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THE MEASUREMENT OF INCOME AND PRICE ELASTICITIES

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In a complete system of disaggregated demand equations, the econometrician must limit himself to the measurement of a small number of parameters for each commodity. Since interest often focuses on the income and own-price elasticities for each good, it is natural to look for models which allow independent measurement of these while providing plausible assumptions about the less essential responses. This paper surveys a number of common theoretical specifications which purport to do this, and argues that these are unsatisfactory. The conclusions are supported by an empirical comparison on British data of the results of three models, one of which is new and is designed to remedy some of the deficiencies of standard practice.

1. Introduction

Although economists have been measuring demand elasticities since the seventeenth century, many important problems of methodology remain unsolved. These are particularly acute when the investigator wishes to analyse a reasonably disaggregated classification of consumers' expenditures. Estimation difficulties and lack of information require the use of a model containing a great deal of prior information, and here there is a wide range of possibilities, each embodying its own particular assumptions. One of the most common and attractive simplifications is to attempt to characterise behaviour in terms of two responses, income elasticity and own-price elasticity, allowing assumption, rather than direct measurement, to determine appropriate values for the cross-price responses. In this paper, various ways in which this simplification can be implemented are examined and an attempt is made to assess the likely effects of adopting rather different models. This involves separating the effects of choice of functional form from the effects of the more fundamental assumptions about economic behaviour. Finally, some conclusions are offered on just how far the 'two-response' hypothesis can take us; the numerical results are specific to British experience but the qualitative conclusions are likely to hold elsewhere.

2. Alternative strategies

It is important in designing an appropriate model of consumers' behaviour to begin by examining the type of data which it is to be asked to explain. In the context of estimating price and income elasticities, time series data are used.

These are usually confined to the postwar period so that some twenty annual observations are available. As is well known, the independent variables, income (or total expenditure) and the prices, exhibit strong positive intercorrelation so that, over so short a period, we can only expect to identify a limited number of independent responses. If we accept that only *real* income and *relative* prices are relevant, most of the information available relates to the former with relatively little variation in the latter. In consequence, it will usually be possible to measure income responses with quite reasonable precision, whereas measurement of price responses will be subject to larger error and will be more affected by prior assumptions. Although there are exceptions, we should expect most goods to be more sensitive to their own prices than to the prices of other goods, so it is natural to devote most attention to the measurement of the own-price response. This can be done in a number of different ways.

The first, and oldest, methodology is to specify quantities demanded as a double logarithmic function of two arguments: real income, and price relative to some overall index, i.e.,

$$\log q_i = a_i + b_i \log(\mu/\pi) + c_i \log(p_i/\pi), \tag{1}$$

where q_i is the quantity of the ith good demanded, p_i is its price, μ is total expenditure, π is an index of all prices, and a, b and c are vectors of the parameters of the system. Since b_i is the income elasticity of good i, and c_i is approximately its own-price elasticity, eq. (1) offers a direct and very simple way of measuring the responses in which we are most interested. However, the model is not consistent for any individual with the hypothesis of utility maximization, and if used for projection will eventually lead to gross over-prediction of income elastic goods, since if $b_i > 1$ the budget share of the ith good will increase without limit as real income rises. Even so, the model remains a useful means of summarizing the evidence over the sample period.

At a more formal level, the theory of utility maximization has been much used to derive restrictions on demand functions. At its most general, the theory does not contain sufficient prior information to allow estimation on any but the broadest of groupings of the data, but the use of particular assumptions can take us much further. Some of these bear considerable resemblance to the 'two response' hypothesis, specifically, direct and indirect additivity of preferences. The first of these, which is as old as utility theory itself, asserts that goods satisfy independent wants, so that the utility function $v(q)$ can be written

$$v(q) = \theta\left(\sum_{i=1}^{n} v_i(q_i)\right), \tag{2}$$

for some monotone increasing function θ, and where each function $v_i(q_i)$ is a function of q_i alone. Indirect additivity, invented by Leser (1942) but first systematised by Houthakker (1960a, b), asserts that the indirect utility function $\psi(p\,\mu^{-1})$ can be written

$$\psi(p\,\mu^{-1}) = \phi\{\sum \psi_i\,(p_i\,\mu^{-1})\}. \tag{3}$$

As might be expected, each of these places strong restrictions on the cross-price responses. Direct additivity implies that compensated cross-price derivatives are proportional to the product of the corresponding income derivatives, while indirect additivity implies that all cross-price elasticities are independent of the good affected, depending only on the good whose price has changed. It might be thought that this leaves the own-price elasticities unaffected, and many investigators have used these models to measure income and own-price elasticities. However, as I have shown elsewhere [see Deaton (1974b)], both (2) and (3) enforce an approximate linear relationship between income and own-price elasticities, so that these models allow the measurement of only one response per commodity. Given the dominance of income variation in most data, the price elasticities measured on the basis of either (2) or (3) are unlikely to bear any close relation to their true values. In defence of additivity, it may be argued that it is more appropriate for broad groups of commodities than for a disaggregated classification. However, it is not clear that price elasticities should be directly related to income elasticities even for broad groups, and such evidence as exists, nearly all on a broad classification, would reject the validity of the assumption [see, for example, Barten (1969), Byron (1970), Deaton (1974a)]. However, the main point for the discussion in this paper is that either type of additivity is inconsistent, by assumption, with the 'two-response' hypothesis.

An even more explicit case of the two-response assumption is provided by the model examined by Fourgéaud and Nataf (1959). They assume that the quantity demanded can be written as a function of real income and real price, i.e.,

$$q_i = f_i(\mu/\pi,\, p_i/\pi), \tag{4}$$

where π is a linear homogeneous price index. They then derive an exhaustive catalogue of those functional forms which allow (4) to be consistent with utility maximization. I have shown elsewhere [see Deaton (1974c)] that, in spite of the apparent generality of eq. (4), the income and price elasticities of the Fourgéaud and Nataf model, like those derived from direct and indirect additivity, lie approximately along a straight line. Consequently such a model cannot be used for the independent measurement of income and own-price elasticities. Since the Fourgéaud–Nataf model, as well as direct and indirect additivity, are special cases of Pollak's (1972) concept of generalised additive separability (GAS), it is interesting to enquire whether or not GAS itself allows independent price and income responses. The answer appears to be in the affirmative, although I have not succeeded in constructing an actual example of GAS which does so.

In summary, models which are general enough to allow independent estimation of price and income responses are too general to estimate on disaggregated data, whereas models which can be estimated in these circumstances either embody restrictions which make them applicable only to broad groups, if at all, or are inconsistent with the theory of demand.

3. The experiments

In Deaton (1974b), in order to illustrate the effects of additivity, I compared the elasticities generated by the constant-elasticity model (CEM), i.e., eq. (1), with those obtained from the linear expenditure system (LES), which is directly additive. Although I do not believe the results to be seriously misleading, the procedure adopted has a number of failings. First, the two models have quite different functional forms for the demand equations, so that at least some of the divergence between them can be ascribed to this rather than to the assumption of additivity itself. Second, and again because of the functional form, the two models are non-nested so that it is impossible to make direct inferences about the superiority of either one. These two problems were inevitable given the absence of a model which is non-additive and which contains the LES as a special case. Since then, I have discussed a new simple non-additive model (SNAM) [see Deaton (1974c)] which has these properties, so that in this paper I shall compare all three systems over the same data in an attempt to quantify more directly the effects of additivity and of choice of functional form.

The CEM has already been described by eq. (1). The LES can be derived from the cost (or expenditure) function

$$g(p, v) = \sum p_k \gamma_k + v \prod p^{\alpha_i}, \tag{5}$$

where v denotes the level of utility (or any monotone increasing transform thereof) and the parameters α are normalized to sum to unity. Differentiation of (5) with respect to p_i and substitution for v gives the demand functions themselves, i.e.,

$$p_i q_i = p_i \gamma_i + \alpha_i \{\mu - \sum p_k \gamma_k\}. \tag{6}$$

The SNAM cost function has an extra term over that of the LES and may be written

$$g(p, v) = \sum p_k \gamma_k - \sum \beta_k p_k \log(p_k/\pi) + v \prod p^{\alpha_i}, \tag{7}$$

where π is a base-weighted price-index with the β-parameters as weights, i.e.,

$$\pi = \sum \beta_k p_k / \sum \beta_k. \tag{8}$$

This model and its properties are more fully described elsewhere [see Deaton (1974c)]; most important in this context is its embodiment of the two-response hypothesis and the fact that it reduces to the LES when $\beta = 0$. Estimates of α determine the income responses – as in the LES these are marginal propensities to spend – whereas the β-parameters, together with the α's, give the own-price responses; the γ-parameters exert no decisive influence and are basically intercept terms. The demand functions corresponding to (6) may be written

$$p_i q_i = p_i \gamma_i - p_i \beta_i \log(p_i/\pi) + \alpha_i\{\mu - \sum p_k \gamma_k - \sum p_k \beta_k \log(p_k/\pi)\}. \quad (9)$$

The data used were U.K. annual time series from 1954–1972 covering 37 goods and services; the list of these is given in table 1. These deliberately exclude durable goods. Parameters were estimated by OLS for the CEM and by the hierarchic maximum-likelihood method discussed in Deaton (1974c) for the LES and the SNAM. Price and income elasticities for 1970 are presented in table 1; the year 1970 has no special significance but its use aids computation since it is the base year for the price series. Before discussing these, it is worth noting that, on a likelihood criterion, the SNAM is very markedly superior to the LES for all of the groups distinguished in the hierarchy, so that the evidence suggests that there is more information in the data than can be accommodated by the LES and that this is put to good use by the SNAM.

The relationship between the three sets of total expenditure elasticities are illustrated in fig. 1, which shows superimposed scatter diagrams for the SNAM against the LES and the SNAM against the CEM. Dealing with a minor point first, it is clear that there is a tendency for the SNAM elasticity to be less than the CEM elasticity for income elastic goods. This is simply a consequence of taking 1970 rather than an earlier year as a basis for the comparisons, and reflects the tendency of a linear model to understate, and a loglinear model to overstate, the response of a rapidly rising expenditure near the end of the sample period. In general, the correlation between the three sets of estimates is good and the slope of the scatter is close to the ideal 45 degrees. Six commodities are well out of line for one system or another: the four fuels, vehicle operation and domestic service. The fuels are almost certainly the most highly substitutable commodities in the classification since, unlike the other goods, they provide no utility per se but are inputs in the production of power, heat and light. It is thus to be expected that the two response hypothesis will be unable to model them satisfactorily. Domestic service is similarly difficult; it is a good which is invariably highly income elastic in cross-section studies and invariably inferior in time series. This situation presumably reflects changes in the technology of household consumption which have encouraged the substitution of mechanical power for human power. Vehicle operation, like electricity, is anomalous only for the SNAM–CEM comparison while the two linear models give similar estimates;

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Table 1
Elasticities for three alternative models (1970).

		Income elasticities			Price elasticities		
		CEM	LES	SNAM	CEM	LES	SNAM
1.	Bread and cereals	−0.499	−0.599	−0.519	−0.221	+0.229	−0.452
2.	Meat and bacon	0.289	0.347	0.235	−0.439	−0.178	−0.692
3.	Fish	−0.034	−0.145	−0.328	−0.094	+0.068	−0.225
4.	Oils and fats	−0.322	0.063	−0.288	−0.266	−0.030	−0.060
5.	Sugar and sweets	−0.446	−0.306	−0.402	−0.192	+0.149	+0.218
6.	Dairy produce	0.534	0.470	0.598	−0.031	−0.229	−0.009
7.	Fruit	0.273	0.518	0.168	−0.359	−0.245	−0.406
8.	Potatoes and vegetables	0.870	0.767	0.700	−0.171	−0.368	−0.372
9.	Beverages	1.369	0.696	0.783	+0.300	−0.329	−0.134
10.	Other man. food	0.298	0.893	0.398	−1.180	−0.417	−0.696
11.	Footwear	1.201	0.984	0.992	+0.223	−0.471	−0.733
12.	Clothing	0.678	0.968	0.885	−0.720	−0.481	−0.687
13.	Rents	1.201	1.772	1.664	−0.147	−0.857	−0.714
14.	Maintenance	2.010	1.703	2.157	−2.212	−0.797	−1.980
15.	Coal	−2.022	−4.081	−1.088	−0.321	+2.036	−2.183
16.	Electricity	3.761	2.628	2.454	−0.962	−1.203	−0.299
17.	Gas	1.740	2.688	1.276	−2.644	−1.236	−2.900
18.	Other fuel	2.512	2.881	−2.009	+0.546	−1.331	−2.735
19.	Beer	1.221	1.272	1.051	+0.238	−0.612	+0.281
20.	Wines and spirits	2.599	1.976	2.459	−0.339	−0.920	−0.034
21.	Cigarettes and tobacco	−0.031	−0.092	−0.142	+0.117	+0.048	+0.519
22.	Communication	2.356	2.516	1.628	−0.430	−1.160	−0.716
23.	Vehicle operation	5.393	2.978	3.252	−0.011	−1.316	+0.982
24.	Rail travel	−0.400	−0.554	−0.593	−0.541	+0.262	−0.922
25.	Other travel	1.051	0.489	0.577	−1.509	−0.237	−1.030
26.	Exp. abroad	1.143	1.352	0.959	−1.626	−0.634	−2.119
27.	Textiles and hardware	1.992	1.530	1.799	−0.050	−0.719	−0.098
28.	Matches soap etc.	0.070	0.289	0.020	−0.052	−0.136	−0.008
29.	Books and magazines	−0.038	0.407	0.036	−0.131	−0.191	−0.212
30	Newspapers	−0.211	−0.974	−0.060	−0.337	+0.464	−0.455
31.	Recreational goods	1.988	1.684	1.704	−0.672	−0.789	−0.569
32.	Chemists' goods	1.435	1.463	1.238	−0.890	−0.685	−0.988
33.	Other goods	0.493	0.949	0.546	−1.201	−0.447	−1.288
34.	Domestic service	−0.987	−1.133	−2.896	−0.393	+0.533	−0.388
35.	Catering	1.638	0.141	1.105	−2.605	−0.072	−1.990
36.	Entertainment	0.893	1.013	0.494	−1.399	−0.479	−1.121
37.	Other services	1.452	1.287	0.955	−0.366	−0.632	+0.013

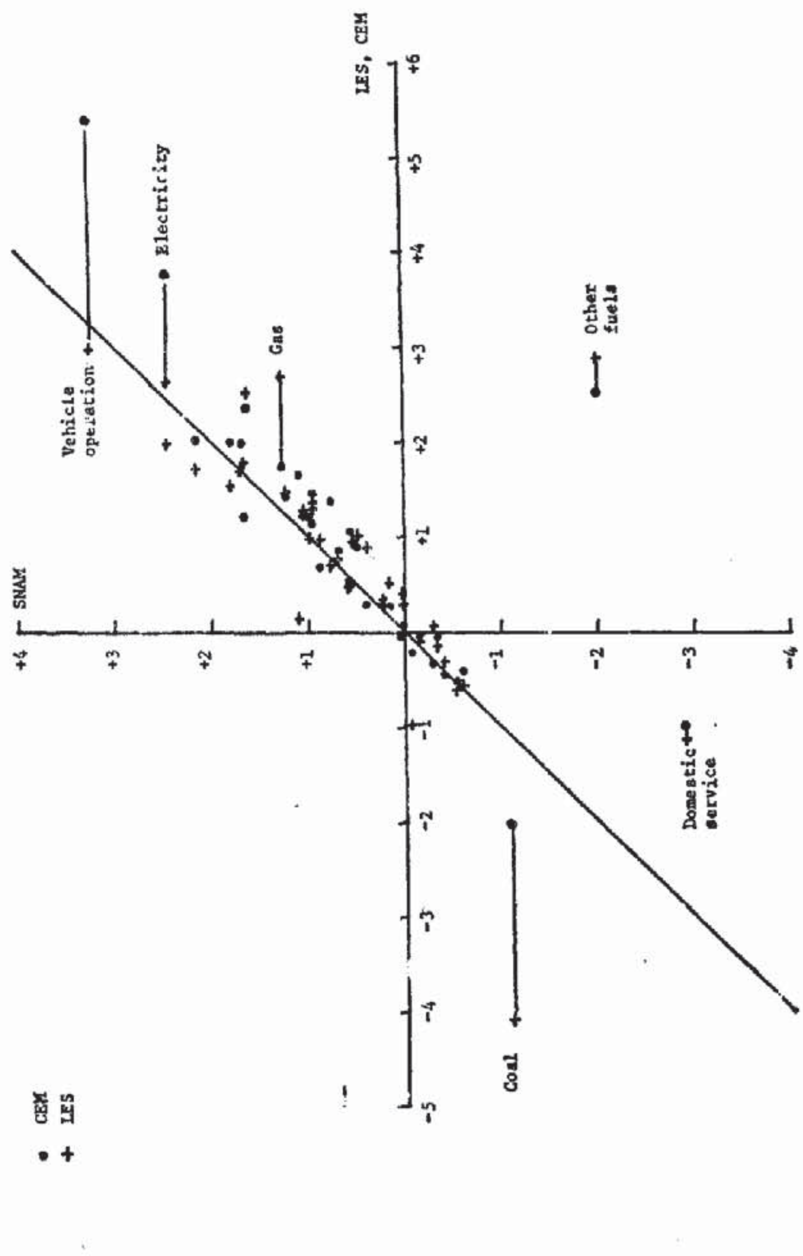

Fig. 1. 1970 income elasticities for SNAM, LES and CEM models.

some of this is undoubtedly due to the underestimation–overestimation phenomenon discussed above, given the very high income elasticity of both commodities. However, a more complete model of vehicle operation would certainly include the stock of motor cars as an explanatory variable and we cannot expect to get sensible results without it.

Apart from these six commodities, this picture is what might be expected given the strength of the income information in the data. This appears to be strong enough to overcome the effects of both functional form and the additivity assumption, except for those commodities where, for other reasons, the demand function is clearly misspecified. It is also worth noting that there is no evidence for my (1974b) contention that the inability of the LES to measure price responses independently of income responses gives rise to significant biases in the income elasticities. This obviously remains as a theoretical possibility but it does not appear to occur in this sample.

Figs. 2 and 3 illustrate the corresponding relationships for the price elasticities;

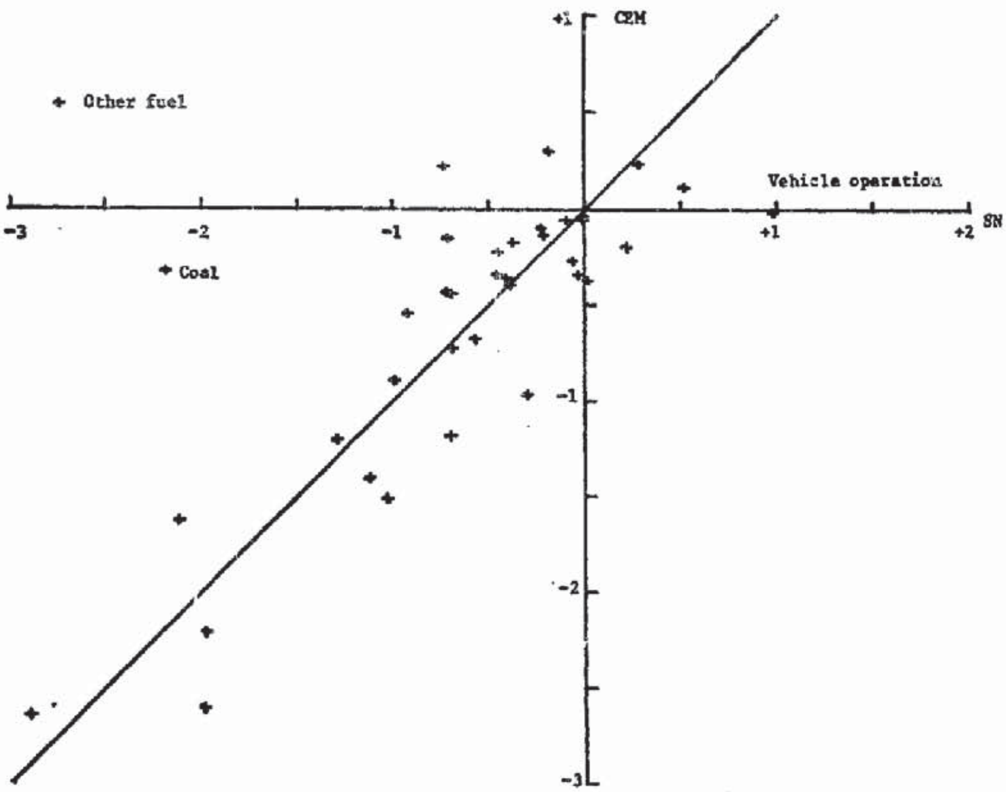

Fig. 2. 1970 price elasticities for CEM and SNAM models.

those for the CEM and the SNAM are illustrated in fig. 2 and those for the SNAM and LES in fig. 3. Looking at fig. 2 first, the correlation between the two sets of estimates is much less close than that between the income elasticities, but it is still quite marked and once again the slope is close to 45 degrees. Of the six goods which were anomalous in the income elasticity comparisons, three reappear here (namely coal, other fuels, and vehicle operation), although a good many other commodities are some distance from the line of equality. Nevertheless, it is clear from this diagram that, in spite of the differences of functional form between the models, there is sufficient information in the data to allow inferences to be made about own-price elasticities as well as about income-elasticities. The looseness of the scatter indicates that such inferences are more difficult since they are more dependent upon the choice of functional form and presumably more sensitive to misspecifications about substitution possibilities and technical change. Many of these could be corrected in further

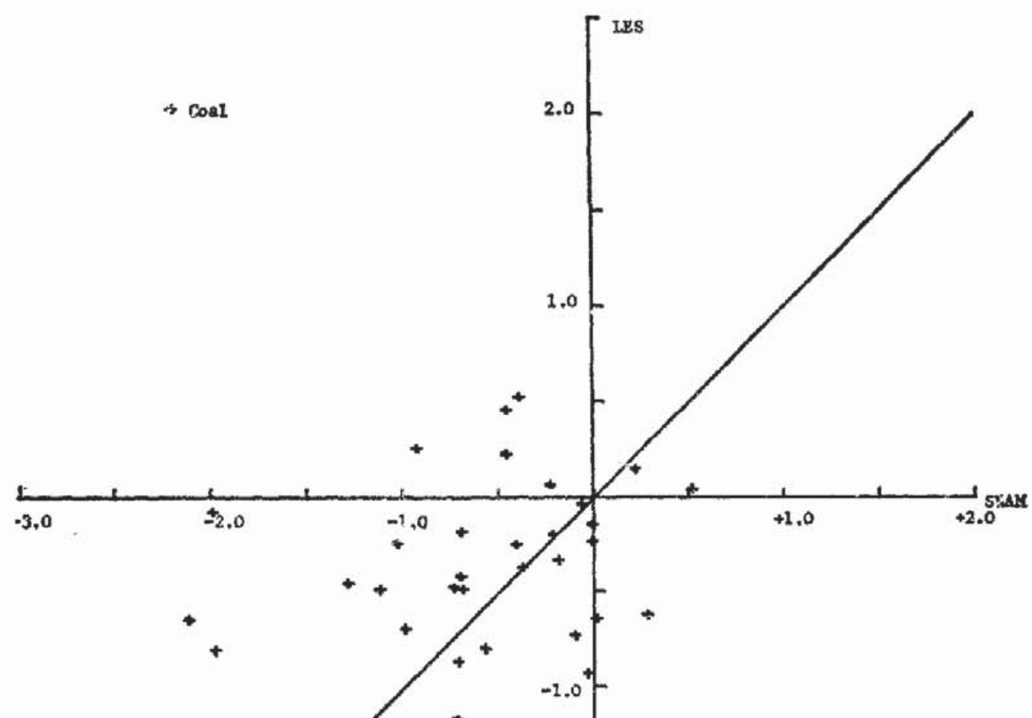

Fig. 3. 1970 price elasticities for LES and SNAM models.

work with the SNAM and it would obviously be desirable to remove those five commodities which exhibit *positive* price elasticities. This could perhaps be done by allowing factors other than price and income to affect the γ-parameters of eq. (9), and it may even be possible to introduce a type of weak-separability into the SNAM by introducing some group-specific prices; these are topics for further research.

Examination of fig. 3 shows that the additivity assumption inherent in the LES effectively destroys the price information in the data. The scatter is almost random and we know that it is the LES rather than the SNAM which is at fault, partly because the latter is much superior in terms of likelihood and partly because the SNAM elasticities are given considerable support by the independently estimated CEM elasticities. Why this happens is quite clear. Both the SNAM and the LES show that, in reality, there is no link between price and income elasticities, but the LES, by construction, enforces proportionality between them. Consequently, one or both responses will be measured incorrectly. In this case, it appears from the diagrams that real income variation is so dominant over relative price variation in the sample that it is the price responses which bear little or no relation to their true values, while the income responses suffer relatively little distortion. The scatter is so wide in fig. 3 that it is hardly worth documenting particular examples of bias. Even so, a number of regularities appear. The number of 'Giffen' goods in the LES is much higher than in either the SNAM or CEM; this occurs because all inferior goods must have positive price elasticities even to the point of giving coal a price elasticity of more than (plus) 2. It can also be seen that the LES produces relatively few price elastic goods – only 5, compared with 9 for the SNAM and 8 for the CEM. For a good to be modelled by the LES as price elastic it must be very income elastic and it is clear that there exist genuinely price elastic goods which are relatively insensitive to changes in income. Clearly, the LES is not an adequate tool for measuring price elasticities.

4. Some index number comparisons

The extra price-sensitivity of the SNAM over the LES implies that the consumer substitutes more readily between goods under the new formulation. Consequently, he is more readily able to protect himself against price rises than is the consumer whose preferences are represented by the LES. It is thus of considerable interest to compare the cost-of-living index numbers associated with the two models, and this final section is devoted to this. Both cost functions can be written in the form

$$g(p, v) = A(p) + B(p)v, \tag{10}$$

so that an index representing the cost-of-living at time t, based on the utility level of time $t-1$, may be constructed as

$$I_{t,t-1} = [A(p_t)+B(p_t)v_{t-1}]/g_{t-1}, \tag{11}$$

where v_{t-1} is calculated from (10). The formulae for A and B in the LES and SNAM can be read off from eqs. (5) and (7). Chain indices based on 1954 = 100 were then constructed by setting $I_0 = 100$, where subscript zero refers to 1954, and then calculating

$$I_\theta = I_0 \prod_{t=1}^{\theta} I_{t,t-1}. \tag{12}$$

It is also convenient to have a third index which assumes zero substitutability to give a standard with which the difference between the LES and SNAM indices can be compared. This is provided by the Laspeyres index, i.e.,

$$I_{t,t-1} = (\sum q_{t-1}p_t)/\sum q_{t-1}p_{t-1}, \tag{13}$$

with a corresponding chain index.

Since not all α_i's of the LES nor all the α_i's and β_i's of the SNAM are positive, the cost functions associated with the models are not concave in the prices as they should be. This does not necessarily prevent their use in this context, since it is always possible to add up groups of commodities – both systems aggregate in this way – so that the group parameters are positive and the aggregated cost functions concave. However, it is not worth doing this explicitly since the calculation of (11) does this automatically. This procedure does mean, however, that the value of $I_{t,t-1}$ from the Laspayres' formula does not always exceed that from the other two formulae as it theoretically should. Divergences of this sort may also occur because eq. (13) uses *actual* quantities consumed rather than the *predicted* quantities implied by the two cost functions. These difficulties did occur in the calculations but in no cases were the discrepancies other than very small.

The chain indices are presented in table 2. The three series are close to one another, as is usual for different indices within a single country, and the difference between the highest and lowest figure for 1972 is very small compared with the total change since 1954. However, the Laspeyres and LES indices are virtually identical throughout the period, with a cumulative difference of only 0.4 points out of 200 by the end of the period. This illustrates very well just how limited are the substitution possibilities permitted by the LES. By contrast, the SNAM index is below the LES by much more than the LES is below the Laspayres. Clearly then allowing greater price flexibility, even within the narrow bounds of the SNAM, can alter calculations about the cost-of-living even though these alterations may be small relative to the total rate of inflation.

Table 2
Alternative cost-of-living chain indices 1954–1972.

Year	Laspeyres	LES	SNAM	Year	Laspeyres	LES	SNAM
1954	100	100	100	1964	134.0	133.8	131.3
1955	103.3	103.3	102.6	1965	140.9	140.7	138.3
1956	107.5	107.4	106.1	1966	146.9	146.7	144.1
1957	111.2	111.1	109.7	1967	151.3	151.1	148.4
1958	114.5	114.4	112.4	1968	158.6	158.3	155.6
1959	115.7	115.5	113.5	1969	167.8	167.5	164.7
1960	117.1	116.9	114.8	1970	177.5	177.1	173.9
1961	120.8	120.6	118.6	1971	192.2	191.8	188.1
1962	126.0	125.8	123.5	1972	205.6	205.2	201.7
1963	129.2	129.0	126.6				

5. Conclusions

We have presented the results of some extremely simplistic calculations based on three alternative models of consumer demand. These are in no sense the best that could be done with the models, and the elasticities presented are obviously deficient from many points of view. However, it is hoped that these calculations have demonstrated some important points. It is possible, for a fairly wide range of commodities, to measure price and income elasticities independently of one another, and except for a few commodities which are obviously highly substitutable, considerable explanation can be achieved on the basis of the simple assumption that behaviour can be described in terms of income and own-price responses. The effects of choice of functional form on measurement depend upon the amount of information in the data, but in any case are small relative to the effects of assumptions, such as additivity, which destroy much of the available evidence.

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