ECO 317 – Economics of Uncertainty – Fall Term 2009
Slides to accompany
19. Price Discrimination by Self-Selection

Demand

Single consumer: quasilinear utility:

$$u = H(Q) + Y \,,$$

where Q is the quantity (or quality ...) of good in question
Y is aggregate of all the other goods, measured in dollars
Budget constraint $P\,Q + Y = I$. So

$$u = H(Q) - P\,Q + I \,.$$

Optimum choice of Q yields the inverse demand function
(quasilinear utility implies zero income effect):

$$P = H'(Q) \equiv D(Q) \,.$$

The second-order condition is $H''(Q) < 0$ or $D'(Q) < 0$; we assume it is satisfied.

Figure 1 illustrates; demand curve shown straight line purely for convenience.

Total utility from this good is area under the curve:

$$H(Q) = \int_0^Q D(q)\,dq\,.$$

Revenue is rectangle, consumer surplus is triangle above it.

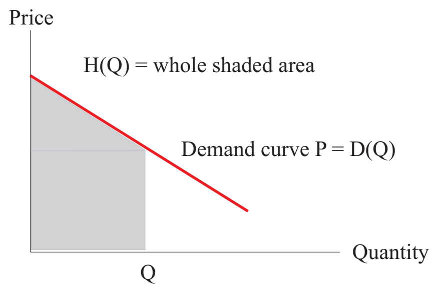

Figure 1: Utility and Demand

Assume constant marginal cost of production c.

Perfect Price Discrimination: One Consumer

If monopolist knows $D(Q)$,can extract full $H(Q)$ (all-or-nothing offer or two-part pricing).

revenue $R(Q) = H(Q) = \int_0^Q D(q)\,dq\,,$ profit $\Pi(Q) = R(Q) - c\,Q = \int_0^Q D(q)\,dq - c\,Q$

Optimum $Q = Q^*$ given by $\Pi'(Q) = D(Q) - c = 0\,,$

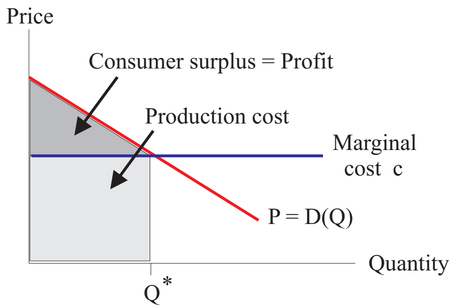

Figure 2: Perfect Price Discrimination with One Consumer

Multiple Types; Complete Information

Types of consumers labeled $i = 1$, 2, $\dots$ n. Population proportions θ_i.
Inverse demand functions are written $D_i(Q) = H_i'(Q)$.
ASSUME these are uniformly ranked:

$$D_1(Q) < D_2(Q) < \dots < D_n(Q) \quad \text{for all } Q\,,$$

or

$$H_1'(Q) < H_2'(Q) < \dots < H_n'(Q) \quad \text{for all } Q\,.$$

This will be the Mirrlees-Spence single crossing condition:
Along an indifference curve of Type i in (Q, Y) space,

$$Y = u_i - H_i(Q)$$

Marginal rate of substitution $-dY/dQ = H_i'(Q)$.
So indifference curve of the higher type is steeper:
indifference curves of different types cross in only one direction.

If the monopolist knows individual consumer's type (and can use this), can extract all of $H_i(Q)$ from each type by selling Q_i^* defined by

$$D_i'(Q_i^*) = c_i \,, \qquad \text{and} \qquad R_i^* = H_i(Q_i^*) = \int_0^{Q_i^*} D_i(q)\,dq \,.$$

Economically fully efficient (first-best), but the consumers get no surplus. Figure 3 illustrates this with three types.

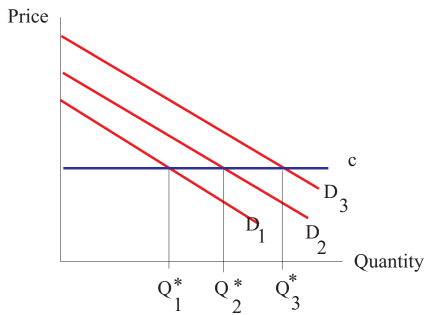

Figure 3: Perfect Price Discrimination with Multiple Types of Consumers

Incomplete Information – Unavoidable Inefficiency

When monopolist does not know individual consumers' types, or can't use this.
If he tries perfect discrim., consumer can gain by pretending to be lower types.

Let $u(i,j) =$ utility of a consumer of Type i from offer intended for Type j.

$$u(i,j) = H_i(Q_j^*) + [\ I_i - H_j(Q_j^*)\ ]\,.$$

So

$$\begin{aligned} u(i,j) - u(i,i) &= H_i(Q_j^*) - H_j(Q_j^*) \\ &= \int_0^{Q_j^*} D_i(q)\,dq - \int_0^{Q_j^*} D_j(q)\,dq \\ &= \int_0^{Q_j^*} [\,D_i(q) - D_j(q)\,]\,dq \end{aligned}$$

which is positive when $j < i$.

Figure 4 illustrates this.

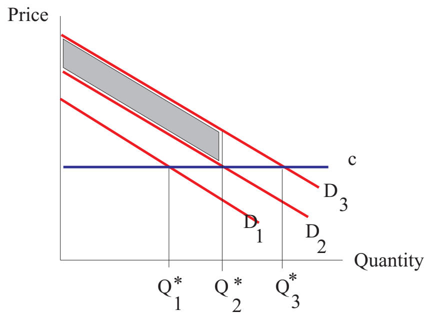

Figure 4: Utility Gain From Mimicking Lower Demand Types

Shaded area = extra utility of Type 3 pretending to be Type 2.

Monopolist's Optimal Contracts Under Incomplete Information

Let (Q_i, R_i) be contract intended for Type i. If Type i buys contract j,

$$u(i,j) = H_i(Q_j) - R_j + I_i \, .$$

$$\begin{aligned} u(i,i) - u(i,j) &= H_i(Q_i) - H_i(Q_j) - R_i + R_j \\ &= \int_0^{Q_i} D_i(q)\, dq - \int_0^{Q_j} D_i(q)\, dq - R_i + R_j \\ &= \int_{Q_j}^{Q_i} D_i(q)\, dq - R_i + R_j \, . \end{aligned}$$

No imitation (self-selection or incentive-compatibility constraint):

$$R_i \leq \int_{Q_j}^{Q_i} D_i(q)\, dq + R_j \qquad \text{for all } j \neq i \, . \tag{IC$_{\text{all}}$}$$

And *individual rationality* or *participation* constraints

$$R_i \leq H_i(Q_i) = \int_0^{Q_i} D_i(q)\, dq \, . \tag{PC$_i$}$$

This general formulation allows $Q_i = Q_j$ and $R_i = R_j$; that is, pooling.
Also allows corner solutions $Q_i = 0$ for some i (not serving some type).

Steps in analysis:

Lemma 1: For any two types $j < i$, if IC's hold for this pair, then $Q_j \le Q_i$. So if all incentive constraints hold, then

$$Q_1 \le Q_2 \le \ldots \le Q_n. \tag{Order}$$

Proof: Choose labels so that $j < i$. ICs for this pair are

$$R_i - R_j \le \int_{Q_j}^{Q_i} D_i(q)\, dq$$
$$R_j - R_i \le \int_{Q_i}^{Q_j} D_j(q)\, dq = - \int_{Q_j}^{Q_i} D_j(q)\, dq$$

Adding these together,

$$0 \le \int_{Q_j}^{Q_i} [\, D_i(q) - D_j(q))\,]\, dq\,.$$

Since $D_j(q) < D_i(q)$ for all q, this implies $Q_j \le Q_i$.

Next, single-crossing property reduces number of ICs from $(n-1) * n$ to $(n-1)$.

Lemma 2: If (a) the quantities satisfy (Order), and (b) if reduced ICs

$$R_i \leq \int_{Q_{i-1}}^{Q_i} D_i(q)\,dq + R_{i-1}\,, \tag{IC$_i$}$$

are binding (hold as exact equalities), then all ICs hold.

Proof: By example. Consider types 2, 3, 4.

$$\begin{aligned}
u(4,4) - u(4,2) &= \int_0^{Q_4} D_4(q)\,dq - \int_0^{Q_2} D_4(q)\,dq - R_4 + R_2 \\
&= \int_{Q_2}^{Q_4} D_4(q)\,dq - R_4 + R_2 \\
&= \int_{Q_2}^{Q_3} D_4(q)\,dq + \int_{Q_3}^{Q_4} D_4(q)\,dq - R_4 + R_2 \\
&= \left[\int_{Q_2}^{Q_3} D_4(q)\,dq - R_3 + R_2\right] + \left[\int_{Q_3}^{Q_4} D_4(q)\,dq - R_4 + R_3\right] \\
&= \left[\int_{Q_2}^{Q_3} D_3(q)\,dq - R_3 + R_2\right] + \left[\int_{Q_3}^{Q_4} D_4(q)\,dq - R_4 + R_3\right] \\
&\quad + \int_{Q_2}^{Q_3} D_4(q)\,dq - \int_{Q_2}^{Q_3} D_3(q)\,dq
\end{aligned}$$

$$= \; [\, u(3,3) - u(3,2)\,] + [\, u(4,4) - u(4,3)\,] + \int_{Q_2}^{Q_3} [\, D_4(q) - D_3(q)\,]\, dq\,.$$

So ICs ruling out 3 mimicking 2 and 4 mimicking 3 also rule out 4 mimicking 2.

Next consider the possibility that Type 4 wants to mimic Type 5. We have

$$\begin{aligned}
u(4,4) - u(4,5) &= \int_0^{Q_4} D_4(q)\, dq - \int_0^{Q_5} D_4(q)\, dq - R_4 + R_5 \\
&= \int_0^{Q_4} D_4(q)\, dq - \int_0^{Q_5} D_4(q)\, dq - R_4 + \left[\, \int_{Q_4}^{Q_5} D_5(q)\, dq + R_4 \,\right] \\
&= -\int_{Q_4}^{Q_5} D_4(q)\, dq + \int_{Q_4}^{Q_5} D_5(q)\, dq \\
&= \int_{Q_4}^{Q_5} [\, D_5(q) - D_4(q)\,]\, dq\,,
\end{aligned}$$

Note: going from first to second line assumes that (IC_5) holds as an equality.

The monopolist's profit per capita is

$$\Pi = \sum_{i=1}^{n} \theta_i \, (R_i - c \, Q_i) \, . \qquad \text{(Profit)}$$

Choose contracts to maximize this subject to:

one-step downward ICs (IC_i) holding as equalities, and

PC for the lowest type, (PC_1).

Then verify that all the other PCs for types 2, 3, ... n also hold.

Assume that quantities satisfying (Order); then by Lemma 2 all ICs hold.

So our solution also solution to the full problem.

The problem with fewer constraints we solve is called the relaxed problem.

At the end, see if the quantities in the solution to the relaxed problem satisfy (Order).

Result: for $k = 1$ 2, $\ldots$ $(n-1)$, FOC is

$$\frac{\partial \Pi}{\partial Q_k} = \theta_k \, [\, D_k(Q_k) - c \,] - (\theta_{k+1} + \theta_{k+2} + \ldots + \theta_n) \, [\, D_{k+1}(Q_k) - D_k(Q_k) \,] = 0 \, .$$

Therefore

$$D_k(Q_k) = c + \frac{\theta_{k+1} + \theta_{k+2} + \ldots + \theta_n}{\theta_k} \, [\, D_{k+1}(Q_k) - D_k(Q_k) \,] > c \, .$$

So $Q_k < Q_k^*$: the quantities distorted downward.

Purpose: To reduce rent-loss to higher types.

For $k = n$, there are no types $h > k$. FOC is

$$\frac{\partial \Pi}{\partial Q_n} = \theta_n \, [\, D_n(Q_n) - c \,] = 0 \, , \quad \text{so} \quad D_n(Q_n) = c \, ,$$

no distortion for the best type.

Want to keep all the R_i as high as possible. So set R_1 at its max:

The worst type gets no rent. Others get just enough to meet successive ICs.

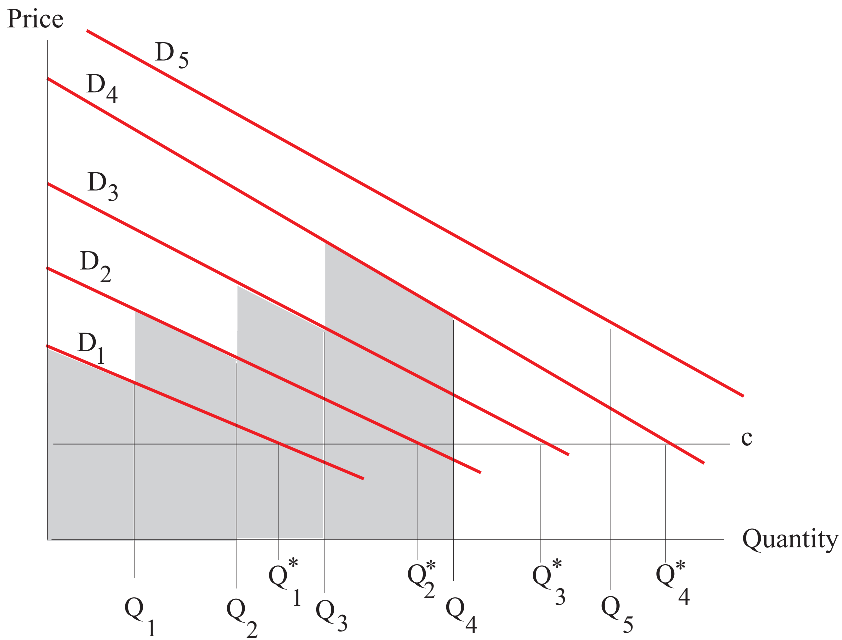

Figure 5: Quantities and Payments for Incentive Compatibility

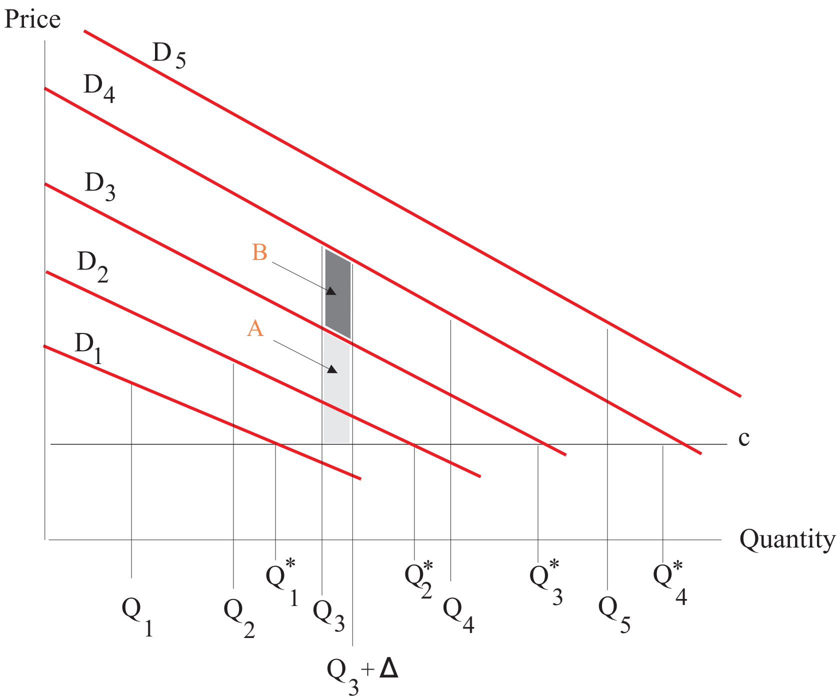

Figure 6: Effects of Marginal Changes in Quantities