11. REVIEW OF ECO 310 – GENERAL EQUILIBRIUM AND PARETO EFFICIENCY

1. GEOMETRIC TREATMENT

General equilibrium

Look at all markets simultaneously,

look for price vector such that quantity supplied = quantity demanded in all markets

Tool of analysis: indifference and transformation curves, MRS etc

Pareto efficiency or Pareto optimality

No one can be made better off without making someone else worse off

For ease of exposition only, we

[1] treat case with 2 consumers, 2 goods, 2 inputs to production,

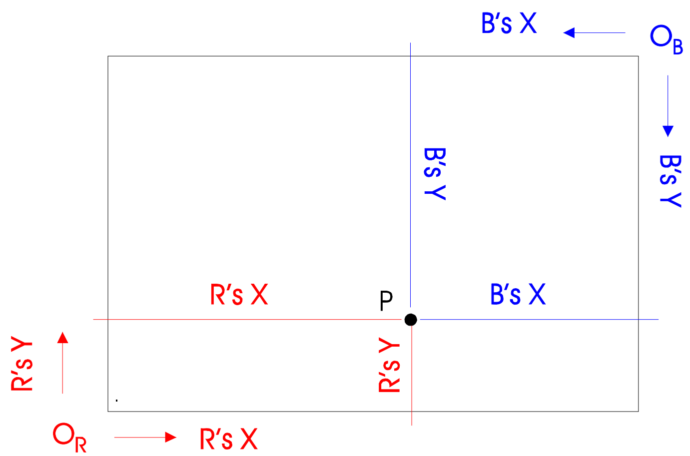
[2] separate analysis of exchange and production

here focus mostly on exchange

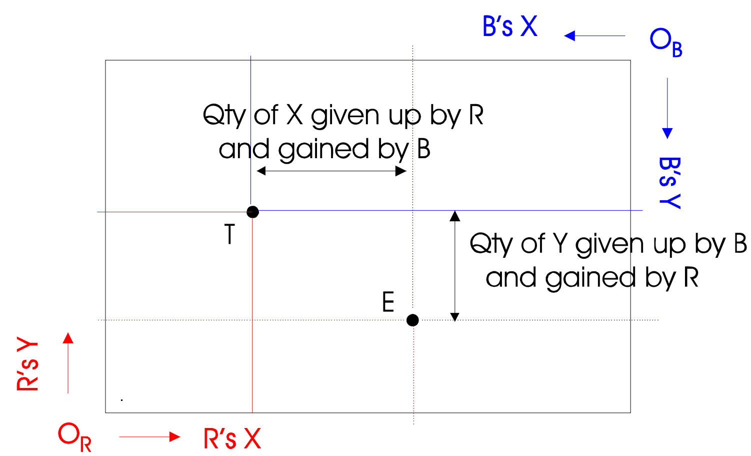
EXCHANGE – EDGEWORTH BOX DIAGRAM

Two goods X, Y, and two consumers R, B
Analyze exchange when total amounts of 2 goods are fixed
Rectangular box, lengths of sides X, Y equal to the fixed quantities of the two goods
R's quantities read from origin O_R ; B's from origin O_B in the reverse direction

Each point P in the box shows an allocation of X and Y between R and B (4 quantities)

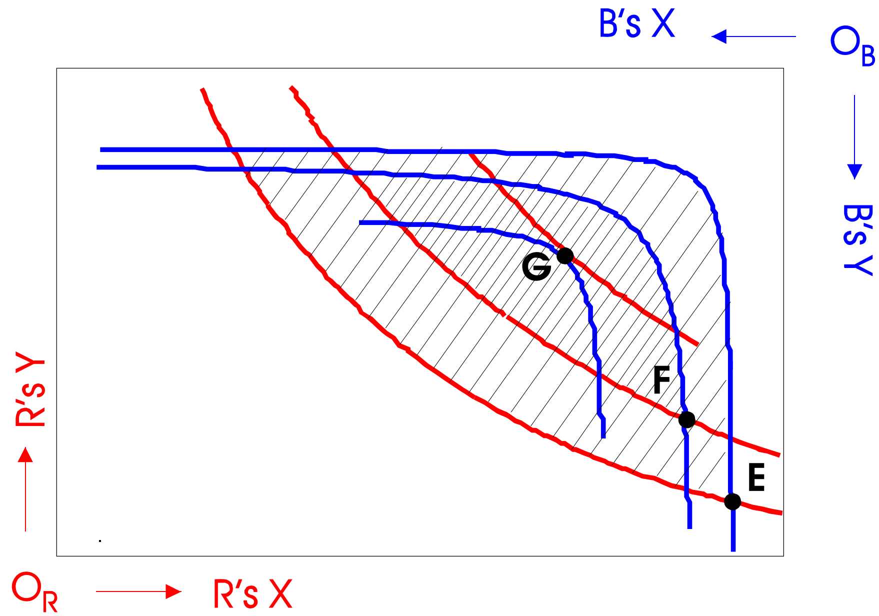


Move from one point E to another point T is a reallocation or exchange or trade



MUTUALLY BENEFICIAL AND EFFICIENT TRADES

Initial allocation E (endowment or ownership)
Move to F is mutually beneficial -
    lies above the indifference curve
    through E for both R and B
(Remember B's quantities are measured
    from O_B in reverse direction, so B's utility
    increases toward the south-west and
    B's indifference curves are rotated 180^{O})
Trade from E to any point in the shaded area
    is also mutually beneficial
Move to F still leaves open the possibility of
    further mutually beneficial trades in the
    similar smaller double-shaded area



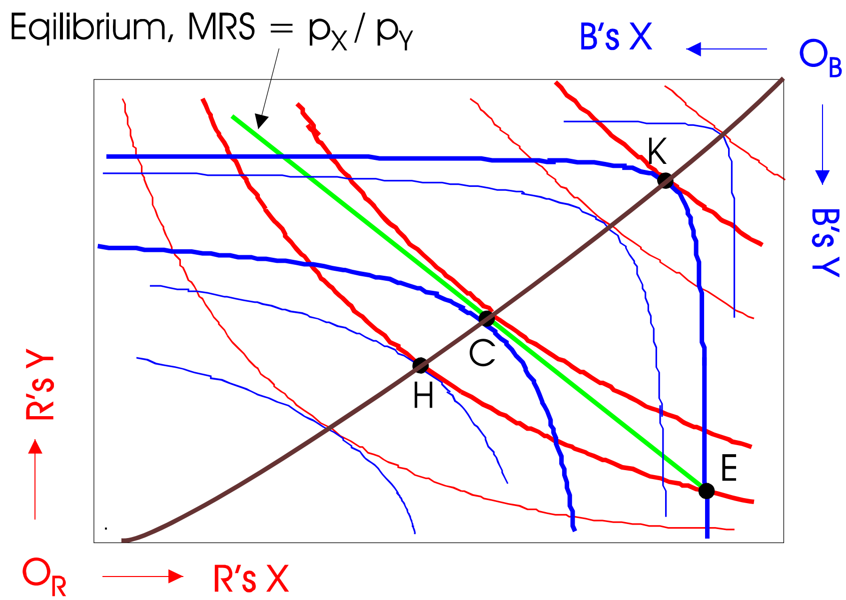
Now consider G as shown. If a further trade from F to G (or a direct trade from E to G) is made,
    then there remains no possibility for further mutual benefit
  Any further reallocation that increases R's utility must decrease B's utility and vice versa
    This is just the definition of Pareto efficiency, so G is Pareto efficient
How can we characterize a Pareto efficient allocation in the exchange Edgeworth box?
    When the shaded area of beneficial trades starting at this point vanishes
    Or when indifference curves for R and B through that point are mutually tangential
    That is, MRS between X and Y for R = MRS between X and Y for B
More generally, take any two goods; MRS between them should be same for all consumers

The “contract curve” consists of ALL Pareto efficient allocations in the exchange Edgeworth box
ignoring initial ownership / endowment ,
as if the government can seize and
redistribute goods among people
Contract curve extends from O_R to O_B

If initial ownership E must be respected,
see where indifference curves of R, B
through E intersect the contract curve
The figure shows this at H, K, respectively
Then only the portion HK becomes relevant
This is called the “core” of the exchange:
trades that are voluntary and efficient

In the core, there must be at least
one point C such that the line of
R and B’s common MRS between
X and Y at C passes through E

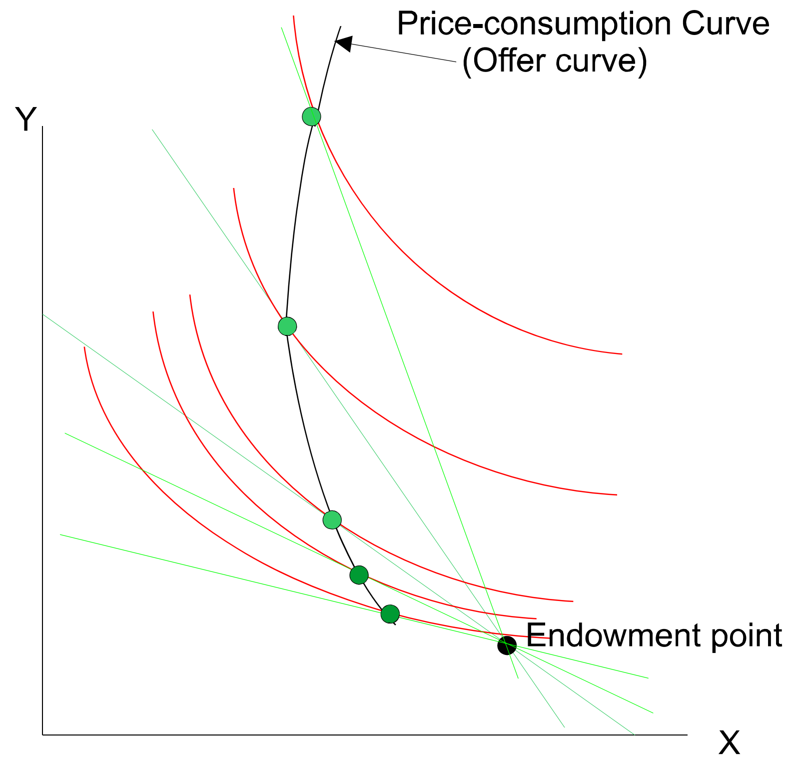
E = initial endowment, $O_R HCKO_B$ = contract curve
HK = core, C = equilibrium



This gives a way of achieving the allocation C as a competitive market equilibrium
Set the relative price of X in terms of Y equal to the slope of this line.
Also the line passes through the point E showing the endowments of both people
Therefore it becomes the common budget line for the two
The optimal choice of each is at the point of tangency with his/her indifference curve, namely C
Therefore both want to trade from E to C; that is the price-taking (perfect compet’n) equilibrium

Can show general equilibrium analogs of supply/demand curves to construct equilibrium

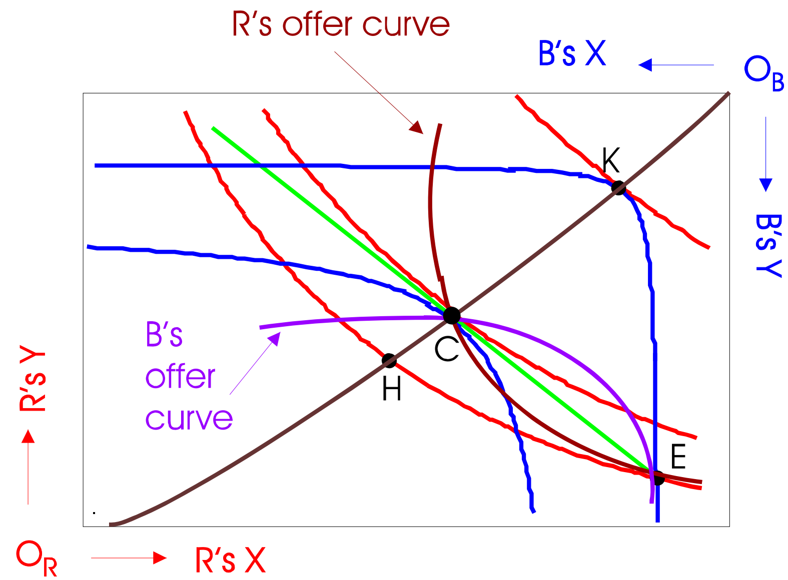
Consider just one consumer
Take budget lines of different slopes
all through endowment point E
Connect up all their tangencies
with indifference curves
This is the “price-consumption curve”
or “offer curve” : locus of all trades
the consumer optimally chooses
when facing different relative prices



Normal case: steeper budget line
(higher relative price of X) causes
the consumer to keep less X out
of endowment; trade away more
This is substitution effect

But offer curve can “bend back”
due to income effects:
when P_X / P_Y very high,
consumer can get a lot of Y by
giving up very little X, and so consume more of both X and Y than at a lower price

Now put the two consumers' offer curves
together in the exchange Edgeworth box
Where they intersect is equilibrium
Must be on contract curve, because the
two consumers' indifference curves are
tangent to the same budget line



Mathematically, such an equilibrium exists
But should we expect it to arise in reality?
If literally just two consumers, then they can
try to exercise market power or to bargain;
R wants outcome close to K and
B wants outcome close to H
If actually there are many consumers, and
each of R, B stands for many of that type, then each must compete with others of
the same type, therefore has less market power. In the limit, only C can be sustained
This is the rigorous formulation of connection between large numbers and perfect competition

Limitations of perfect competition:
[1] Says nothing about distribution – some may do much better than others depending on
initial endowments and on whether the endowment is valued highly in the market
[2] Efficiency requires numerous traders / freedom of entry so no one has market power
[3] Efficiency requires symmetric information (absence of moral hazard or adverse selection)
[4] Efficiency requires everything to be tradeable in the market
If there are external economies or diseconomies or public goods/ bads,
some benefits or costs are not priced in markets, so individuals lack correct incentives

Subject to these limitations, general equilibrium framework has wide application. Examples:

1: INTERNATIONAL TRADE

Replace consumers in above analysis by countries

A country that has relatively large endowment of a good will export it
in exchange for others of which it has less

Competitive free trade equilibrium will be Pareto efficient for the world as a whole

Each country will gain from trade. But within each country, there can be winners and losers,
raising question of whether / how to compensate losers

Countries don't usually have given endowments of goods, but
will relate production & pattern of trade soon

2: INSURANCE

Interpret the goods as wealth contingent on random events,
e.g. X = my wealth if I have good luck, Y = my wealth if I have bad luck

Then my endowment has a lot of X and very little Y

Others' endowments of X and Y are nearly equal if their luck is uncorrelated with mine

In equilibrium I will give up some X in exchange for some Y
Others will take up some of my risk for a suitable relative price

Even better if my luck is negatively correlated with others' luck

Other financial markets are essentially a generalization of this idea, in conjunction with:

3: BORROWING AND LENDING

Interpret X as this year's income and Y as next year's income

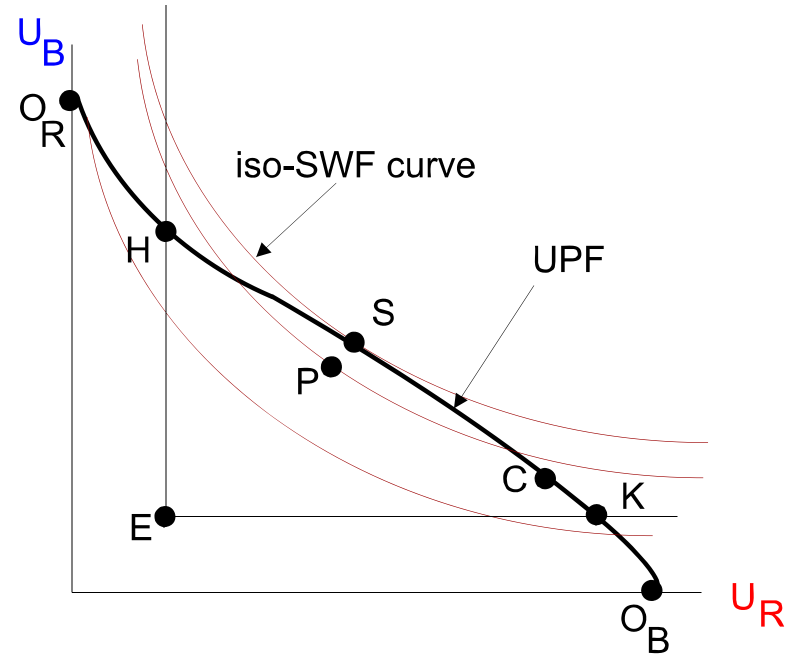
Relative price of X equals 1 plus the one-year interest rate

DISTRIBUTION

Along contract curve, R has lowest utility at O_R and highest utility at O_B ; B is other way round

"Utility Possibility Frontier (UPF)" in figure
shows the levels of utilities of the two

Cannot always have concave frontier
because utilities are ordinal



A social welfare function (SWF) is a
normative or ethical valuation of
the two utilities: $W(U_R , U_B)$

The figure shows iso-welfare curves
Social optimum at tangency with UPF

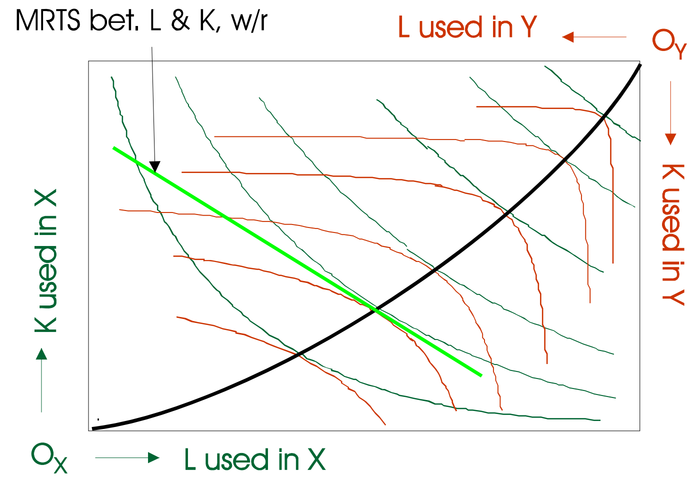
Who chooses SWF? Implicitly, the
society's political process does
Philosophers debate the merits

Special problems if

[1] Optimum leaves someone worse off than at endowment
Requires some coercion or expropriation to implement such a policy

[2] Available policies for redistribution are inefficient (create some dead-weight loss),
so need efficiency-equity tradeoff in judging whether a policy is socially desirable
In the example shown in figure, the efficient competitive equilibrium at C
is worse in SWF evaluation than the inefficient point P

PRODUCTION



Two inputs, fixed total quantities L, K to be
allocated between two outputs, X and Y

Production Edgworth box:
Lengths of sides = total qtys of L, K
Isoquants of X from origin O_X ,
of Y from O_Y in reverse direction

Allocation is technically efficient if cannot increase
output of one good without decreasing that of the other
Efficiency if an isoquant of X is tangential to an isoquant of Y
MRTS between L and K in X production = MRTS between L and K in Y production
More generally, for two specified inputs, MRTS should be same in production of all goods
Common MRTS (slope of each isoquant) will equal input price ratio w/r
Efficient allocation can be achieved using perfectly competitive factor markets

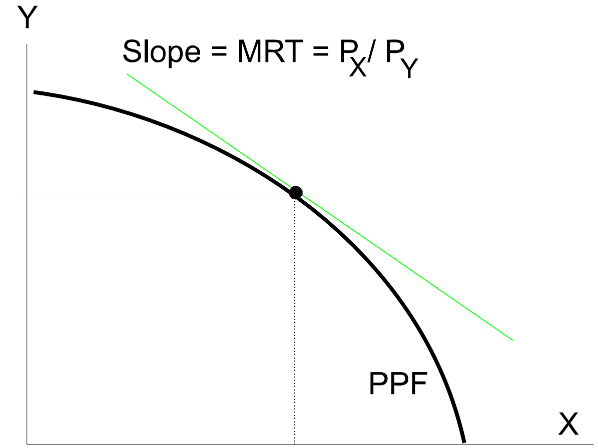
Note in figure the curve of efficient input allocations is below diagonal of box
Efficient to use lower ratio of K/L in X production than in Y production -
X is relatively less K-intensive (relatively more L-intensive) than Y
In international trade, a country that has a lower K/L ratio will have
comparative advantage in the production of X; the other will have comp. adv. in Y

From production Edgeworth box, can construct production possibility frontier (PPF)
exactly as utility possibility frontier came from exchange Edgeworth box

At each point on the PPF, slope = MRT between outputs = P_X / P_Y

General equilibrium of production and exchange:
Draw exchange box rectangle of outputs in PPF figure
Full equilibrium: MRT in production = MRS in exchange

Even more general: input quantities not fixed
Consumers choose L (income-leisure tradeoff)
Saving/investment increases K over time



OVERVIEW – CONDITIONS FOR ECONOMIC EFFICIENCY

1. Efficient exchange: For each pair of goods, MRS same for all consumers
2. Efficient use of inputs in production: For each pair of inputs, MRTS same in all goods
3. Efficiency in output market: For any pair of goods, MRT along PPF = MRS for all consumers
   These conditions are satisfied if markets are complete and perfectly competitive

REASONS FOR MARKET FAILURE

1. Market power – prices are kept higher than marginal costs; quantity inefficiently low
2. Incomplete information – inefficient outcomes due to adverse selection, moral hazard
3. Externalities (incomplete markets) – some goods are not traded in markets, so
   one consumers' or one firm's actions can create unpriced spillovers on others
4. Public goods – non-payers cannot be excluded from enjoying benefits

In the context of uncertainty, we will be concerned with 2 and 3

2. ALGEBRAIC TREATMENT

Assumptions

Competitive (price-taking) behavior;

(when production, this rules out scale economies, fixed costs)

No information asymmetries (for people or planner)

Complete markets. No externalities, pure private goods

Exchange economy only (production in longer handout)

Notation

Individuals (households) $h = 1,\, 2\, \ldots\, H$

Goods $g = 1,\, 2\, \ldots\, G$

$$\begin{aligned} \mathbf{X}_h = (x_{hg}) &= \text{consumption quantities of household } h \\ \mathbf{X}^0_h = (x^0_{hg}) &= \text{initial endowments of household } h \\ U_h(\mathbf{X}_h) &= \text{utility function of } h \end{aligned}$$

Total quantities available of the goods:

$$X^0_g = \sum_{h=1}^{H} x^0_{hg}$$

Optimization by central planner

CONSTRAINTS: Material balance for goods g :

$$x_{1g} + x_{2g} + \ldots + x_{Hg} \leq x_{1g}^0 + x_{2g}^0 + \ldots + x_{Hg}^0 = X_g^0$$

SOCIAL OBJECTIVE (Pareto efficiency + interpersonal weights)

$$W = W(U_1,\, U_2 \, \ldots \, U_H)$$

Lagrangian

$$\mathcal{L} = W + \sum_{g=1}^{G} \alpha_g \left[X_g^0 - \sum_{h=1}^{H} x_{hg} \right]$$

FONCs for interior optimum:

$$\frac{\partial W}{\partial U_h} \, \frac{\partial U_h}{\partial x_{hg}} - \alpha_g = 0$$

Therefore

$$\frac{\partial U_h / \partial x_{h1}}{\partial U_h / \partial x_{h2}} = \frac{\alpha_1}{\alpha_2} \text{ for all } h$$

Tangency of indifference curves : contract curve in Edgeworth box.

Equilibrium

$\mathbf{p} = (p_g)$, vector of prices of goods

HOUSEHOLDS' OPTIMIZATION

$$\mathbf{p} \cdot \mathbf{X}_h \leq \mathbf{p} \cdot \mathbf{X}_h^0$$

Conditions for maxing $U_h(\mathbf{X}_h)$

$$\frac{\partial U_h}{\partial x_{hg}} - \lambda_h \, p_g = 0$$

These yield demand functions, $x_{hg} = x_{hg}(\mathbf{p})$

They are homogeneous of degree zero: only relative prices matter

They satisfy the budget constraint for all $\mathbf{p}$

EQUILIBRIUM

$$\sum_{h=1}^{H} x_{hg}(\mathbf{p}) = \sum_{h=1}^{H} x_{hg}^0 \quad \text{for all } g$$

Existence etc.

System has $(G-1)$ unknowns and equations:

(1) All demand functions homogeneous degree zero in $\mathbf{p}$
    Only relative prices matter. Can use freedom to set any one p_g equal to 1
    Then all prices measured in units of this good
    It is called *numeraire* – can be composite bundle

(2) Walras' Law - at all $\mathbf{p}$ (equilibrium or not)
    total value of excess demands is $\equiv 0$

$$\begin{aligned}
\sum_{g=1}^{G} p_g \sum_{h=1}^{H} \left(x_{hg}(\mathbf{p}) - x_{hg}^0 \right) &= \sum_{h=1}^{H} \sum_{g=1}^{G} p_g \left(x_{hg}(\mathbf{p}) - x_{hg}^0 \right) \\
&= \sum_{h=1}^{H} \left[\sum_{g=1}^{G} p_g \, x_{hg}(\mathbf{p}) - \sum_{g=1}^{G} p_g \, x_{hg}^0 \right] \\
&= \sum_{h=1}^{H} 0 = 0
\end{aligned}$$

Existence of solution proved by fixed point theorem
Uniqueness, dynamic stability not guaranteed

Equivalence

In equilibrium, consumers' FONCs are

$$\frac{\partial U_h}{\partial x_{hg}} - \lambda_h \, p_g = 0$$

At social optimum, planner's FONCs are

$$\frac{\partial W}{\partial U_h} \, \frac{\partial U_h}{\partial x_{hg}} - \alpha_g = 0$$

These coincide if

$$\partial W / \partial U_h = 1/\lambda_h, \qquad p_g = \alpha_g$$

So equilibrium is an optimum with particular social weights
  It is "Pareto efficient"
Conversely, planner's social optimum can be implemented as equilibrium if
  lump sums can be transfered between people to make

$$\lambda_h = 1/(\partial W / \partial U_h)$$

Slick proof of Pareto efficiency of competitive equilibrium

Denote equilibrium choices by $\mathbf{X}_h^e = (x_{hg}^e)$. Suppose this is not Pareto efficient.
So there exists feasible allocation $\mathbf{X}_h^a = (x_{hg}^a)$, which is
at least as good for all h, and strictly better for at least one h, say $h = 1$.
Why was (x_{hg}^a) not chosen? Must have been unaffordable.

$$U_1(\mathbf{X}_1^a) > U_1(\mathbf{X}_1^e) \qquad \text{implies} \qquad \mathbf{p} \cdot \mathbf{X}_1^a > \mathbf{p} \cdot \mathbf{X}_1^0$$

and for $h = 2$, 3, $\ldots$ H, so long as these consumers are not fully satiated,

$$U_h(\mathbf{X}_h^a) \geq U_h(\mathbf{X}_h^e) \qquad \text{implies} \qquad \mathbf{p} \cdot \mathbf{X}_h^a \geq \mathbf{p} \cdot \mathbf{X}_h^0$$

Summing the budget inequalities

$$\mathbf{p} \cdot \sum_{h=1}^{H} \mathbf{X}_h^a > \mathbf{p} \cdot \sum_{h=1}^{H} \mathbf{X}_h^0$$

which contradicts the requirement for feasibility, namely

$$\sum_{h=1}^{H} \mathbf{X}_h^a \leq \sum_{h=1}^{H} \mathbf{X}_h^0$$