ECO 199 – GAMES OF STRATEGY
Spring Term 2004
PRECEPTS WEEK 5 (March 1-2)

We will discuss Question 1, which illustrates two concepts: [1] combined sequential and simultaneous moves and [2] mixed strategies. We will also do Question 2 as far as time permits.

Question 1

Consider the following simplified version of baseball. The pitcher can throw either a fastball or a curveball, the batter can either swing at the pitch or take (not swing). These choices are simultaneous for each pitch. On the first pitch, if the batter swings at a curveball or takes a fastball, he strikes out and gets 0. If the batter swings at a fastball, he has probability 0.75 of hitting a home run and getting 1, and probability 0.25 of hitting a fly ball and getting 0. If the batter takes a curveball, there is a second pitch.

On the second pitch, the first three combinations (swing at a curveball, take a fastball, and swing at a fastball) work as before; if the batter takes a curveball second pitch, he walks and gets 0.25.

This is a zero-sum game; the batter tries to maximize his expected score (probability weighted average payoff), and the pitcher tries to minimize the batter's expected score. Note that this is a sequential move game (the two pitches) containing a simultaneous move game in each pitch.

(a) Solve this game using backward induction; construct a table of payoffs for the second pitch and use these to determine the table of payoffs for the first pitch. Show that on the first pitch, the batter should take with probability 0.8.

(b) What is the pitcher's strategy in the subgame perfect equilibrium?

(c) What is the batter's expected score in this equilibrium?

(d) Explain intuitively why the batter's probability of swinging is so small.

Question 2

Do Games of Strategy, Chapter 6, Exercise 5, pp. 181-2

Offer an alternative interpretation for this game.

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PRECEPTS WEEK 5 (SOLUTION)

Question 1

(a) Table of payoffs for second pitch:

		Pitcher	
		Fast	Curve
Batter	Swing	0.75	0
	Take	0	0.25

Batter's p-mix (probability p on Swing) must satisfy: $0.75p = 0.25 (1-p)$ so we get $p = 0.25$. Pitcher's q-mix (probability q on Fast) must satisfy: $0.75q = 0.25 (1-q)$, so we get $q = 0.25$. Batter's payoff in the mixed strategy equilibrium on the second pitch is $0.75 \times 0.25 = 0.1875$, or 3/16.

We use the batter's expected payoff from the second pitch as the payoff to the batter from taking a curveball on the first pitch; *this is the crucial step of rollback in this context.* Thus, 3/16 is the payoff in the bottom right cell of the payoff table for the first pitch, as seen below.

		Pitcher	
		Fast	Curve
Batter	Swing	0.75	0
	Take	0	0.1875

Batter's p-mix here satisfies: $0.75 p = 0.1875 (1-p)$ which yields $p = 0.2$. Thus, the batter should take with probability $(1-p) = 0.8$ on the first pitch and take with probability 0.75 on the second pitch.

(b) The pitcher's equilibrium q-mix on the first pitch must satisfy: $0.75q = 0.1875 (1-q)$ which yields $q = 0.2$. The pitcher's equilibrium strategy is: Throw a curve with probability 0.8 on the first pitch. If the game goes to a second pitch, then on that pitch throw a curve with probability 0.75.

(c) The batter's expected payoff in equilibrium here is the expected payoff calculated from the equilibrium mixtures at the first pitch; $0.75 \times 0.2 = 0.15$. (The pitcher's equilibrium payoff is thus -0.15.)

(d) The batter's probability of swinging is so low because the pitcher is less likely to throw a fastball, which in turn is so because fastballs are "expensive" for the pitcher. If fastballs are thrown with greater probability, the batter will swing more often to take advantage and hit several home runs. (One player's strategy in a game cannot be understood inisolation; you must examine the game from the perspectives of both players.)

This greatly simplified "model" of baseball can be enriched in various ways. By introducing multiple stages, and more strategies at each stage, we can extend the analysis of one at-bat to the real case of three strikes and four balls, greater variety of pitches etc. And we can think of one at-bat as a part of a longer game consisting of an inning and then a whole game. Most importantly, such generalization will enable us to replace the 0.25 value of a walk by a better "intermediate value", derived by considering the situation where this at-bat occurs (how many outs, how many men on base), and possible future developments (later batters get hits, opportunities to steal bases, ...) Of course the game quickly gets beyond paper-and-pencil analysis and needs computer solutions.

Question 2

(a)

		Vendor 2				
		A	**B**	**C**	**D**	**E**
Vendor 1	**A**	85 , 85	100 , 170	125 , 195	150 , 200	160 , 160
	B	170 , 100	110 , 110	150 , 170	175 , 175	200 , 150
	C	195 , 125	170 , 150	120 , 120	170 , 150	195 , 125
	D	200 , 150	175 , 175	150 , 170	110 , 110	170 , 100
	E	160 , 160	150 , 200	125 , 195	100 , 170	85 , 85

(b) Locations A and E are both dominated for both vendors.

(c) In the reduced 3-by-3 matrix, cell-by-cell inspection reveals two pure-strategy Nash equilibria: both have one vendor located at B, and the other at D. To arrive at one of these equilibria requires coordination of some type. (If both players tried to locate at B or D, payoffs would be only 110 each instead of 175.)

(d) In sequential play of the game, the vendor that moves first will choose either B or D and the vendor that moves second will choose the other location (D or B). In the sequential game, the move order establishes the necessary coordination so that each vendor receives a payoff of 175 in equilibrium.

The game can be interpreted as positioning along the political left-right spectrum by two power-motivated candidates in an election. The reluctance to walk captures the idea that some voters may stay at home if the position of the candidate(s) is too far from their own ideal. This stops the candidates short of moving all the way to the center. However, in the political context the objective may be to maximize not your own vote total, but the difference between your vote total and that of the other. Then, ignoring the two end-locations which remain bad strategies even in this version, the payoff matrix of this zero-sum game is

		Candidate 2		
		B	C	D
Candidate 1	B	0 , 0	- 20 , 20	0 , 0
	C	20 , - 20	0 , 0	20 , - 20
	D	0 , 0	- 20 , 20	0 , 0

Here (C,C) is the pure strategy Nash equilibrium.

By the way, Gambit reveals that the original 5-by-5 ice cream game has other Nash equilibria in mixed strategies:

1. Each vendor mixes between B, C, and D with probabilities 0.26087, 0.47826, and 0.26087 respectively.

2. One vendor chooses C, the other mixes between B and D with probabilities that can form a continuous range from 0.9231 on 0.0769 on D to the other way round.