GAME THEORY CONCEPTS

Players $\{1, 2, \ldots n\}$
Strategies s_1, $s_2 \ldots s_n$
Payoff functions $\Pi_1(s_1, s_2, \ldots s_n)$, $\Pi_2(s_1, s_2, \ldots s_n)$, ...

Simultaneous moves: Nash equilibrium

Definition 1 – Each chooses own best strategy given the others' strategy.

Two players: (s_1^*, s_2^*) is NE if for any other s_1, s_2

$$\Pi_1(s_1^*, s_2^*) \geq \Pi_1(s_1, s_2^*),\ \Pi_2(s_1^*, s_2^*) \geq \Pi_2(s_1^*, s_2)$$

"Best responses" – given s_2, $s_1 = BR_1(s_2)$ maxes Π_1. Nash equilibrium is intersection of best responses. But what does "response" mean when moves simultaneous? So

Definition 2 – Each chooses own best strategy given his belief about others' strategy; AND these beliefs are correct.

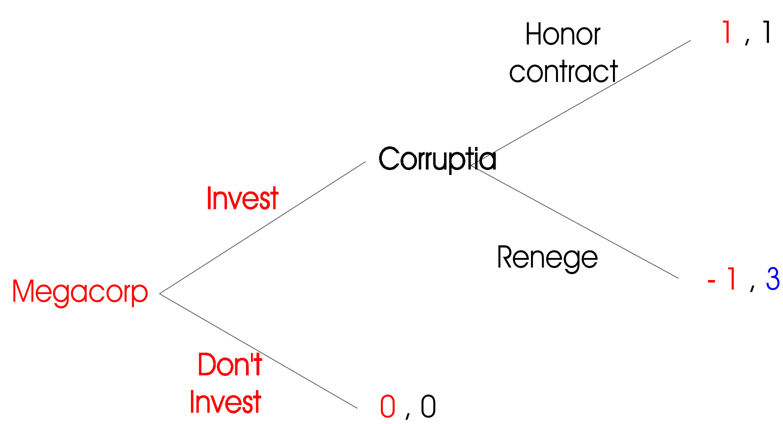
Sequential moves: Backward induction or rollback reasoning, leading to subgame perfect equilibrium:
For simple two-player, two-stage game, this means

For any s_1, response $R_2(s_1)$ maxes $\Pi_2(s_1, s_2)$
s_1 maxes $\Pi_1(s_1, R_2(s_1))$

Example of simultaneous-move game

		Column		
		Left	Middle	Right
Row	Top	3 1	2 3	10 2
	High	4 5	3 0	6 4
	Low	2 2	5 4	12 3
	Bottom	5 6	4 5	9 7

Example of sequential-move game



Our economic application will have
continuously variable strategies (price etc.)

QUANTITY-SETTING (COURNOT) DUOPOLY

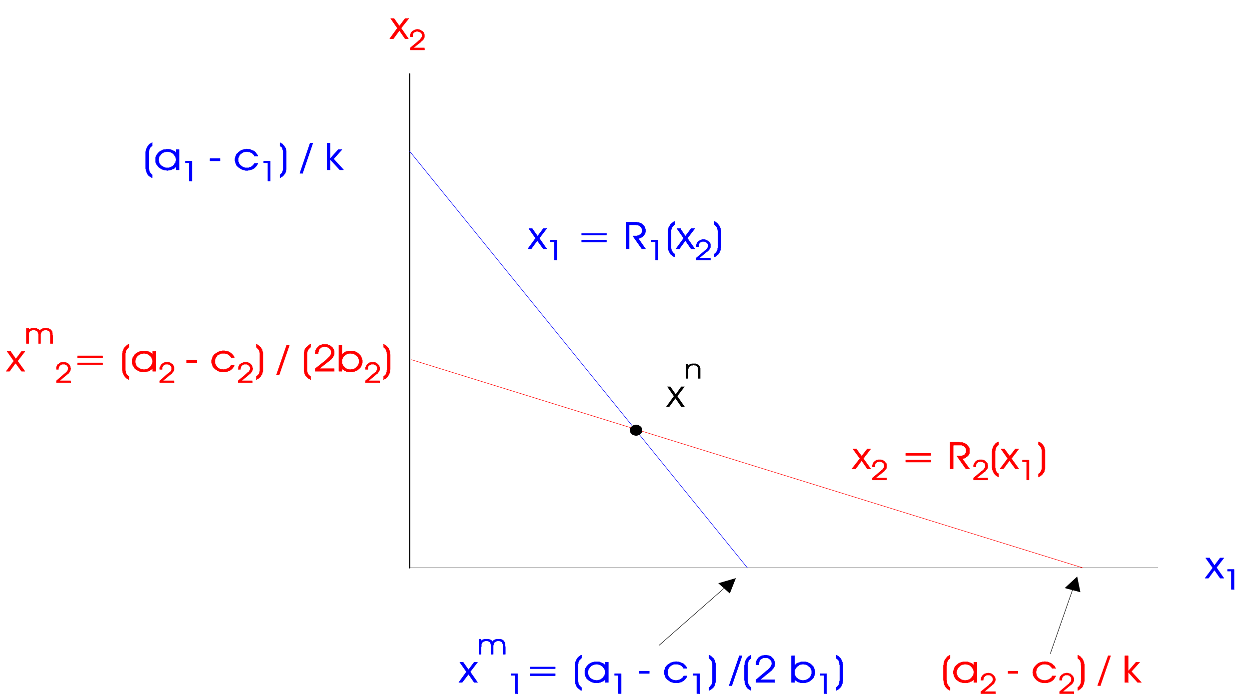
$$\Pi_1(x_1, x_2) = (p_1 - c_1)\, x_1 = [(a_1 - c_1) - b_1\, x_1 - k\, x_2]\, x_1$$

Firm 1's best response

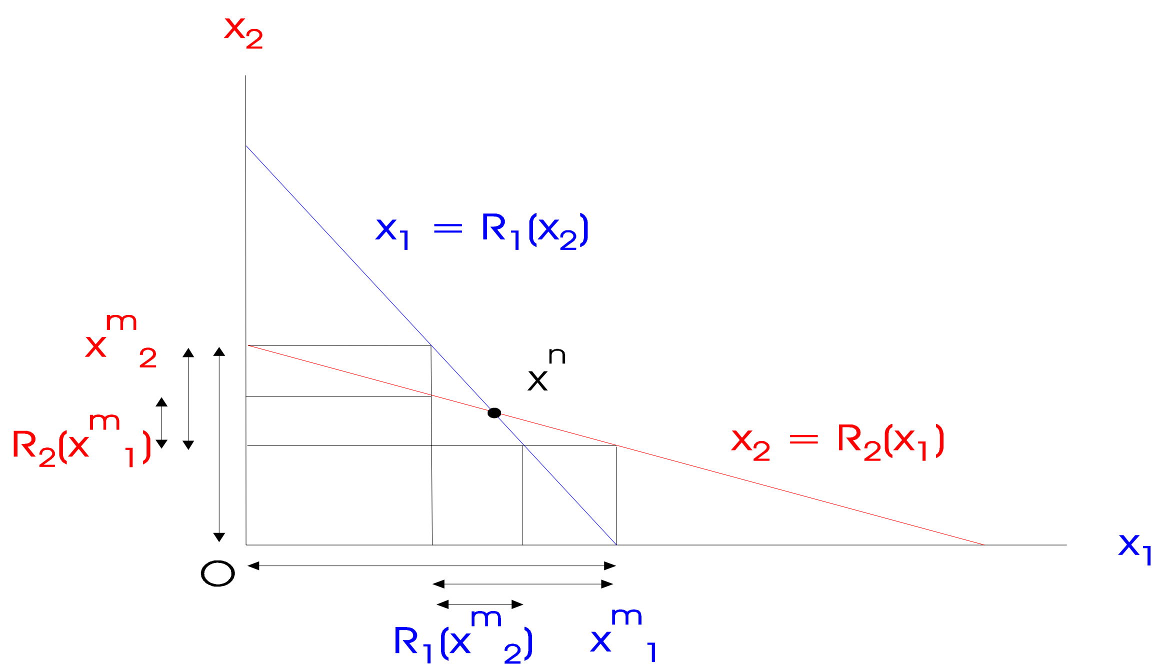
$$(a_1 - c_1) - 2\, b_1\, x_1 - k\, x_2 = 0$$

Similarly firm 2's. Solve jointly for Cournot-Nash eqm:

$$\begin{aligned} x_1^n &= [\, 2\, b_2\, (a_1 - c_1) - k\, (a_2 - c_2)\,] \,/\, (4\, b_1\, b_2 - k^2) \\ x_2^n &= [\, 2\, b_1\, (a_2 - c_2) - k\, (a_1 - c_1)\,] \,/\, (4\, b_1\, b_2 - k^2) \end{aligned}$$

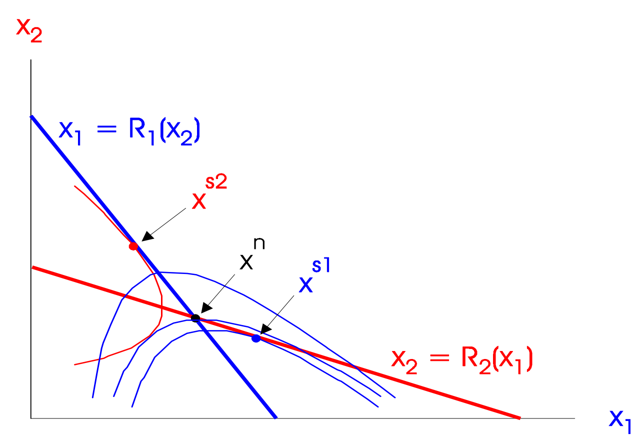


STABILITY – RATIONALIZABILITY



STACKELBERG LEADERSHIP

Sequential: firm 1 chooses x_1; then firm 2 chooses x_2



COURNOT OLIGOPOLY

Homog. product, n identical firms

Constant marg. cost c, fixed cost f for each

Linear industry demand : $p = a - b\,X$

Firm i profit:

$$\Pi_i = [a - b\,(x_1 + x_2 + \ldots + x_n)]\; x_i - c\; x_i - f$$

FONC : $a - b\,(x_1 + x_2 + \ldots + x_n) - c - b\,x_i = 0$

Adding FONCs : $n\,[a - b\,X - c] - b\,X = 0$.

Solution for eqm.

$$X = \frac{n}{n+1}\;\frac{a-c}{b}, \qquad x = \frac{1}{n+1}\;\frac{a-c}{b}, \qquad p = \frac{a + n\,c}{n+1}$$

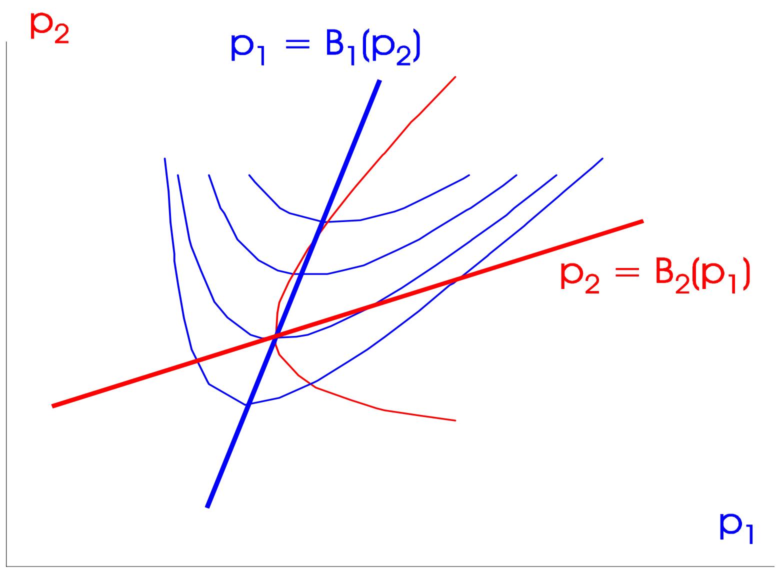
As $n \uparrow \infty$, $p \downarrow c$ (competitive limit).

But max n compatible with $\Pi > 0$

$$\overline{n} \equiv \frac{a-c}{\sqrt{b\,f}} - 1$$

PRICE-SETTING (BERTRAND) DUOPOLY

Profit $\Pi_1(p_1, p_2) = (p_1 - c_1)\ (\alpha_1 - \beta_1\, p_1 + \kappa\, p_2)$
Best response $p_1 = [\,(\alpha_1 + \beta_1\, c_1) + \kappa\, p_2\,]\,/\,(2\,\beta_1)$



COMPARISONS

For substitute products, ranked by Prices ↑, Quantities ↓,
Firms' profits ↑, Cons. surplus and Social efficiency ↓

1. Marginal cost pricing
2. Bertrand
3. Cournot
4. Cartel (Joint profit max)