ECO 352 – Spring 2010 – Precepts
Weeks 1, 2 – Feb. 1, 8

REVIEW OF MICROECONOMICS

Concepts to be reviewed

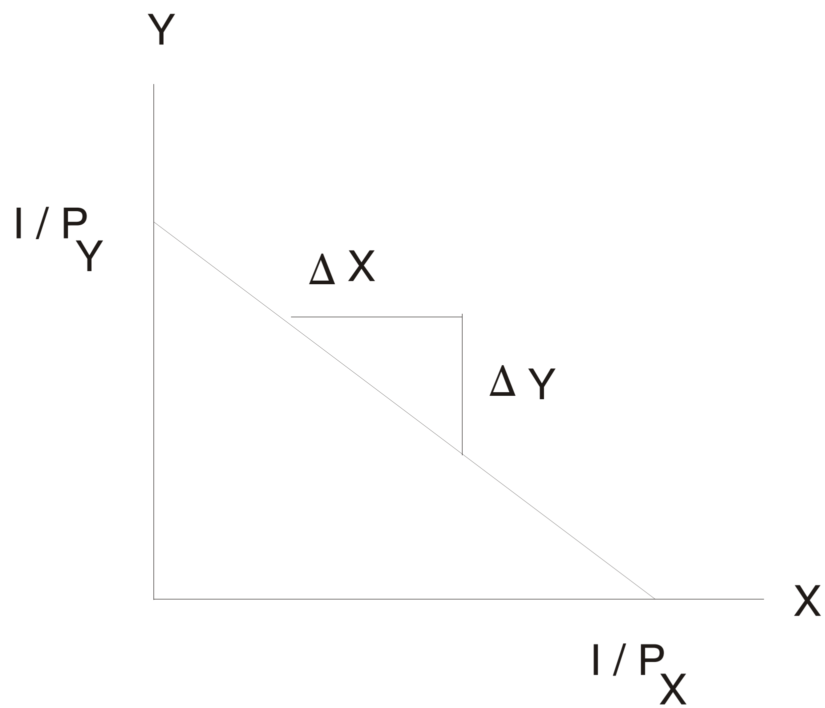
Budget constraint: graphical and algebraic representation
Preferences, indifference curves. Utility function
Marginal rate of substitution (MRS), diminishing MRS
    algebraic formulation of MRS in terms of the utility function
    Utility maximization: Tangency, corner, and kink optima
Demand functions, their homogeneity property
Homothetic preferences. Form of demand functions for these
Aggregation of demand over consumers
Relative demand, elasticity of substitution
Special cases: Linear and Leontief preferences; Cobb-Douglas
Related concepts for production: Production function. Isoquants.
    Marginal products. Marginal rate of technical substitution (MRTS)
Output transformation frontier. Marginal rate of transformation (MRT)
    Achieving the optimum as a market equilibrium

BUDGET CONSTRAINT

Equation : $P_X X + P_Y Y = I$
Moving along line, $P_X \Delta X + P_Y \Delta Y = 0$

So slope $\Delta Y/\Delta X = - P_X / P_Y$

Or solve for Y in terms of X:
$Y = (I / P_Y) - (P_X / P_Y) X$
Derivative = $dY/dX = - P_X / P_Y$



Economic interpretation:
price of X "relative to Y" or
"measured in units of Y":
How much Y you have to give up to get 1 more of X
(Opportunity cost of consuming more X)

Intercepts: (I / P_Y) on Y-axis, (I / P_X) on X-axis
Economic interpretation: how much of each good could you buy
if you bought nothing of the other

PREFERENCES AND UTILITY

Preferences represented geometrically by indifference map: complete set of indifference curves

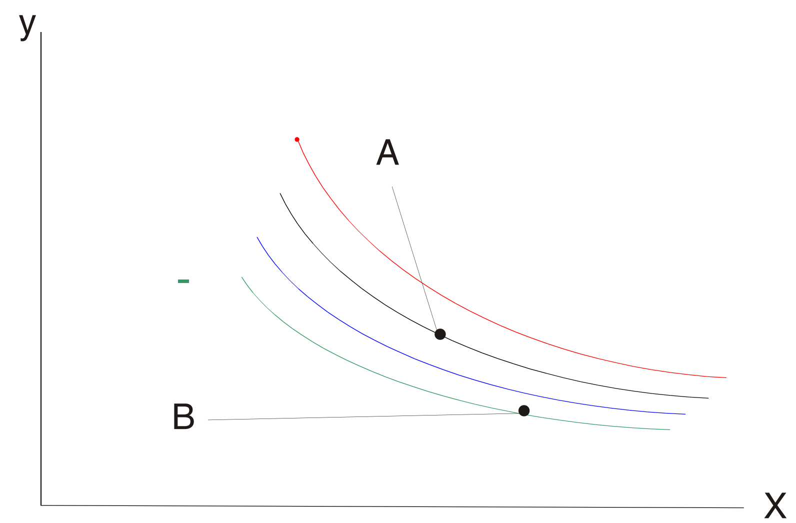
All consumption bundles on any one indifference curve are equally good (or bad!)

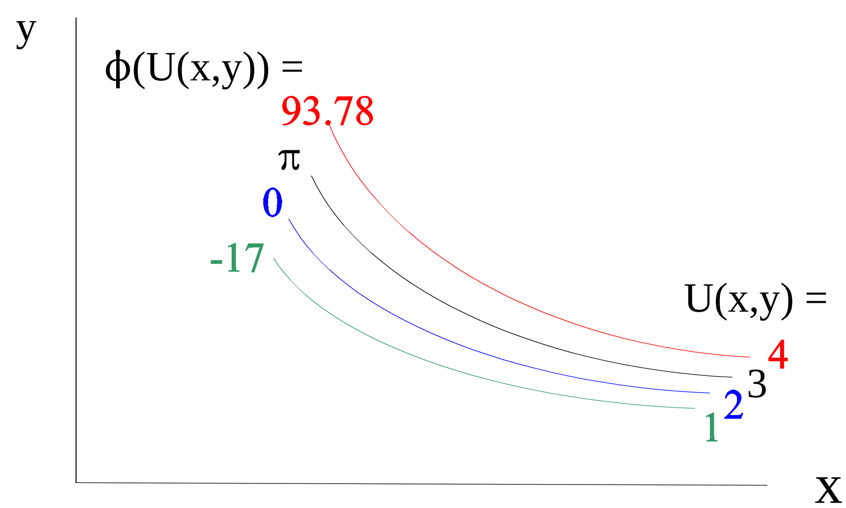
One on higher curve e.g. A is preferred to one on a lower e.g B.

Numerical / algebraic representation: utility function. Higher indifference

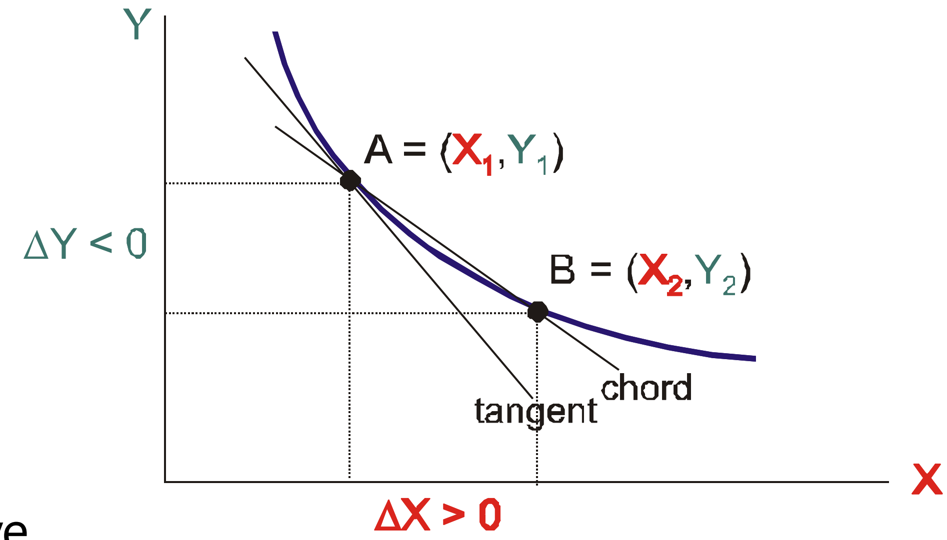
curves get higher utility number.

But arbitrariness in choice of scale: utility is ordinal, not cardinal.





MARGINAL RATE OF SUBSTITUTION (MRS)



MRS along an indifference curve

How much Y is the consumer willing to give up in order to get 1 more of X

Usually shown positive (numerical value)

Arc: Slope of chord

$MRS = (-\Delta Y)/(\Delta X) = -\Delta Y / \Delta X$

Point: slope of tangent

$MRS = -dY/dX$ along indifference curve

If U(X,Y) represents preferences,

then for a small move along an indifference curve,

$(\partial U/\partial X)\,\Delta X + (\partial U/\partial Y)\,\Delta Y = 0$, therefore $MRS = (\partial U/\partial X) / (\partial U/\partial Y)$

Diminishing MRS: As X increases and Y decreases along an indifference curve, the curve becomes flatter, and MRS decreases. Indiff. curves are convex

Intuition – As consumer has more X and less Y (retaining indifference) values X **relatively** less: willing to give up less of Y to get even more X

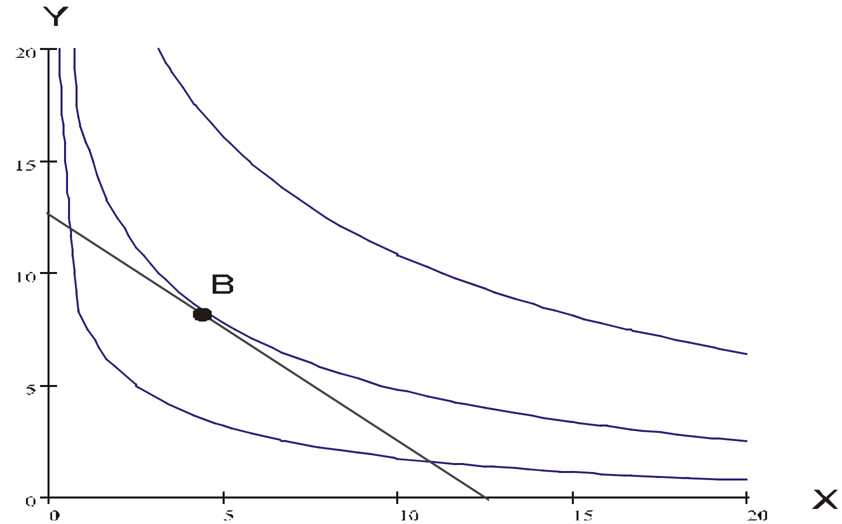
This may not always be true, but failure of this assumption is not so important in international trade context so we assume it holds.

UTILITY MAXIMIZATION

Regular case: Tangency

At optimum choice B, slopes of budget line & indiff. curve equal

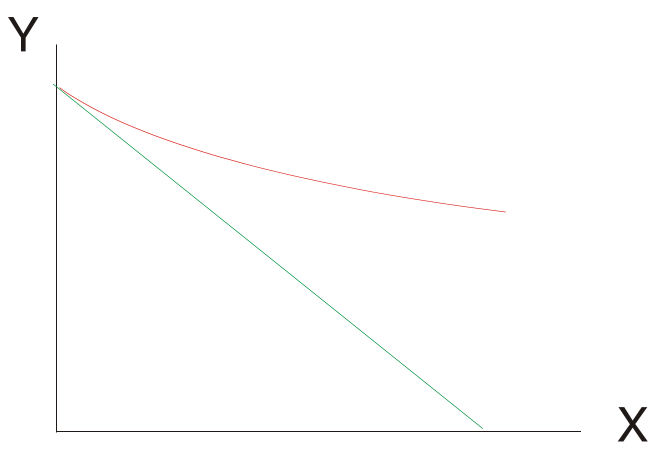
$(\partial U/\partial X) / (\partial U/\partial Y) = P_X / P_Y$



Other possibilities:

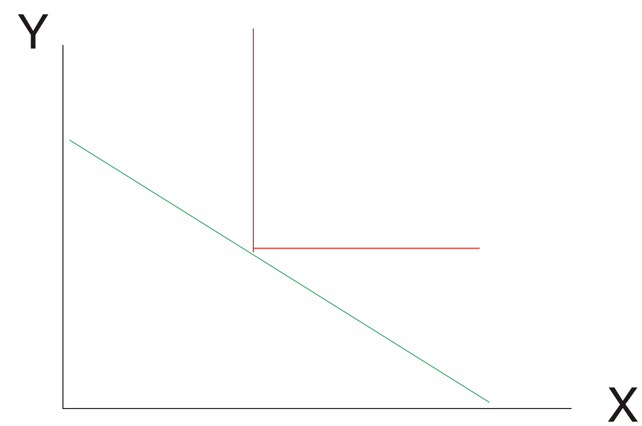
Corner solution:

MRS at X = 0 smaller than P_X / P_Y, so don’t want to buy any X



Indifference curve has kink:

Most important: Leontief preferences



DEMAND FUNCTIONS

Result of utility maximization: $X = D_X(P_X, P_Y, I)$, $Y = D_Y(P_X, P_Y, I)$

Homogeneity of degree zero: $D_X(2P_X, 2P_Y, 2I) = D_X(P_X, P_Y, I)$

Economic interpretation: absence of money illusion

Income and substitution effects when prices change:

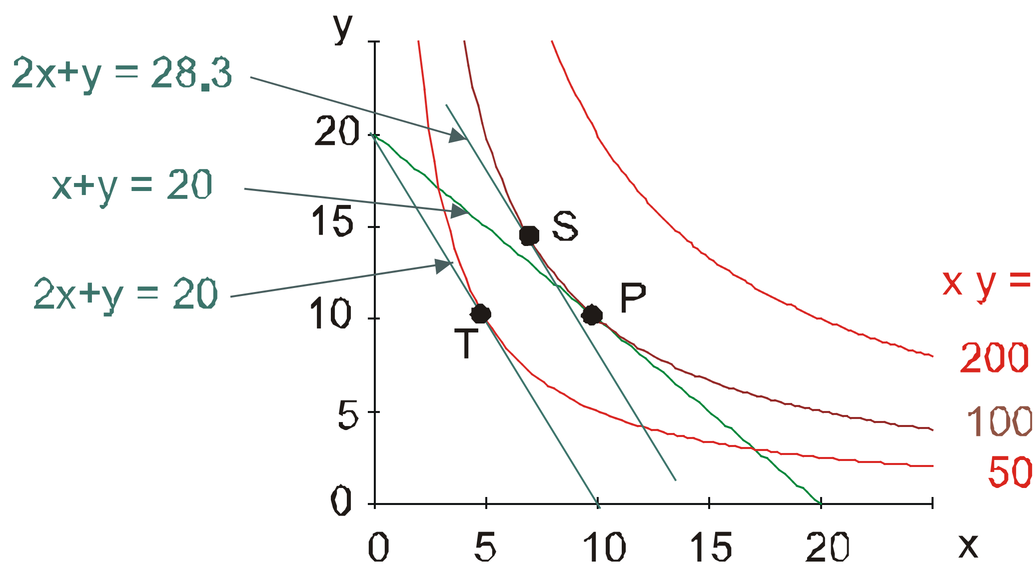
Example: $U(X,Y) = XY$

Initially $P_X = P_Y = 1$, $I = 20$

Optimum at P. Then P_X rises to 2.

Pure substitution effect: to S

Actually achieving this needs income compensation: increase to $I = 20\sqrt{2} \approx 28.3$ (Explanation next page)



Income effect of price increase: Move from S to T

Total effect: move from P to T; actual effect in absence of income compensation

COBB-DOUGLAS UTILITY FUNCTION

$U(X,Y) = X^{\alpha} \ Y^{\beta}$, or equally valid representation $\ln U(X,Y) = \alpha \ln(X) + \beta \ln(Y)$

$MRS = (\alpha / X) / (\beta / Y) = (\alpha Y) / (\beta X) = P_X / P_Y$

$P_X X / \alpha = P_Y Y / \beta$, so each $= (P_X X + P_Y Y) / (\alpha+\beta) = I / (\alpha+\beta)$

Demand functions: $X = [\alpha/(\alpha+\beta)] I / P_X$, $Y = [\beta/(\alpha+\beta)] I / P_Y$

Expenditure shares: $P_X X / I = [\alpha/(\alpha+\beta)]$, $P_Y Y / I = [\beta/(\alpha+\beta)]$

Example on previous page: $\alpha = 1$, $\beta = 1$. Therefore

When $P_X = P_Y = 1$, $I = 20$, $X = 10$, $Y = 10$, $U = 100$

When $P_X = 2$, $P_Y = 1$, $I = 20$, $X = 5$, $Y = 10$, $U = 50$

When $P_X = 2$, $P_Y = 1$, for general I, $X = I/4$, $Y = I/2$, $U = I^2 /8$,

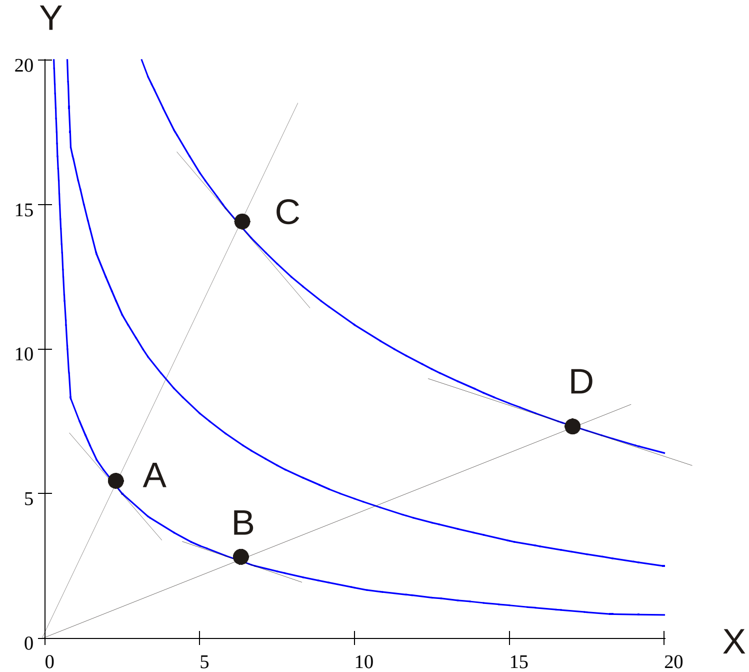
so to achieve old $U = 100$ needs $I^2 = 800$, or $I = \sqrt{800} = 20 \sqrt{2}$

HOMOTHETIC PREFERENCES

Indifference is preserved by equiproportionate scale changes for all goods:
indifference curves are radial magnifications or reductions of each other.

In figure, MRS at A same as at C
MRS at B same as at D
So MRS depends only on ratio Y/X,
not on absolute scale
Cobb-Douglas is an example:

$$MRS = (\alpha / \beta) (Y / X)$$



Useful properties:

[1] Tangency condition solves to
Relative demand function

$$Y / X = f(P_X / P_Y)$$

The elasticity of this function is the elasticity of substitution in consumption.
For Cobb-Douglas it = 1. For Leontief, it = 0.
Straight line preferences (perfect substitutes) is the limiting case, el. of sub. = ∞

[2] Demand functions $X = d_X(P_X/P_Y) I$, $Y = d_Y(P_X/P_Y) I$

Unitary income elasticities of demand.

[3] Suppose the consumers 1, 2, … in an economy have different incomes but identical homothetic preferences.

Their demands for good X are

$X_1 = d_X(P_X/P_Y) I_1$, $X_2 = d_X(P_X/P_Y) I_2$ etc.

Total demand in the economy for good X:

$X_1 + X_2 = d_X(P_X/P_Y) (I_1 + I_2)$

Depends only on total income, not its distribution

Similarly for good Y

So we have "exact aggregation" on the demand side of the economy

This is very useful in international trade.

While demand matters, the more important question in that context is how production responds to trade liberalization or trade barriers.

Therefore we will assume identical homothetic preferences most of the time.

PRODUCTION

Concepts are analogous, with simple reinterpretations of those for consumption:

Preference map ☐ Isoquant map
Utility function ☐ Production function (PLUS output quantities are cardinal)
MRS ☐ Marginal rate of technical (input) substitution (MRTS)

Additional useful concept:

Marginal product. If output $Q = F(K,L)$, marginal products are $\partial Q/\partial K$, $\partial Q/\partial L$

Cobb-Douglas production function $Q = K^{\alpha} L^{\beta}$

Exercise: calculate its marginal products

Returns to scale: If both inputs are doubled, output becomes

$$(2K)^{\alpha} (2L)^{\beta} = 2^{\alpha+\beta} K^{\alpha} L^{\beta} \ > , = \text{or} < 2 K^{\alpha} L^{\beta} \text{ as } \alpha + \beta > , = \text{or} < 1$$

Perfect competition requires constant or diminishing returns to scale.

OUTPUT TRANSFORMATION FRONTIER

TT' = Output transformation or
 production possibility frontier

MRT = its slope at a point.

Assume exact aggregation,
 social indifference curves
 generate aggregate demands.

Then social optimum maximizes
 social preference over TT'

Optimum can be achieved as a
 competitive market equilibrium,
Relative price = slope of PP'
 the common tangent