

## EQUITY, EFFICIENCY, AND THE STRUCTURE OF INDIRECT TAXATION

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Optimal tax formulae are derived and optimal tax rates calculated for the case where there are many consumers, an income tax is impossible and the government has to trade off efficiency in order to improve the real distribution of income. Consistent aggregation assumptions are used to permit the derivation of simple tax rules depending only on the behaviour of the average consumer and of a socially representative consumer. Calculations for the U.K. in 1972 show how subsidies and taxes vary with the government's revenue requirement and with the degree of egalitarianism in the social welfare function.

### 1. Introduction

This paper derives and calculates optimal commodity taxes in the case where the government is not only constrained to raise a certain revenue but is also concerned with the resulting distribution of real income among its citizens. It has recently been shown by Atkinson and Stiglitz (1976) that, provided individual utility functions are separable between leisure and other goods, governments will not wish to use commodity taxes, relying instead on an optimal (nonlinear) income tax. In the further special case where labour supply is exogenous, this solution is clearly not only optimal but also non-distortionary, even without the separability assumption. However, these recommendations may not always be practicable, especially when there is difficulty administering and collecting an income tax. The most obvious example is that of an underdeveloped country where a large proportion of the population has no direct contact with government agencies. In many such cases, however, the government may have extensive control over commodity prices either because many industries are state owned or because many important commodities are imported or exported through government controlled ports or frontier points. Indirect taxation can thus be a practical, albeit 'third-best,' instrument for influencing real income distribution as well as for collecting revenue.

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This emphasis on commodity taxation is likely to produce a particularly acute conflict between equity and efficiency. To minimize distortion, it is necessary to tax goods which have few substitutes and are in inelastic demand. Since there is a general tendency for price inelasticity to be associated with income inelasticity, a tendency which becomes inevitable under direct (or indirect) additivity of preferences [see Deaton (1974b)], efficiency demands that necessities should be taxed more heavily than luxuries. This is of course the opposite of what one would hope for from the equity point of view.

Formulae for optimal indirect taxes with allowance for equity as well as for efficiency are given by Diamond and Mirrlees (1971), Diamond (1975) and, in a different form, by Atkinson and Stiglitz (1976). These formulae are reasonably complicated, and by requiring information on each individual consumer, set formidable problems for empirical implementation. Indeed, seemingly the only authors who have actually calculated optimal taxes on the basis of empirically estimated relationships [Atkinson and Stiglitz (1972)] did so only for the one-consumer case. As indicated above, this emphasizes efficiency at the expense of equity and their results are unlikely to be acceptable as a solution to the more general problem. In this paper I shall adopt an alternative approach which, by using strategic aggregation assumptions, permits a considerable simplification of the original formulae as well as reducing the amount of information required for computation. Thus, instead of having to calculate welfare weights for each consumer, the solution can be stated in terms of the behaviour of two consumers, an average consumer and a socially representative consumer. Exactly what position the latter occupies in the income distribution will depend on the degree of egalitarianism built into the social welfare function.

I shall work throughout with the assumption that labour is supplied exogenously so that the money distribution of income is given. In the one-consumer case, this assumption would produce the trivial result of a uniform commodity tax [see e.g. Dixit (1970) and Sandmo (1976)]. In our case, with many consumers and a progressive income tax ruled out by assumption, this is no longer necessarily so, and we shall see from the calculations that even a mildly egalitarian social welfare function can produce significant departures from the uniform taxation rule.

In section 2 below, the model is set out in detail, the aggregation assumptions are made explicit and the optimal tax formulae are derived. Section 3 contains some illustrative calculations for British data for a particular form of preferences and for a range of degrees of egalitarianism in the social welfare function. Perhaps surprisingly, it is possible to produce quite complex tax schedules on the basis of the simple formulations considered here.

## **2. The model and the optimal tax formulae**

We assume all consumers have identical tastes and differ only in incomes

$m$ . Preferences are given by the indirect utility function  $v(m, z)$ , where  $z$  is the vector of prices faced by all consumers. The government is concerned to maximize the social welfare function

$$\int_a^\infty f(m) h\{v(m, z)\} dm, \quad (1)$$

where  $f(m)$  is the p.d.f. describing the distribution of income. The parameter  $a$  is minimum income and the function  $h(\cdot)$  is a normalization function chosen by the government which summarizes its attitudes to inequality. (Note that  $h' \partial v / \partial m$  is the *social* marginal utility of income to a consumer with income  $m$ .) The government's revenue constraint will be written

$$\int_a^\infty f(m) \sum_k t_k q_k(m, z) dm = \rho \bar{m}, \quad (2)$$

where  $\rho$  is the percentage of average income going in tax. The vector  $t$  is a vector of taxes;  $z - t$  is the vector of producer prices and, following convention, we shall assume constant returns and the nonsubstitution theorem so that these remain fixed in face of changes in the pattern of demand.

The first-order conditions may be written, via Roy's theorem and the Slutsky equation as

$$\int_a^\infty f(m) \lambda(m, z) q_i(m, z) dm = \xi \int_a^\infty f(m) \left\{ q_i(m, z) (1 - \beta' \theta) + \sum_k s_{ik} t_k \right\} dm, \quad (3)$$

where  $\xi$  is a Lagrange multiplier,  $\lambda(m, z) = h' \partial v / \partial m$  is the marginal social utility of income,  $S$  is the Slutsky substitution matrix,  $\theta$  is the vector of tax rates ( $\theta_i = t_i / z_i$ ), and  $\beta$  is the vector of marginal propensities to spend ( $\beta_i = z_i \partial q_i / \partial m$ ). Eq. (3) is analogous to the results of Diamond and Mirrlees (1971) and Atkinson and Stiglitz (1976), although the latter are more general since  $m$  is itself a variable. In any case, it is clear that (3) gives a balance between equity and efficiency. On the left-hand side, we have quantities weighted by marginal social utilities, on the right-hand side marginal tax yield allowing for distortionary substitution. To simplify further we must choose more particular forms for  $v(m, z)$  and for  $h(v)$ .

To facilitate aggregation, we assume that consumers have linear Engel curves so that  $v(m, z)$  takes the Gorman (1953, 1961) form

$$v(m, z) = \{m - a(z)\} / b(z), \quad (4)$$

where  $a(z)$  and  $b(z)$  are homogeneous of degree one in the prices  $z$ . We assume that  $m \geq a(z)$  for all consumers and for all relevant tax rates  $t$ . The utility

function (4) has the important property that the demand functions associated with it aggregate consistently. From Roy's theorem,

$$q_i(m, z) = a_i(z) + b_i(z)\{m - a(z)\}/b(z), \quad (5)$$

from which it follows immediately that

$$\int_a^\infty f(m)q_i(m, z)dm = q_i(\bar{m}, z), \quad (6)$$

where  $\bar{m}$  is average income, and  $a_i$  and  $b_i$  denote the  $i$ th partial derivatives of  $a(z)$  and  $b(z)$ . If we define  $w$  as the vector of value shares ( $w_i = z_i q_i/m$ ), then from (5),

$$w_i(m) = \frac{a_i z_i}{a} \left( \frac{a}{m} \right) + \frac{b_i z_i}{b} \left( 1 - \frac{a}{m} \right), \quad (7)$$

i.e. the vector of value shares is a weighted sum of  $a_i z_i/a$  – the expenditure pattern of the very poor ( $m = a$ ) – and  $b_i z_i/b$  – the expenditure pattern of the very rich ( $m = \infty$ ).

Finally, we follow Atkinson (1970), and define  $h(v)$  by

$$h(v) = v^{1-\varepsilon}/(1-\varepsilon), \quad (8)$$

for some  $\varepsilon \geq 0$ . Note that while Atkinson (1970) worked with money incomes we shall consider real incomes or utilities  $v$ . Even so,  $\varepsilon$  has the same interpretation as in Atkinson's work and represents a measure of the government's aversion to inequality. If  $\varepsilon$  is zero, (1) is simply average real income and the government is unconcerned about distribution. As  $\varepsilon$  tends to infinity, social welfare is sensitive only to the real incomes of the very poorest, i.e. (1) becomes maximin.

Substituting (4) and (8) in the first-order conditions (3) gives immediately

$$b^{-1} \int_a^\infty f(m) \left( \frac{m-a}{b} \right)^{-\varepsilon} q_i(m, z) dm = \xi \left\{ q_i(\bar{m}, z)(1 - \beta'\theta) + \sum_k s_{ik} t_k \right\}, \quad (9)$$

where  $S$  is now measured at average income. It is straightforward to show that (9) may be written as

$$q_i(m_0, z) = \alpha \left\{ q_i(\bar{m}, z)(1 - \beta'\theta) + \sum_k s_{ik} t_k \right\}, \quad (10)$$



where  $m_0$  is given by

$$m_0 = a(z) + \frac{\int_a^\infty f(m)(m-a)^{1-\varepsilon} dm}{\int_a^\infty f(m)(m-a)^{-\varepsilon} dm}, \quad (11)$$

and  $\alpha = \xi b^{1-\varepsilon} I^{-1}$ , where  $I$  is the integral on the denominator of the right-hand side of (11). From (11), it is obvious that when  $\varepsilon$  is zero,  $m_0 = \bar{m}$ ; it is also true and is demonstrated in the Appendix that as  $\varepsilon$  increases, with given  $z$ ,  $m_0$  declines monotonically, tending to subsistence income  $a$  as  $\varepsilon$  tends to infinity. Hence,  $m_0$  is always less than  $\bar{m}$ , the difference depending on the government's degree of inequality aversion  $\varepsilon$ . For reasons that will become apparent below, we shall refer to  $m_0$  as *socially representative income*.

The quantity  $\alpha$  may be found by noting that  $z'q(m_0, z) = m_0 \cdot z'q(\bar{m}, z) = \bar{m}$  and  $z'S = 0$ . Hence  $m_0 = \alpha \bar{m}(1 - \beta'\theta)$ , so that (10) may be written

$$\frac{\sum_k s_{ik} t_k}{q_i(\bar{m})} = (1 - \beta'\theta) \left( \frac{w_i(m_0)}{w_i(\bar{m})} - 1 \right), \quad (12)$$

or defining  $c_{ij} = -z_i z_j s_{ij} / \bar{m}$ , in the more elegant matrix form,

$$C\theta = (1 - \beta'\theta)\{w(\bar{m}) - w(m_0)\}. \quad (13)$$

Eqs. (12) and (13) give the tax rule which we shall calculate in section 3 below. Eq. (12) gives the rule in terms of quantities. If the tax rates are small, the left-hand side is the compensated proportionate reduction in the demand for good  $i$  as a result of imposing the taxes. Since socially representative income is below average income, this must be negative for luxuries and positive for necessities. Taxes must thus encourage the substitution of necessities for luxuries to an extent which depends on the egalitarianism of the social welfare function. Note that this does *not* imply that luxuries necessarily attract higher rates of tax; if they are relatively price sensitive, only a low tax may be necessary to induce the required shift in demand.

The conflict between equity and efficiency is even more clear in eq. (13) which states the rule directly for the tax rates  $\theta$ . If substitution possibilities are limited, the  $C$  matrix will be close to diagonal and the tax rates will be determined by the 'equity' part of the rule, i.e. luxuries will be relatively heavily taxed. Efficiency aspects are represented by the  $C$  matrix so that highly elastic goods, which have large elements of  $C$  associated with them, will tend to have low tax rates. Clearly, since luxuries tend to be substitutable and hence highly elastic, the equity and efficiency parts of the rule operate in opposite directions and the optimal tax structures must strike a balance between the two.

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One special case is of some interest. If preferences are directly additive – and there are a number of such cases consistent with Gorman utility functions [see Pollak (1971)] – the  $C$  matrix takes the form [see Barten (1969) or Deaton (1974a)],

$$C = \phi(\beta - \beta\beta'), \quad (14)$$

for some  $\phi \geq 0$ , so that

$$\theta_i \geq \beta' \theta \quad \text{iff} \quad w_i(\bar{m}) \geq w_i(m_0), \quad (15)$$

so that relative to the weighted average of tax rates  $\beta' \theta$  (note  $\beta' \mathbf{1} = 1$ ), luxuries are taxed more heavily than necessities. But note that the case of additive preferences is a very special one, and if substitution between goods is not limited by (14), the ‘inverse elasticity’ implications of (13) may dominate.

At this point it must be emphasized that the formulae (12) and (13), involving as they do only average income  $\bar{m}$  and socially representative income  $m_0$ , depend crucially on the assumption of linear Engel curves. This can be relaxed somewhat by the adoption of ‘generalized linearity’ and the associated utility functions [see Muellbauer (1975, 1976)]. In this formulation, average value shares are no longer in aggregate a function of average income, but rather of some *market representative income*. This means that in (12) and (13), the critical quantity is the difference between *market representative income* (which deviates from average income because Engel curves are nonlinear) and *socially representative income* (which deviates from average income because governments are averse to inequality). Since in most empirical cases, market representative income turns out to be above average income, the equity implications of (12) will be accentuated compared with the present formulation. For further details of this approach, see Muellbauer (1976).

Note finally that (12) and (13) will produce uniform taxes under one of two possible assumptions. Since  $z$  spans the nullspace of  $S$ , the unit vector  $i$  spans the nullspace of  $C$ ; hence taxes will be uniform if the right-hand side of (13) is zero. This will happen if  $m_0 = \bar{m}$ , i.e. when  $\varepsilon = 0$  and the government is unconcerned about equity. A similar result holds if  $w_i(\bar{m}) = w_i(m_0)$  for all  $i$ , whatever the values of  $\bar{m}$  and  $m_0$ , i.e. when preferences are homothetic. In either of these cases, we have effectively a one-consumer problem.

### 3. Illustrative calculations for the United Kingdom

In order to use the preceding theory to calculate tax rates it is necessary not only to give a general specification of the context in which the tax is to operate but also to specify the income distribution, the utility function, i.e. the functions  $a(z)$  and  $b(z)$ , and the government’s inequality aversion parameter  $\varepsilon$ .

Further, since (12) and (13) are highly nonlinear in the tax rates  $\theta$ , it is necessary to specify some computational algorithm.

In the first place, we shall assume that income tax is given at its actual 1972 structure and yield so that attention is confined to the problem of calculating optimal indirect taxes to replace the actual taxes then in force. The main difficulty with this view of the problem is whether or not it is sensible to apply the social welfare function approach *after* a progressive income tax has been levied. However it would be assuming too much to pretend that the existing income tax structure results in an optimal post-tax distribution of income, so it is reasonable to allow some redistributive role to indirect taxation. In line with our general approach, we shall continue to assume that these redistributive goals can be summarized by maximization of an Atkinson-type social welfare function. The choice of the inequality aversion parameter will be dealt with by calculating optimal tax schedules for a range of values of  $\varepsilon$ , so that the question of the 'correct' value need never arise.

We shall further restrict the problem by excluding savings and purchases of durable goods. This is done largely for the convenience of being able to use immediately available parameter estimates and although such a procedure would be indefensible in a study making policy recommendations, it is perhaps not too serious in an illustrative context. There are, of course, difficult problems in extending the analysis since consumers' demands for assets must be considered in an intertemporal context whereas the social welfare function as defined is essentially static and timeless.

The distribution of income in which we are interested is thus the distribution of money expenditures on nondurable goods. We shall assume that these are lognormally distributed above the minimal level  $a(z)$ . Eq. (11) can be written as

$$m_0 = a(z) + \lambda'_{1-\varepsilon} / \lambda'_{-\varepsilon}, \quad (16)$$

where  $\lambda'_j$  is the  $j$ th moment around zero of the distribution of  $m - a(z)$ . For a lognormal distribution with parameters  $\mu$  and  $\sigma^2$ ,  $\lambda'_j$  is equal to  $\exp \{j\mu + \frac{1}{2}j^2\sigma^2\}$  [see Aitchison and Brown (1957, p. 8)] so that

$$m_0 = a(z) + \{\bar{m} - a(z)\} \exp \{-\sigma^2\varepsilon\}. \quad (17)$$

It can immediately be seen that eq. (17) satisfies the general conditions on  $m_0$  proved in the Appendix, so that  $m_0$  declines from  $\bar{m}$ , when  $\varepsilon$  is zero, to  $a(z)$ , as  $\varepsilon$  tends to infinity. The particular form (17), associated with the lognormal distribution, is both of analytical interest and of great computational convenience. The quantities  $\sigma^2$  and  $\varepsilon$  appear only as a single product, so that a high degree of dispersion in the original distribution associated with a low inequality aversion results in the identical tax structure produced by low initial dispersion coupled with high inequality aversion. On the computational side, since we are calculating



taxes for different values of  $\varepsilon$ , there is no need to have precise information on  $\sigma^2$ . Further, since  $a(z)$  is a function of the tax rates, the distribution of  $m - a(z)$  will shift as the tax rates change. In order to calculate anything at all, it is necessary to assume that the eq. (17) continues to hold good as an approximation, but within this framework, shifts in the parameters can easily be dealt with; changes in  $\mu$  are automatically handled by changes in  $a(z)$ , whereas by ignoring the effects of  $\theta$  on  $\sigma^2$  (which are likely to be of second order importance) we need only reinterpret the value of  $\varepsilon$ .

For the utility and demand functions we shall use those associated with the linear expenditure system. The indices  $a$  and  $b$  are then given by

$$a(z) = \sum \gamma_k z_k \quad \text{and} \quad b(z) = \beta_0 \prod z_k^{\beta_k}, \quad (18)$$

where the notation for  $\beta$  is consistent with earlier usage since these parameters are the marginal budget shares in the linear expenditure system. In accordance with (5) and the usual results,

$$q_i(m) = \gamma_i + \beta_i z_i^{-1} \{m - \sum z_k \gamma_k\}. \quad (19)$$

From eq. (14) we can see that the linear expenditure system, which is directly additive, will guarantee that luxuries will be taxed more heavily than necessities, and we know that this result is not generally true. However, we shall be considering only a broad disaggregation of expenditures into eight groups so that it is reasonable to suppose some limitation of substitution possibilities. In any case, we shall see that even this simple case permits a complicated pattern of development in the taxes as inequality aversion increases.

An interesting possibility for future work would be to make use of the many existing sets of parameter estimates derived for the Rotterdam demand system [see e.g. Barten (1969) and Deaton (1974a)]. This is particularly attractive since in that model the matrix  $C$  is estimated directly and values are available for a range of countries and time periods. However, although the Rotterdam model possesses linear (differential) Engel curves, it can at best be thought of only as a finite approximation to the first derivatives of a set of demand equations, since neither utility function nor demand equations exist. In consequence, there are obvious difficulties in evaluating  $m_0$  and  $w(\bar{m}) - w(m_0)$ . Even so, it may be possible to overcome these, in the spirit of the Rotterdam model, by suitable approximations.

The estimates of the parameters of the linear expenditure system were obtained from data on consumers' expenditures on eight broad groups from 1954 to 1972 taken from successive issues of the National Income and Expenditure Blue Book. The estimates for  $\beta$  and for  $\gamma$  (scaled to sum to unity) are given in table 1; these numbers correspond to eqs. (5) and (6), i.e. they are the budget shares of extremely rich and of extremely poor consumers. The third column



in the table shows the actual values of  $\theta$ , the gross tax rates, actually prevailing in 1972. These were calculated from Table 25 of CSO (1973); the unallocated items in that table were allocated *pro rata* over the eight groups (excluding commodities known to be exempt) in line with the general methodology outlined in Maurice (1968).

Note that the major differences between the first two columns lie in three commodity groups, food, which is a necessity, housing, and travel and communication, which are luxuries. This result for housing may seem surprising but the constant price series for this has risen rapidly in the postwar years. Partly this is due to the presence in the data of expenditure on housing maintenance and improvements by owner-occupiers, which can be expected to be highly responsive to income, but the major influence is the upward movement in the

Table 1  
Extremal budget shares and 1972 indirect tax rates.<sup>a</sup>

	'Misery' budget = $\gamma_i/\Sigma\gamma$	'Bliss' budget = $\beta_i$	Tax rate
1. Food	0.3529	0.0542	0.0290
2. Clothing	0.0931	0.0885	0.1043
3. Housing	0.0476	0.2428	0.1414
4. Fuel	0.0507	0.0503	0.0396
5. Drink & tobacco	0.1534	0.1156	0.4916
6. Travel & Communication	0.0255	0.2271	0.2296
7. Other goods	0.0975	0.1101	0.1212
8. Other services	0.1793	0.1114	0.0979

<sup>a</sup>The 'misery' budget share is at 1970 prices, i.e. when  $z_i = 1$ .

real value of imputed rents. The precise meaning of this is unclear, and although much may be due to the increase in owner-occupation, it might be preferable to ascribe this to factors other than increasing real income. For the moment, we shall take the figures as given and interpret the shift from the first to the second column as representing the fact that poor consumers live in low quality rented accommodation and that rich consumers own their homes. The high degree of luxury displayed by travel and communication is clearer, and is mainly due to the presence of expenditure on the operation of motor vehicles.

The tax rates in the last column show some association between income elasticity and taxation since food is the lowest taxed item while travel and communication and housing (through rates) are relatively heavily taxed. However, the highest tax rate is associated with drink and tobacco which is more heavily consumed by poorer households.

We shall use eq. (13) as a basis for calculation. As it stands there are two problems; first, since  $C$  and the  $w$ 's are functions of  $\theta$ , the formula is heavily

nonlinear; second, the revenue constraint is not allowed for. This latter can conveniently be written as

$$w(\bar{m})'\theta = \rho, \quad (19)$$

which can be pre-multiplied by  $w(\bar{m})$  and added to (13) to give

$$\{C + w(\bar{m})w(\bar{m})'\}\theta = (1 - \beta'\theta)\{w(\bar{m}) - w(m_0)\} + \rho w(m). \quad (20)$$

This is just as nonlinear as (13), but whereas the matrix  $C$  is singular, the matrix  $\{C + ww'\}$  is not.

Hence we may write

$$\theta = (1 - \beta'\theta)\{C + w(\bar{m})w(\bar{m})'\}^{-1}\{w(\bar{m}) - w(m_0)\} + \rho\iota. \quad (21)$$

Eq. (21) may be used as the basis for a Gauss–Siedel iterative procedure to solve for  $\theta$ ; a given value of  $\theta$  is used to evaluate the right-hand side which in turn gives a new value of  $\theta$ , and so on. It is also straightforward to see that any such estimate of  $\theta$  satisfies the revenue requirement (19). The equation was used to calculate optimal tax rates for 1972 for a range of values of  $\sigma^2\varepsilon$  and  $\rho$ . In practice the right-hand side of (21) turned out to be quite insensitive to changes in  $\theta$  so that convergence was, in most cases, extremely rapid.

The results are given in tables 2, 3 and 4, each corresponding to a different revenue requirement; table 3, with  $\rho$  set to 19.3%, produces the same revenue as the actual 1972 tax rates. In each case, the calculations are repeated for different values of  $\varepsilon$ , starting from zero which always gives a uniform tax rate. The interpretation of the values of  $\sigma^2\varepsilon$  in terms of  $\varepsilon$  is complicated by the problems discussed above; however,  $\sigma^2$  is presumably rather less than one, say around 0.8, so that the highest inequality aversion considered corresponds to a value of  $\varepsilon$  of around unity. This is not a high value, and corresponds to Atkinson's logarithmic case, but is nevertheless sufficiently high to produce fairly extreme results in this model.

The broad conclusions following from these numbers are that, at low values of  $\varepsilon$ , food should be taxed at a very low rate with an approximately uniform tax on all other goods, (columns 2 of tables 2 and 3 and column 3 of table 4), whereas at high levels of inequality aversion, food should be subsidized more or less heavily, the revenue being raised by taxing housing and transport and communication. This latter is perhaps the prescription to be recommended, since table 3 is closest to reality in terms of revenue, and we should probably not want to consider the lowest values of  $\varepsilon$ . The instruments of this policy are clear: food subsidies accompanied by much higher property taxes and road fund licences.

Table 2  
Optimal tax schedules for 1972: Revenue = 10% of expenditure.

Commodity	Inequality aversion ( $\sigma^2\epsilon$ )				
	0	0.1	0.2	0.4	0.8
Food	0.10	-0.052	-0.219	-0.352	-0.731
Clothing	0.10	0.127	0.142	0.146	0.129
Housing	0.10	0.158	0.217	0.262	0.384
Fuel	0.10	0.126	0.139	0.140	0.118
Drink & tobacco	0.10	0.139	0.171	0.189	0.217
Travel & communication	0.10	0.162	0.227	0.279	0.431
Other goods	0.10	0.134	0.158	0.169	0.176
Other services	0.10	0.107	0.096	0.079	0.001

Table 3  
Optimal tax schedules for 1972: Revenue = 19.3% of expenditure.

Commodity	Inequality aversion ( $\sigma^2\epsilon$ )				
	0	0.1	0.2	0.4	0.8
Food	0.193	0.048	-0.017	-0.228	-0.565
Clothing	0.193	0.223	0.239	0.243	0.229
Housing	0.193	0.253	0.312	0.355	0.469
Fuel	0.193	0.221	0.235	0.237	0.219
Drink & tobacco	0.193	0.235	0.267	0.285	0.311
Travel & communication	0.193	0.257	0.322	0.372	0.524
Other goods	0.193	0.229	0.254	0.265	0.273
Other services	0.193	0.203	0.194	0.178	0.110

Table 4  
Optimal tax schedules for 1972: Revenue = 30% of expenditure.

Commodity	Inequality aversion ( $\sigma^2\epsilon$ )				
	0	0.1	0.2	0.4	0.8
Food	0.30	0.165	0.025	-0.081	-0.364
Clothing	0.30	0.333	0.351	0.355	0.346
Housing	0.30	0.363	0.422	0.464	0.566
Fuel	0.30	0.332	0.347	0.350	0.337
Drink & tobacco	0.30	0.345	0.378	0.395	0.420
Travel & communication	0.30	0.367	0.432	0.480	0.608
Other goods	0.30	0.340	0.365	0.377	0.385
Other services	0.30	0.314	0.308	0.295	0.239

Changes in the revenue requirement do not exert a major qualitative change in the results. The main effect of increasing  $\rho$  is to shift to a higher inequality aversion level the point at which food is subsidized.

Commodities other than those discussed do not conform to immediate prior expectations. The tax rates on clothing, fuel and other services increase with increases of  $\varepsilon$  at first, but at higher values of the parameter, this increase is reversed. These three groups are slightly more important in the budget of poor than rich households, but it seems that it is necessary to increase these taxes in the first instance in order to permit the low taxation of food. The category other goods, which is a luxury, is taxed more heavily as  $\varepsilon$  increases, but the same is also true of drink and tobacco which is a necessity according to table 1. Clearly, even in this simple case, there is no direct relationship between inequality aversion, tax rates and the luxury–necessity distinction except for the commodities which most obviously fall into one or other category.

#### 4. Conclusions

In this paper we have derived a simple result for optimal commodity taxation in the case where there are many consumers and where the government is concerned with the distribution of real income among them. The result rests on strong simplifying assumptions in order to avoid the complexity of the general case; in particular, consumer behaviour has been restricted by the use of linear Engel curves and by permitting only very limited substitution between commodities. Relaxing either of these could alter quite fundamentally the nature of the empirical results. As it is, however, the examples calculated here allow us to see clearly the trade-off between equity and efficiency in constructing commodity taxes. While much of the basic structure of the one consumer model is retained, our result, instead of assuming away all distributional considerations, makes explicit recognition of the problems of equity and inequality. We have also applied the model to commodity taxation in Britain and the conclusions suggest that, within existing revenue requirements, equity could be improved by the greater use of food subsidies accompanied by much higher taxes on housing and on the use of private motor vehicles. It is of interest to note that recent relative price changes have gone very much in this direction. All this is of course conditional on the inability to remove commodity taxation by the use of a less distortionary optimal income tax.

#### Appendix

Eq. (11) in the text defines

$$m_0 = a(z) + \frac{\int_a^\infty f(m)(m-a)^{1-\varepsilon} dm}{\int_a^\infty f(m)(m-a)^{-\varepsilon} dm}. \quad (\text{A1})$$



Write  $\sigma = m - a(z)$ , so that

$$\sigma_0 = m_0 - a(z) = \left\{ \int_0^\infty g(\sigma) \sigma^{1-\varepsilon} d\sigma \right\} \left\{ \int_0^\infty g(\sigma) \sigma^{-\varepsilon} d\sigma \right\}^{-1}, \quad (\text{A2})$$

where  $g(\sigma)$  is the p.d.f. of  $\sigma$ , i.e.

$$g(\sigma) = f(\sigma + a). \quad (\text{A3})$$

We wish to prove that  $\sigma_0$  is a monotone decreasing function of  $\varepsilon$ , taking all values from  $\bar{\sigma}$  when  $\varepsilon$  is zero, to zero as  $\varepsilon$  tends to infinity;  $m_0$  then has the properties described in the text.

First, it is obvious by inspection of (A2) that  $\sigma_0 = \bar{\sigma}$  when  $\varepsilon = 0$ . To prove the other propositions, define

$$M_\varepsilon = \int_0^\infty g(\sigma) \left( \frac{1}{\sigma} \right)^\varepsilon d\sigma. \quad (\text{A4})$$

We know from Hardy, Littlewood and Polya (1934. p. 143, proposition 192, and p. 146, proposition 197) that  $\log M_\varepsilon$  is an increasing, convex function of  $\varepsilon$  for  $\varepsilon$  strictly positive. But

$$\log \sigma_0 = \log M_{\varepsilon-1} - \log M_\varepsilon, \quad (\text{A5})$$

so that for  $\varepsilon > 1$ ,  $\log \sigma_0$  is a decreasing function of  $\varepsilon$  since the negative difference on the right-hand side of (A5) becomes steadily larger as  $\varepsilon$  increases. By the same token,  $\log \sigma_0 \rightarrow -\infty$  as  $\varepsilon \rightarrow \infty$  so that  $\sigma_0 \rightarrow 0$  as  $\varepsilon \rightarrow \infty$ . To complete the proof, we need to show that  $\sigma_0$  declines with  $\varepsilon$  when  $0 \leq \varepsilon \leq 1$ . Define

$$\log N_\varepsilon = \log M_\varepsilon \cdot \varepsilon^{-1}. \quad (\text{A6})$$

Then

$$\log \sigma_0 = (\varepsilon - 1) \log N_{\varepsilon-1} - \varepsilon \log N_\varepsilon. \quad (\text{A7})$$

Since we have assumed that moments of all orders of  $g(\sigma)$  exist, we may differentiate  $N_\varepsilon$  and  $N_{\varepsilon-1}$  with respect to  $\varepsilon$ . (A proof without differentiation would not be hard to construct.) Hence

$$\begin{aligned} \frac{\partial \log \sigma_0}{\partial \varepsilon} &= -(\log N_\varepsilon - \log N_{\varepsilon-1}) + (\varepsilon - 1) \frac{\partial \log N_{\varepsilon-1}}{\partial \varepsilon} \\ &\quad - \varepsilon \frac{\partial \log N_\varepsilon}{\partial \varepsilon}. \end{aligned} \quad (\text{A8})$$

Hardy, Littlewood and Polya (1934, p. 144) show that the first bracketed expression on the right-hand side of (A8) is positive, as are the two derivatives. Consequently, for  $0 \leq \varepsilon \leq 1$ , (A8) is negative, and  $\sigma_0$  declines with  $\varepsilon$ .

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