Thus, as with both the LES and NLES, all income elasticities are positive, no complementarity can be accepted. The main difference between the systems here is the greater flexibility of the NLES in dealing with cross price effects. Equations 8 (ii) show that the cross price elasticities can now be positive or negative, depending on the magnitude of the income elasticity of the good whose price is being varied. Secondly, it can be shown from 8 (i) and (vi) that $n_{ii} < -1$; thus the individual demand curves are elastic for the NLES but inelastic for the LES.

Finally, it may be noted that the NLES also satisfies the three conditions imposed on the LES. Additivity is built-in; 8(i)–(iii) confirm that

$$\Sigma_j n_{ij} + n_i = 0 \tag{9}$$

hence homogeneity; and from 8 (v) we see that the ijth element of the substitution matrix m_{ij} is given by

$$m_{ij} = \frac{x_i x_j}{\sigma(1-\beta_i)(1-\beta_j)} \quad \text{which is symmetric.} \tag{10}$$

Estimation and Results

In order to estimate the parameters of the system we allow one good to be specific, thus rewriting (4)

$$\log \frac{e_i}{e_1} = \log \frac{\alpha_i}{\alpha_1} + \beta_i \log x_i - \beta_1 \log x_1 \tag{11}$$

We may, without loss of generality, let $\alpha_1 = 1$ so that the only parameter common to each equation is β_1. Equations (11) then represent a system of $n-1$ equations for i running from 2 to n. The β's and α's can then be estimated by ordinary least squares. We then must solve the system for the values of real consumption. We may re-write (5) as

$$x_i^{1-\beta_i} = \frac{\alpha_i}{f} \frac{e}{p_i} \text{ where } f = \Sigma \alpha_k \, x_k^{\beta_k} \tag{12}$$

Thus, once having obtained values for α and β we may solve the system by iterating on the value of f, at each iteration solving for x_i then minimizing the distance function.

$$S = \{\Sigma \, \alpha_k \, x_k^{\beta_k} - f\}^2 \tag{13}$$

It can be shown that if β_i is less than unity for all i (i.e. if convexity holds) this yields a unique positive solution for f. This done, R^2 statistics can be calculated in the usual way, and the standard errors calculated subject to modifications due to the assumed heteroscedacity.

We have not yet attempted to programme this system at forty-three commodity disaggregation; however it was tested on the quarterly six-commodity data and on the annual fuel data. The quarterly results were much less satisfactory than those yielded by the LES; several of the β's were greater than unity and we have had trouble finding a root of f which will give reasonable values for the expenditures. Thus from a computational point of view at least it is desirable that convexity be satisfied. However for the fuel model, the NLES was much more satisfactory. We were rather less successful here with the weighting technique so that the R^2 statistics varied rather more from commodity to commodity; nevertheless the overall fit was similar for both systems. The trouble encountered with solid fuel in the LES was not met here, for the NLES all the β's were less than unity (and greater than zero) and all the parameters were rather better determined than for the LES. The total expenditure elasticities were comparable for both systems but this was of course not true of the uncompensated cross or own price elasticities, the former being largely negative for one and positive for the other, the latter greater than and less than minus one, all by construction. Thus here we have two systems, each providing a good approximation to reality, each using income and prices as explanatory variables, yet each predicating an entirely different response by consumers to changes in relative prices.

The resolution of this seeming paradox is immediate when we realise the lack of relative price movements in the data. There is very little variation in either consumers' expenditure or prices which is independent of variations in income; thus the data can give us little or no information about price responses. The elasticities which come out of these systems have the appearance of having been determined by the juxtaposition of theory and evidence, but this is largely illusory. Their magnitudes are largely determined before we look at the data; they are determined by the construction of the system and not by inference from the data. This situation is mirrored almost precisely in production function estimation and similar conclusions to our own have been reached by Medershausen [7] and Phelps-Brown [8].

In conclusion, anyone searching for *the* expenditure system should first find a set of data containing more price information than any now available; for the present, if we wish to keep to simple systems such as so far described we must be prepared to recognize that the relative superiority of any such system will depend on the purpose for which it is required.

5. *A Permanent-Income Linear Expenditure System (PLES)*

Both systems so far described are static; the parameters are fixed in time as is the utility function if we wish to bring it into the analysis. This may not be a serious defect over short period analysis but it becomes much more so if we are trying to predict some distance into the future. The simplest way to dynamize the LES is to make the c's explicitly or implicitly functions of time or of some other variable which moves through time. This violates none of the conditions imposed on the LES.

The dynamism built into the system described below is very weak; it has very little of the generality of some other versions of the LES (e.g. Phlips [9]). Nevertheless this system is basically very simple, it is capable of accommodating saving behaviour, it has some rather appealing properties, and it is a very natural extension of the original LES.

In the static LES, expenditure is either committed or supernumerary. Real committed expenditure on each category is constant, thus all the fluctuations in the total are mirrored in the residual supernumerary category. This has an obvious parallel in the behaviour of permanent and transient incomes in consumption function theory. We have extended the parallel to its logical conclusion by letting committed real expenditure on each commodity, instead of remaining constant, be proportional to real permanent income, the constants of proportionality summing to unity. Thus we write

$$c_i = a_i z \qquad \Sigma a_i = 1^1 \tag{14}$$

and the permanent income linear[2] expenditure system as

[1] This constraint is not strictly necessary but (i) it enables us to use the permanent and transient income concepts more readily and (ii) when the constraint was relaxed the sum of a_i's was very close to unity.

[2] As z involves p inversely, the system is no longer linear in prices and income; we choose to keep the name only because of the strong links with the LES. Note also that for forecasting, the value of p is dependent on the real expenditure forecasts themselves; some iteration would thus normally be required.

$$e_i = a_i z\, p_i + b_i\,(y - \Sigma\, p_k a_k z),\ \Sigma a_i = \Sigma b_i = 1. \tag{15}$$

The quantity inside the brackets on the right hand side of (15) is money income less real permanent income multiplied by a weighted price index and is close enough to the original concept to be called transient income. If saving, including durables, is regarded as the (n+1)th expenditure, (15) is very close to the permanent-income consumption hypothesis and thus we should expect to be reasonably successful in using the PLES to allocate total income rather than just expenditure.

The 'utility' function underlying this system is now negatively dependent upon income and this necessitates some rethinking of the usual concepts. First, it is worth remaking the point that the PLES, i.e. equations (15), can be accepted or rejected as it stands; given the difficulties of aggregation, there is little point in challenging an aggregate demand system because it is based on an unlikely utility function. Nevertheless, it is possible to rationalise the PLES for a single consumer; we can perhaps regard the levels of committed expenditure as being optimal with respect to some planning horizon, the 'utility' function is then rather a rule for getting as close as possible to the longer-run levels within a short-run budget constraint.

The price and income elasticities of the PLES are those of the LES plus terms to take account of the derivatives of permanent income with respect to money income and prices. The general formulae are not particularly elucidating and we shall delay discussion of these until the results of estimation have been presented.

Estimation and Results

We first must specify the mechanism whereby permanent income adjusts to changes in income; we use the usual partial adjustment or expectational form:

$$z = \lambda\left(\frac{y}{p}\right) + (1-\lambda)\,E^{-1}(z) \text{ or } z = \lambda\sum_{o}^{\infty}(1-\lambda)^{\theta}\,E^{-\theta}\left(\frac{y}{p}\right) \qquad \text{(16) (i) and (ii)}$$

Several techniques for estimating the parameters of this system were tried. Note first that applying the Koyck transform to (15) as it stands does not remove the unobservable permanent income; some other method must be used. We tried the method successfully employed by Phlips [9] when faced with a similar problem. Basically, the procedure may be described as follows: leave the transient income term in (15) unspecified, apply the Koyck transform, then iterate over the values of the term until the adding-up constraint is satisfied. This appeared to converge very rapidly but on closer examination we found that further steps, though small (each element less than 10^{-7}), were not diminishing from iteration to iteration. The final values could thus be at some considerable distance from those so far reached. Next we used the same algorithm as for the LES, using (16) (ii) to evaluate z and to calculate the derivatives of the residual sum of squares with respect to λ. This was completely futile; we suspect that non-convergence was due to the heavy non-linearity with respect to λ of (16) (ii) coupled with the inadequacy of a finite-sum approximation to that equation when λ is small.

The method we have finally used employs a correction process for the unobservable term and has yielded credible and interesting results.

If we use the Koyck transform on (15) and rearrange we can write

$$e_i = (1-\lambda)E^{-1}e_i + b_i y - b_i (1-\lambda) E^{-1}y + \lambda y/p \{a_i p_i - b_i \Sigma a_k p_k\}$$
$$+ (1-\lambda) E^{-1}z \{a_i \triangle^* p_i - b_i \Sigma \triangle^* p_k a_k\} \quad (17)$$

The last term on the right hand side of (17) is small relative to the total and we can thus set up the following procedure. Absorb the last term into the error and estimate the parameters using the non-linear algorithm; using (16) (ii) calculate the missing term, subtract from e_i and re-estimate; stop when the estimates do not change from iteration to iteration. This method unfortunately involves one iterative process within another and turns out to be very expensive; consequently we have not been able to proceed to final convergence. Nevertheless, over several iterations the parameter estimates were quite stable (always settling within the previous standard errors) and the missing term was of comparable size to the estimated errors. The provisional results are set out below for the six-commodity, quarterly data.

Grouping	a_i	b_i	R^2
Food, drink, and tobacco	.3570 (.036)	.0213 (.030)	.9930
Clothing and Footwear	.0879 (.072)	.0406 (.030)	.9874
Housing	.1213 (.010)	.0048 (.030)	.9992
Fuel and light	.0560 (.033)	−.0180 (.030)	.9226
Other Goods	.1420 (.037)	.0217 (.030)	.9970
Other Services	.1980 (.021)	.0116 (.030)	.9980
Saving	.0379 (1.58)	.9180 (.032)	.9879
	$\lambda=$ .0876 (.018)		

(Quarterly s.a. data, UK 581–681V [2], assymptotic standard errors in brackets).

The overall fit is similar to that of the LES, with three goods slightly better and three slightly worse; the fit for savings is comparable to that of permanent-income single equation models. (This is a fairly remarkable achievement as savings was very volatile throughout this period). However it is the parameter estimates which are most interesting. Note that each of the a's is well determined except for that for savings, while the situation is reversed for the b's, with the ambiguous exception of footwear and clothing. This implies that each category of consumption varies with permanent income, while savings does not; similarly savings is highly correlated with transient income and purchases of goods are not. The partial exception of footwear and clothing may indicate that, for some purchasers at least, these are durable goods; a similar conclusion was reached by Phlips from a much more complex analysis [10]. We may thus say that there is some evidence for Friedman's permanent-income hypothesis in its strictest form, not only for total consumption and saving, but for disaggregated consumption and saving.

One word of warning about the R^2 statistics; closer inspection of the expected values given by the PLES indicates that the lag coefficient, largely determined by the very volatile savings category, is not entirely appropriate for all the other categories. This applies particularly to the category, fuel and light. This model then, would be appropriate for forecasting some distance ahead when the position in the cycle is of little importance, but a more complex dynamic structure is required if both long and short-term forecasts are envisioned. Our experience would suggest that the data is capable of determining some rather better lag mechanism.

Lastly, we shall examine the implications of the PLES for price and income elasticities. We may now write the system as

$$e_i = a_i p_i z \text{ for goods} \qquad s = y - \Sigma a_k p_k z \qquad \text{for savings} \tag{18}$$

$$\text{Thus } n_i = \lambda\, y/pz \qquad n_{ij} = -\lambda \frac{e_j}{e} \frac{y}{pz} \qquad \text{for goods} \tag{19}$$

$$\text{and} \quad \frac{\partial s}{\partial y} = 1 - \lambda \frac{e}{pz} \qquad \frac{\partial s}{\partial p_i} = x_i \left(\frac{\lambda y}{pz} - 1 \right)$$

These elasticities are readily explained; changes in income affect permanent income only slowly, thus an increase in real income will in the first instance increase each category of consumption by the *same* small percentage leaving some for an increase in saving. In the long run however, when the adjustment has fully taken place, real income is equal to permanent income, and all categories of consumption have increased by the same proportion. A similar, though somewhat more complicated process occurs in response to price changes. An alteration in any price will increase the overall price index by an amount depending on consumption of that good, this reduces real income and thus permanent income. Thus any increase in one price will have an equal proportional effect on *all* goods, even that whose price has increased. This effect is not sufficient in the short run to induce any fall in money expenditure, thus savings will fall; once all adjustments have taken place, money expenditures on each category (though not real expenditures) and thus savings return to their initial levels.

Thus here we have a set of very attenuated price effects which work only through their impact on real income. Once again, we may not like this theoretically, but it is impossible to infer a rejection of this process from the data alone.

5. *Conclusions*

It is time to refer back to our aims of the introduction and to state our conclusions. These empirical models can tell us very little about the indifference mappings or utilities of consumers. Even if individual preference orderings exist, we know that the possibility of aggregating these into a social preference ordering is indeed remote [1]. Consequently, no amount of estimation of aggregate demand equations can help us find something which in all probability does not exist. However, for those models which can be derived from utility functions, we can at least say that, for an individual consumer, the demand functions we postulate are consistent with the usual rules of 'rational' behaviour. Conversely, if there exists a consumer who behaves typically as a consumer, his behaviour is consistent with his possessing a preference ordering of the type indicated. Beyond this we cannot go.

We have already devoted a fair proportion of our space to the discussion of income and price elasticities and we need only summarise here. The data at present available for the UK, and probably for the US [5, Chap. 7], can help us determine income elasticities within a fair margin of error, but can tell us little about price responsiveness.[1] Any price elasticities derived from empirical work will almost inevitably owe their determination more to the construction of the model than to the empirical evidence. It must be emphasized that this is *not* an argument for excluding price sensitivity from consumption models—prices may not always trend with income—but we must fall back heavily on theory and our a priori judgements to help us decide which of the empirically satisfactory models to accept.

As to forecasting, the first two models are probably adequate for short-term work, the third for long-term work though a fully dynamic model is obviously desirable and could perhaps be used for both. However, the behaviour of consumers in an environment, in which relative prices and the relationships between income and prices were very different from those now existing, would almost certainly be inexplicable in terms of any of these models. But this is a caveat which applies to most, if not all, econometric models.

6. *References*

[1] K. J. Arrow: *Social Choice and Individual Values*, J. Wiley 1963.

[2] Central Statistical Office: *Economic Trends*, October 1969.

[3] G. R. Fisher: 'Maximum Likelihood Estimators with Heteroscedastic Errors': *Review of the International Statistical Institute*, Vol. 25, Nos. 1/3.

[4] H. S. Houthakker: 'Additive Preferences' *Econometrica*, Vol. 28, No. 2.

[5] H. S. Houthakker and L. D. Taylor: *'Consumer Demand in the United States* 1929–1970.' Harvard University Press 1966.

[6] D. W. Marquardt: 'An Algorithm for Least-Squares Estimation of Non-linear Parameters,' *Journal of the Society of Industrial and Applied Mathematics* 1963.

[7] H. Mendershausen: 'On the Significance of Professor Douglas' Production Function' *Econometrica*, Vol. 6, No. 2.

[8] E. H. Phelps Brown: 'The Meaning of the Fitted Cobb-Douglas Production Function,' *Quarterly Journal of Economics*, Vol. 71, No. 4.

[9] L. Phlips: 'A Dynamic Version of the Linear Expenditure Model' University of Louvain discussion paper.

[10] L. Phlips: 'Substitution, Complementarity, and the Residual Variation around Dynamic Demand Equations'. University of Louvain discussion paper.

[11] Richard Stone: 'Linear Expenditure Systems and Demand Analysis: An Application to the Pattern of British Demand.' *Economic Journal*, Vol. 64, No. 255.

[12] Richard Stone: 'Models for Demand Projections' in *Essays on Econometrics and Planning*, Pergamon Press 1965.

[1] If there were more price movement in the data, our implicit assumption that producers set prices and then supply any quantity demanded at that price might well not hold. This would involve us in the usual identification problems.

Department of Applied Economics
Cambridge