ECO 199 – GAMES OF STRATEGY
Spring Term 2004
FINAL EXAMINATION MAY 19 – ANSWER KEY

The distribution of grades was as follows:

Range	Numbers
90-99	3
80-89	14
70-79	13
60-69	12
< 60	11

The median was 74, the mean 72.2, and the standard deviation 13.1.

QUESTION 1: (10 POINTS, 15 MINUTES)

2 points each. We do not insist on exact wording, but the key ideas must be present and clearly stated. For example, optimality at each node is key when defining subgame perfect equilibrium, and it is important to specify what tit-for-tat does at the first move.

COMMON ERRORS: Most mistakes came from the definition of a subgame perfect equilibrium: people described an equilibrium in just one subgame, instead of defining it as an equilibrium of the whole game where the continuation of its strategies constituted an equilibrium in every subgame (-2). A few mistakes were made on the evolutionary stable strategy, where students described polymorphic situation as well (-1). Also occasionally forgetting to include a statement of what TFT does on the first play.

[1] Expected payoff – The probability-weighted average (statistical mean or expectation) of the payoffs of one player in a game, corresponding to all possible realizations of a random choice of nature or mixed strategies of the players.

[2] Subgame perfect equilibrium – A configuration of strategies (complete plans of action), one for each player, such that their continuation in any subgame remains optimal (part of a rollback equilibrium), whether that subgame is on- or off-equilibrium.

[3] Tit-for-tat – In a repeated prisoners' dilemma, this is the strategy of (i) cooperating on the first play and (ii) thereafter doing each period what the other player did the previous period.

[4] Evolutionary stable strategy – A phenotype or strategy which can persist in a population, in the sense that all the members of a population or species are of that type and cannot be successfully invaded by a small population of mutant types (static criterion). OR, starting from an arbitrary distribution of phenotypes in the population, the process of selection will converge to this strategy (dynamic criterion).

[5] Screening – Strategy of a less-informed player to elicit information credibly from a more-informed player.

QUESTION 2: (15 POINTS, 30 MINUTES)

5 points each. Identifying the key concepts of game theory that are involved is crucial. Clarity and brevity of the discussion are also important.

COMMON ERRORS: Most students realized the first statement was false but simply described situations where the first player had lower pay-offs irrespective of the order of moves (-3). For the last statement, the most common mistake was to ignore cost differentials between types (-2).

[a] False. In any zero-sum game which would have only a mixed strategy equilibrium when played with simultaneous move, you would do better to move second, because the other cannot keep you guessing: you can observe the other player's actual action and make his best response. Another example is price competition with two sellers selling substitute products.

[b] If the machine is automatic and tamper-proof, the threat of deploying it is irreversible; revealing the existence of the machine makes the action observable. Irreversibility and observability are essential ingredients for credibility of a strategic move.

[c] It cannot directly signal quality. But it may signal the owner's quality. The cost of time and effort in washing and polishing a car (or even taking it to a car wash for this purpose) may be lower for inherently careful owners than for intrinsically sloppy or careless ones. And an owner who does not even bother to do these things when a potential buyer will view the car is likely to have been careless in its maintenance in many different respects throughout his ownership.

QUESTION 3: (15 POINTS, 25 MINUTES)

Generally very well done. Only a few students were unable to write down the system of equations associated with a mixed strategy equilibrium.

(a) (5 points)

		Player II		
		L	S	R
Player I	L	0, 0	-1, 3	-6, -6
	S	3, -1	-6, -6	3, -1
	R	-6, -6	-1, 3	0, 0

(b) (4 points) (L,S), (R,S), (S,L), (S,R)

(c) (6 points) Suppose Player II is choosing probabilities x each on L and R and (1-2x) on S, Since Player I is also assumed to be choosing all three strategies with positive probability, his expected payoff from all of them must be equal. The expected payoffs to player I from each of his pure strategies against player II's assumed mixture are:

L: $-1 * (1-2x) - 6 * x = -1 - 4x$

S: $3 * x - 6 * (1-2x) + 3 * x = 18x - 6$

R: $-6 * x - 1 * (1-2x) = -1 - 4x$

Equating these gives $-1 - 4x = 18x - 6$, or $22x = 5$, or $x = 5/22$. Then the probability of S is $1 - 2x = 1 - 10/22 = 1 - 5/11 = 6/11$.

QUESTION 4: (15 POINTS, 25 MINUTES)

COMMON ERRORS: This was the worst question (average score only 7 out of 15). Many students failed to realize that when each player has two moves in a sequential game, the second mover has four strategies, and therefore the matrix of the game written in its strategic form is 2-by-4, not 2-by-2. This was a crucial error costing a lot of points (-5). Many also failed to recognize the crucial issue about the non-rollback Nash equilibria, namely credibility of threats or promises (-5). Some discussed focal points, which are not pertinent here. A few students got the 5 bonus points, one particularly good about Netscape vs. Microsoft. If you produced an original example but did not get the bonus points, this was because of some error of the kind mentioned above in the analysis.

The additional equilibria usually represent threats or promises that are not credible. They can be strategies in a Nash equilibrium because the logic of this equilibrium requires each player to accept as given the specified strategy of the other player. Examples: from the textbook, pp. 173-5, from the class handouts, Feb. 24, last page.

QUESTION 5: (15 POINTS, 25 MINUTES)

COMMON ERRORS: Most students who wrote down the matrices of pay-offs proposed too many NE (-1 per false NE) and most students who proposed a verbal argument missed the unanimity equilibria (-3). For 5 (c), a few students predicted the sincere voting outcome because Nader voters are irrational or just concerned to make a point and therefore "throw away" their votes. But this is not correct reasoning, since Kerry voters would then be switching.

(a) (3 points) Bush wins with 45% of the vote.

(b) (9 points) The matrix of outcomes is as follows. Only the outcomes are shown in the cells; the payoffs are then obvious. The notation is mostly obvious too, except that outcomes shown in bold and upper-case are Nash equilibria, and those shown in lower case are not Nash equilibria.

	GROUP 1								
	N			B			K		
	GROUP 2			GROUP 2			GROUP 2		
GROUP 3	N	B	K	N	B	K	N	B	K
N	**N**	n	n	**N**	b	b	n	k	k
B	n	b	n	b	**B**	b	k	b	k
K	n	n	k	b	b	**K**	k	k	**K**

A verbal argument is as follows. Truthful voting is not a Nash equilibrium because given the strategies of the other two groups, either Group 2 (Kerry supporters) or Group 3 (Nader supporters) can get a better outcome by switching to vote for the first preference of the other. The configurations where one of the two does switch in this way (Nader supporters vote strategically for Kerry or vice versa) are both Nash equilibria because then Bush supporters cannot do anything about it. In fact they may as well be voting for the winning candidate, that is also an equilibrium. (However, the Bush supporters could not be voting for the other losing candidate in such an equilibrium because then one of the other two groups would deviate). Similarly, everyone voting for Bush is a Nash equilibrium because then neither of the other two can affect the outcome by switching on their own (remember coordination across groups is ruled out by assumption).

In the table, the strategic voting equilibria are the top left and bottom right in the center matrix. The unanimous equilibria are the top left in the left matrix, center in the central matrix, and bottom right in the right matrix.

Thus there are only the following pure strategy Nash equilibria: [1] Three “unanimity” equilibria, where one group gets its best outcome and the other two are separately powerless to change that. [2] Two where Group 1 votes for Bush and the other two groups vote for one of the other candidates, either Group 2 voting strategically for Nader or Group 3 voting strategically for Kerry.

(All this is fairly intricate argument. We don’t deduct points for minor slips in stating the reasoning. But points are taken off for missing equilibria altogether: 3 for missing the unanimity equilibria, and 3 for missing each of the other two with strategic voting.)

(c) (3 points) Of these, the last one (Nader supporters voting strategically for Kerry) seems most plausible in practice as a focal point, because of the common expectation that the minor candidates’ supporters switch strategically to vote for the major one, not the other way round. (Important to mention focal point and/or common expectations.)

QUESTION 6: (30 POINTS, 45 MINUTES)

<u>EITHER</u>

A good essay will cover the following points. But merely listing them is not enough for a full or near-full score; the arguments you offer in support of your assertions, and the clarity and organization of your answer, also matter. Therefore the grading has some subjective elements, and scores of 25 or more are quite rare. The grading was roughly as follows: Good essays - all the key game-theoretic concepts presented clearly and with pertinent examples - between 20 and 25; essays with numerous examples or very precise definitions of the concepts got >25; similarly, essays where some concepts were missing or misunderstood got generally between 15 and 19. A common failure was not linking general statements about this market to the precise game-theoretic concepts.

LIST OF POINTS:

The pure bargaining issues are similar to those we saw in connection with buying or selling a house. For example, if you are a buyer, find the seller’s BATNA by checking the prices of used cars of this make and year in the available price guides, find your BATNA by checking in the same sources the prices of other makes and years that interest you. If you have a good BATNA or are patient, look for credible signals of these attributes; for example do not improve an initial offer very quickly. If your bargaining attributes are not so good, see if you can mimic the behavior of someone with good ones.

Strategies in buying and selling used cars are made particularly complicated by the existence of asymmetric information – the seller knows much more about the quality of a particular car than does the buyer. This introduces signaling and screening strategies. For a private seller, it is quite difficult to credibly signal good quality, because someone selling a poor-quality car can mimic most of these signals. The cost of washing and polishing a car does not differ by quality. Anyone can *offer* to have the car checked by the buyer’s mechanic – if the buyer is satisfied by the mere offer, well and good; if he actually goes ahead with a check, you are no worse off than before. And if his mechanic points out problems that he then wants to use in bargaining down the price, how do you know whether his mechanic is colluding with him? You can get your own mechanic to certify to the quality, but then how does a buyer know whether your mechanic is colluding with you. Warrantees from a private person are often unenforceable; the seller may move and it may be very costly to track him down. And so on. For a buyer, good screening strategies are similarly difficult to find. You can get the car inspected by your mechanic, but then you bear the cost. If he finds a defect, you cannot credibly use that in bargaining because the seller may suspect collusion between you and your mechanic.

If the seller is a dealer who is in this business for a long time, then his concern for his reputation may make his assertion of quality credible. But that depends on the existence of sufficiently good information flow or communication among potential buyers. A third party such as local better business bureau may facilitate this. “Lemon Laws” also serve such a purpose.

<u>OR</u>

COMMON ERRORS: In part (b), some students missed that for joining to be a dominant strategy for 1, he must prefer to join for all n, not only n=0 (-2). In (d), most students repeated the previous argument starting with k=1 and reasoning forward, whereas rollback analysis requires you to start from player 15's decision and work backwards. Finally, a few students focused on the lack of credibility of the transfer from losers to 1, and missed the issue of the credibility of 1's not joining after the payment (-2).

(a) (3 points) $J(k,n) > S(k,n)$ if and only if $n - k > 3(k-n) - 6$,
or $4k < 4n + 6$, or $k < n + 3/2$

In parts (b) and (c), remember that the game is played with simultaneous moves.

(b) (3 points) When $k = 1$, $k < n + 3/2$ for all $n = 0, 1, 2, \ldots 14$. Therefore regardless of how many (if any) of the others are choosing IN, player 1 gets a higher payoff by choosing IN than by choosing OUT. Thus IN is player 1's dominant strategy.

(c) (4 points) OUT is dominated for player 1. When that strategy is removed, the case $n = 0$ is eliminated. Then for $n = 1, 2, \ldots 14$, we have $2 < n + 3/2$, so for player $k = 2$, IN is dominant or OUT is dominated. We can proceed in this way through the entire list of players. (A mathematically more rigorous statement uses “induction,” but here we will accept simply a verbal “and so on.”)

In parts (d) and (e), remember that the game is played with sequential moves.

(d) (5 points) Here player 15 moves last. Since $15 < 14 + 3/2$ but $15 > 13 + 3/2$, he will choose IN if and only if all the previous players have chosen IN. At the move before that, player 14 knows this. And $14 < 13 + 3/2$ but $14 > 12 + 3/2$. Therefore 14 will choose IN if all the 13 who went before him have chosen IN, because he knows that when he chooses IN so will 15 following him, but not if 12 or fewer have chosen IN, because in that case he knows that even if he chooses IN, 15 will choose OUT and so only 12 of the others will be IN. And so on through to player 2. Finally, at the initial move, player 1 will choose IN – he would do so even if no one were to follow, but in fact he knows others will follow. Then all the others will follow.

(e) (3 points) No matter when player 1 moves, he will choose IN. Then no matter when player 2 move, he will have seen player 1's choice if it went before his, or know what player 1 will choose if that move comes after player 2's move. Since $2 < 1 + 3/2$, it is optimal for player 2 to choose IN, regardless of what or when players 3, 4, ... choose. And so on.

In parts (f)-(h), remember that the status quo is the situation where the club does not exist at all ($n = 0$ and everyone has chosen OUT). The equilibrium outcome refers to the common outcome of all the cases (c)-(e) above, where everyone chooses IN, so for any one player, $n = 14$. The

(f) (4 points) Player k’s payoff in the status quo is $S(k,0) = 3k - 6$, and his payoff in the equilibrium outcome is $J(k,14) = 14 - k$. Player k is better off in the equilibrium than in the status quo if
$14 - k > 3k - 6$, or $4k < 20$, or $k < 5$.
The opposite is true if $k > 5$. Player 5 gets the same payoff in the two situations (is indifferent).

(g) (4 points) The payoff sums are

Status quo:

S(1,0) + S(2,0) + ... S(15,0) = 3 (1+2+ ... + 15) - 6 * 15

= 3 * ½ * 15 * 16 - 6 * 15 = 360 - 90 = 270

Equilibrium:

J(1,14) + J(2,14) + ... J(15,14) = 14 * 15 - (1+2+ ... 15)

= 14 * 15 - ½ * 15 * 16 = 210 - 120 = 90

Therefore the total payoff is lower in the equilibrium than in the status quo.

(h) (4 points) Players 6-15, who fare worse in the equilibrium, would be willing to pay players 1-4 who fare better in the equilibrium in exchange for a credible promise not to start the process that leads to everyone joining the club. It may be difficult to extract the required contributions from the players who gain to create the fund that compensates those who lose. But credibility is the key problem – player 1, once he has received the money, may renege on the promise and launch the club anyway.
