ECO 317 – Economics of Uncertainty

Lectures: Tu-Th 3.00-4.20, Avinash Dixit

Precept: Fri 10.00-10.50, Andrei Rachkov

All in Fisher Hall B-01

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RISK MARKETS

Intrade: http://www.intrade.com Current price of health care “public option” is $19 for ticket that pays $100 if this happens, has come down from $50.

Iowa Electronic Markets: http://www.biz.uiowa.edu/iem/
Party presidential nomination markets in mid-September 2007: Price of $1 ticket on

Democratic		Republican	
Clinton	0.647	Giuliani	0.306
Obama	0.206	Romney	0.257
Edwards	0.075	Thomson	0.248
Rest of field	0.063	McCain	0.080
		Rest of field	0.100
Sum	0.991	Sum	0.991

Rough idea: [1] The prices reflect a “market estimate” of probability of the event.
[2] The price will aggregate everyone’s information, so capture “wisdom of crowds”.
Neither is rigorously true: [1] problematic because of risk-aversion,
[2] because of strategic manipulation by informed participants

TYPES OF PROBABILITY

[1] Physical or classical probability:
- Radioactive decay; quantum physics; statistical mechanics
- Deterministic but very complex phenomena may look random and be modeled as such

[2] Independent repeated trials, frequentist

[3] Beliefs, subjective probability
- Similar math can handle all; won't distinguish unless necessary
- Will see how to quantify/estimate subjective probability by offering suitable bets

PROBABILITY CONCEPTS

Sample space or probability space S
- Examples: all possible outcomes of coin tosses, card deals etc.

Singletons are called elementary events – cannot be decomposed any further
- Others are composite events

What is elementary may depend on context – e.g. when two coins tossed
- "1-head, 1-tail" elementary if indistinguishable coins simultaneously tossed
- (but still need care about assigning probabilities)

Elementary events called “states of the world” in microtheory, “scenarios” In finance
Once you know which of these has occurred, all uncertainty is resolved
Partial resolution: knowledge of composite event that contains the actual state

Two things to note in economic applications:
[1] Once you know the true state of the world, and therefore conditional on a scenario, can calculate what the equilibrium prices etc in the spot markets in that scenario will be.
[2] List of all elementary events (sample space) should be exogenous, but probabilities may be endogenous, affected by actions of individuals – moral hazard.

PROBABILITY MEASURE

Function from the set of events (set of subsets of S) to non-negative real numbers
Pr(Ø) = 0, Pr(S) = 1, and additive over finite or countable disjoint subsets:

$$\text{If } A_i \cap A_j = \varnothing \text{ for all } i \neq j, \text{ then } \Pr\left(\bigcup_{k=1}^{\infty} A_k\right) = \sum_{k=1}^{\infty} \Pr(A_k)$$

If S is continuum, requiring countable additivity poses measure-theoretic problems
Basically, cannot define probability for all sets, need sigma-fields. Ignored here.

CONDITIONAL PROBABILITIES

If you know that an event E has occurred, you can "update" probabilities of other events. For another event A, only the part $A \cap E$ remains relevant. Define

$$\Pr(A \mid E) = \frac{\Pr(A \cap E)}{\Pr(E)}$$

Events A and B are said to be independent if the occurrence of one gives on additional information about the probability of the other, that is

$$\Pr(B \mid A) = \Pr(B), \quad \text{or} \quad \Pr(A \cap B) = \Pr(A)\Pr(B)$$

For many events, we require such multiplicative property for every collection of every size

Examples: [1] A = Ace or King; $\Pr(A) = 8 / 52 = 2 / 13$. B = Hearts, $\Pr(B) = 13 / 52 = 1 / 4$.
$A \cap B$ = Ace or King of Hearts; $\Pr(A \cap B) = 2 / 52$. Independent.
[2] A = Ace; $\Pr(A) = 4 / 52 = 1 / 13$. B = Ace or King of Hearts, $\Pr(B) = 2 / 52 = 1 / 26$.
$A \cap B$ = Ace of Hearts; $\Pr(A \cap B) = 1 / 52 > \Pr(A) \Pr(B)$. Not independent.
[3] A = Spade; $\Pr(A) = 13 / 52 = 1 / 4$. B = Hearts; $\Pr(B) = 13 / 52 = 1 / 4$.
$A \cap B = \emptyset$; $\Pr(A \cap B) = 0 < \Pr(A) \Pr(B)$. Not independent.

BAYES' THEOREM; "INVERSE PROBABILITY"

Partition the sample space *S* in two different ways: *C* and not-*C*; *E* and not-*E*
Think of C as a cause and E as an effect or outcome
Suppose we know Pr(*C*) (prior probability that the cause is operative) and
Pr(*E* | *C*), Pr(E | Not-*C*) (probabilities of effects conditional on causes);
this may be based on a theory or on experience / observation.
We observe that *E* has occurred. How do we update Pr(*C*) to get
the posterior probability for C, i.e. find Pr(*C* | *E*)

Using the definition of conditional probability $\Pr(C \mid E) = \dfrac{\Pr(C \cap E)}{\Pr(E)} = \dfrac{\Pr(E \mid C)\Pr(C)}{\Pr(E)}$

Also E can be partitioned into two disjoint events $E = (E \cap C) \cup (E \cap \text{Not - } C)$
Therefore

$$\Pr(E) = \Pr(E \cap C) + \Pr(E \cap \text{Not - } C) = \Pr(E \mid C)\Pr(C) + \Pr(E \mid \text{Not - } C)\Pr(\text{Not - } C)$$

This gives Bayes' Formula: $\Pr(C \mid E) = \dfrac{\Pr(E \mid C)\Pr(C)}{\Pr(E \mid C)\Pr(C) + \Pr(E \mid \text{Not - } C)\Pr(\text{Not - } C)}$

Example: Suppose 25% of the population are smokers. Previous observations tell us that smokers have a 40% chance of developing a certain disease, and nonsmokers only 4%. A person has the disease, what is the probability that he was a smoker? (This may be relevant for an insurance company in deciding whether to pay the claim.)

The working of Bayes' Formula is easier to see from the following table:

Person	Prior probability	Conditional probs of outcomes	
		Disease	No Disease
Smoker	0.25	0.40	0.60
Non-smoker	0.75	0.01	0.99

Unconditional probabilities of various cause and outcome combinations:

	Disease	No disease	Row sum
Smoker	0.25 * 0.40 = 0.10	0.25 * 0.60 = 0.15	0.25
Non-smoker	0.75 * 0.04 = 0.03	0.75 * 0.96 = 0.72	0.75
Column sum	0.13	0.87	

Therefore Pr(Smoker I Disease) = 0.10 / 0.13 = 0.77

RANDOM VARIABLES

A random variable is neither random nor variable.
It is just a real-valued function on the sample space: $X : S \mapsto \Re$
Example: Price of an asset, or the rate of return on an asset, in different scenarios
This leads to further concepts:

Cumulative distribution function of a random variable:
$F : \Re \mapsto [0,1]$ defined by $F(t) = \Pr(s \mid X(s) \leq t)$

Density function $f(t) = F'(t)$ if / where the derivative exists

Expected value: $\mathrm{E}(X) = \sum_{s \in S} X(s) \Pr(s) = \int_{-\infty}^{\infty} t\, f(t)\, dt$ when the expression is appropriate

(Otherwise more complicated notions of integration etc. may be needed. For us this will almost never be an issue; will treat it ad hoc if / when needed.)

Variance $\mathrm{V}(X) = \mathrm{E}[(X - \bar{X})^2]$

Etc. Remember and review these from ECO 202 or ORF 245.
Will do some review and application in tomorrow's precept.