

OPTIMALLY UNIFORM COMMODITY TAXES *

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In a many-person economy with an optimal *linear* income tax, weak separability between goods and leisure together with linear Engel curves for goods render discriminatory commodity taxes unnecessary. The result extends to uniform taxation for separable groups of goods.

1. Introduction

Perhaps the most interesting case in optimal tax theory is that where, in a many person economy, the government is allowed a linear income tax supplemented, as necessary, by differential taxes on individual commodities. Atkinson and Stiglitz (1976) showed that, in a number of special cases, differential commodity taxes are unnecessary. Atkinson (1977) showed that this is also true if all consumers have preferences over leisure and goods corresponding to the linear expenditure system. In this paper, I generalise this result further and show that provided preferences are weakly separable between goods and leisure, and provided all consumers have parallel linear Engel curves for goods in terms of income, then differential commodity taxation is unnecessary given an optimal linear income tax. Further, if there exists a separable group of goods for which the within-group Engel curves are linear, then all goods within the group should be taxed at the same rate. These results are clearly related to that of Atkinson and Stiglitz whereby, given an optimal *non*-linear income tax, weak separability (*without* linear Engel curves) is sufficient for uniform commodity taxation. Since linear Engel curves can provide a local first-order approximation to any demand functions, calculations of optimal differentiated taxes would require knowledge of higher-order derivatives if weak separability were true. Conversely, we can learn nothing about commodity taxes from consumer demand studies in which commodity demands are explained conditionally on total expenditure and commodity prices and which assume linear Engel curves. The results of all such studies are consistent with uniform commodity taxation.

Section 2 of the paper derives the usual many person tax rule in a form suitable

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for demonstrating the results. The rule is based on first-order maximization conditions and to the extent that these do not characterise the optimum, none of the results in the paper are likely to be valid. This proviso should be borne in mind throughout. Section 3 presents and proves the main result, that if individual h 's utility function $v^h(q_0, q)$ defined over leisure q_0 and a vector of goods q , takes the form

$$u^h = v^h(q_0, q) = f^h\{q_0, \theta^h(q)\}, \quad (1)$$

where the maximized indirect utility function corresponding to $\theta^h(q)$ has the form

$$u_1^h = \{x^h - a^h(p)\}/b(p), \quad (2)$$

for total good expenditure x^h and prices p , then differential commodity taxes are unnecessary. Section 4 strengthens the result to show uniform commodity taxation for any separable subgroup of commodities.

2. The problem and first-order conditions

The government's problem is to maximize social welfare W given by

$$W = W(u^1, u^2, \dots, u^h, \dots, u^H). \quad (3)$$

The individual u^h 's must satisfy the cost-function income equality

$$c^h(u^h, p_0^h, p) = p_0^h T + b, \quad (4)$$

where p_0^h is the h th wage rate, T is the time endowment, and b is the uniform benefit level paid to (by) each individual. The revenue constraint is written

$$\sum_h \sum_{k=1}^n t_k c_k^h(u^h, p_0^h, p) = R + Hb, \quad (5)$$

where $c_k^h(u^h, p_0^h, p)$ is the partial derivative of $c^h(u^h, p_0^h, p)$ with respect to p_k (and equals the quantity demanded q_k^h), t_k is the tax rate on good k , R is necessary revenue (denominated in wage units) and H is the number of consumers. The n tax rates t_1, \dots, t_n and b are the control variables.

Routine manipulation of the relevant Lagrangean and first-order conditions gives the following formula:

$$\sum_{k=1}^n \bar{s}_{ik} t_k = -(\bar{q}_i - q_i^*), \quad (6)$$

where bars denote arithmetic means, so that

$$\bar{s}_{ik} = \frac{1}{H} \sum_h s_{ik}^h = \frac{1}{H} \sum_h \frac{\partial^2 c^h(u^h, p_0^h, p)}{\partial p_i \partial p_k}, \quad (7)$$

$$\bar{q}_i = \frac{1}{H} \sum_h q_i^h = \frac{1}{H} \sum_h \frac{\partial c^h(u^h, p_0^h, p)}{\partial p_i}, \quad (8)$$

$$q_i^* = \sum_h \frac{\lambda^h}{H\bar{\lambda}} q_i^h, \quad (9)$$

where the weights λ^h are given by

$$\lambda^h = \theta^h + \bar{\theta} r^h / (1 - \bar{r}), \quad (10)$$

where θ^h is the marginal social utility of one unit of currency to household h ($= \partial W / \partial u^h \div \partial c^h / \partial u^h$) and r^h is the marginal tax revenue resulting from giving the unit of currency.

Equations similar to these are familiar enough. λ^h is the *net* social marginal utility of money to h , so that, for an equality respecting social welfare function, the weighting in (7) will be biased towards low welfare households. Hence, the bracketed term on the right-hand side of (4) will be largest for luxuries and smallest for necessities so that the rule will tend to discourage consumption of the former by more than that of the latter.

An *informal* idea of the power of separability can be gauged by the usual device of assuming $\bar{s}_{ik} \simeq 0$ for $i \neq k$. If so, (6) becomes

$$t_i / p_i = (1 - q_i^* / \bar{q}_i) / (-\bar{e}_{ii}), \quad (11)$$

where \bar{e}_{ii} is the average compensated own-price elasticity. Clearly the tax rate is the ratio of the ‘social’ luxury index $(1 - q_i^* / \bar{q}_i)$ for good i to its compensated own price elasticity. Hence, if luxuries are price elastic and necessities price inelastic – a natural tendency which is inevitable under additive preferences [see Deaton (1974)] – the approximation (9) will give uniform taxes. The equity pressure for higher taxes on luxuries is exactly offset by the efficiency requirements operating against necessities.

3. A sufficient condition for uniform taxation

By homogeneity, we have for each individual

$$\sum_{k=1}^n s_{ik}^h p_k + s_{i0}^h p_0^h = 0, \quad (12)$$

so that, averaging,

$$\sum \bar{s}_{ik} p_k = -\bar{s}_{i0} p_0. \quad (13)$$

Hence, for a uniform tax solution to (b) with $t_k = \tau p_k$ say, we require

$$\tau \overline{s_{i0} p_0} = \bar{q}_i - q_i^* . \quad (14)$$

If a tax rate τ exists which satisfies this (so that, for example, we rule out cases where the government is trying to raise more tax than is possible), it must be possible to cancel the factors involving the index i from both sides. Now, if each consumer has preferences given by (1) and (2), commodity demands may be written as a function of p and $x^h (= p_0^h(T - q_0^h) + b)$ alone, and by (2), these take the linear form

$$q_i^h = \alpha_i^h(p) + \beta_i(p)x^h . \quad (15)$$

It can easily be checked that $\beta_i(p) = b_i(p)/b(p)$ (and is not indexed on h) and $\alpha_i^h(p) = a_i^h(p) - a^h(p)b_i(p)/b(p)$. Hence, calculating \bar{q}_i and q_i^* ,

$$\bar{q}_i - q_i^* = \beta_i(p)(\bar{x} - x^*) , \quad (16)$$

where x^* is a weighted average of x^h 's using the same weights as in (9). Weak separability between goods and leisure also links the substitution response s_{i0}^h to the full income derivatives. Hence, for all h , (1) implies

$$s_{i0}^h = \phi^h \beta_i(p) , \quad (17)$$

where ϕ^h is not indexed on i . Hence, averaging

$$\overline{s_{i0} p_0} = \overline{\phi p_0} \beta_i(p) , \quad (18)$$

so that $\beta_i(p)$ factors out of both sides of (14) yielding a uniform tax solution to the original problem (provided a solution exists at all).

It does not seem possible to show that the above conditions are necessary for uniform taxation. Although it can easily be shown that the condition

$$s_{i0}^h p_0^h = c_i^h(u^h, p_0^h, p) - c_i^h(u^{*h}, p_0^{*h}, p) , \quad (19)$$

for all u^* and u implies both weak separability and linear Engel curves, it is not this condition but an averaged version of it which must hold. However, it seems unlikely – i.e., very special cases apart – and given no restrictions on social welfare function (so that q_i^* can be varied relative to \bar{q}_i), that uniform taxation will be possible under weaker conditions.

4. Uniform taxation for a subgroup of goods

Let q_G be a weakly separable subgroup of commodities so that the direct utility function takes the form

$$u^h = v^h(q_0, q_{\bar{G}}, \theta^h(q_G)) , \quad (20)$$

where $q_{\bar{G}}$ denotes goods not in the group. On the assumption that $\theta^h(q_G)$ in indirect form permits linear Engel curves, the analogue of (15) is, for $i \in G$,

$$q_i^h = \alpha_i^h(p_G) + \beta_i(p_G) x_G^h, \quad (21)$$

where x_G^h is expenditure by household h on the group G and p_G is the price vector for the group. It immediately follows that for $i \in G$ and $j \in \bar{G}$,

$$s_{i0}^h = \beta_i(p_G) \delta x_G^h / \delta p_0^h, \quad (22)$$

$$s_{ij}^h = \beta_i(p_G) \delta x_G^h / \delta p_j, \quad (23)$$

where $\delta x_G^h / \delta p_0^h$ and $\delta x_G^h / \delta p_j$ are the derivatives of x_G^h with respect to p_0^h and p_j with utility held constant – see Gorman (1971) for the origin of this notation. Note that both these derivatives are independent of i so that both s_{i0}^h and s_{ij}^h are proportional to $\beta_i(p_G)$.

Eq. (21) also gives [analogously to (16)] for $i \in G$,

$$\bar{q}_i - q_i^* = \beta_i(p_G) (\bar{x}_G - x_G^*), \quad (24)$$

where the asterisks denote weighted averages as before. Homogeneity, (12), implies that for all i ,

$$\sum_{k \in G} \bar{s}_{ik} p_k + \sum_{k \in \bar{G}} \bar{s}_{ik} p_k = -\bar{s}_{i0} \bar{p}_0, \quad (25)$$

so that a necessary condition for the optimal tax rule to give a uniform rate τ_G , say, for the group G is that a solution should exist to

$$\tau_G s_{i0} p_0 - \sum_{k \in \bar{G}} \bar{s}_{ik} (t_k - \tau_G p_k) = \bar{q}_i - q_i^*, \quad i \in G. \quad (26)$$

Once again, terms involving i must cancel from both sides and it is easily checked that eqs. (22), (23) and (24) guarantee this.

The result of this section implies that of section 3 since \bar{G} may be empty. Moreover, the partial result may be the more useful in practice since, although weak separability between all goods and leisure is unlikely to be valid, there are almost certainly groups of goods for which the separability assumption is acceptable.

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