ECO 305 – Microeconomic Theory

Strategic Commitment in Two-Stage Games

(Mathematical Version)

General Theory

Stage 1: Player 1 chooses K, "Investment"

Stage 2: Players 1, 2 choose actions x_1, x_2.

Payoffs $F^1(x_1, x_2, K)$, $F^2(x_1, x_2)$.

We choose to label the actions in such a way that $F^i_j < 0$ for $i \neq j$. An increase in x_1 lowers F^2 and vice versa, thus the actions represent the degree of aggressiveness of rivalry in the Stage 2 game.

Second-Stage Equilibrium

First-order conditions:

[1] $F^1_1(x_1, x_2, K) = 0$, defines firm 1's reaction function $x_1 = R_1(x_2, K)$.

[2] $F^2_2(x_1, x_2) = 0$, defines firm 2's reaction function $x_2 = R_2(x_1)$.

Second-order conditions: $F^1_{11} < 0$, $F^2_{22} < 0$, omitting the arguments for brevity.

Stability condition: $\Delta \equiv F^1_{11}\ F^2_{22} - F^1_{12}\ F^2_{21} > 0$.

Reaction functions

The first-order conditions define the two reaction functions. We need two items of information about them:

[1] The slope of player 2's reaction function is found using the implicit function theorem along its first-order condition:

$$R_2'(x_1) = \left.\frac{dx_2}{dx_1}\right|_{F^2_2=\text{constant}} = -\ \frac{F^2_{21}}{F^2_{22}}\ .$$

The sign of this slope is the same as the sign of F^2_{21} (because $F^2_{22} < 0$). If it is negative, then player 2's reaction function is downward-sloping: if player 1 commits to being more aggressive, player 2 backs off and becomes less aggressive. If it is positive, then player 2 reacts to player 1's aggressiveness with more of his own.

Definition: player 2's action is a *strategic substitute* to player 1's if a larger x_1 lowers the marginal payoff of player 2, that is,

$$F^2_{21} = \frac{\partial(\partial F^2/\partial x_2)}{\partial x_1} < 0$$

and a *strategic complement* if $F^2_{21} > 0$. Similarly we can define player 1's action being a strategic substitute or complement to player 2's according to whether F^1_{12} is negative or positive. (Note: this concept need not be symmetric across players.)

[2] Shift of player 1's reaction function by using the implicit function theorem in its first-order condition:

$$\frac{\partial R_1}{\partial K} = -\frac{F^1_{1K}}{F^1_{11}}.$$

The sign of this is the same as the sign of F^1_{1K} (because $F^1_{11} < 0$). If positive, then more investment in stage 1 constitutes a commitment to act more aggressively in stage 2; if negative, then higher K in stage 1 is a commitment to act less aggressively in stage 2.

Interpretation of Stability Condition

Consider the case where the actions are strategic substitutes: $F^2_{21} < 0$ and $F^1_{12} < 0$. Then the stability condition can be written as

$$-\frac{F^2_{21}}{F^2_{22}} > -\frac{F^1_{11}}{F^1_{12}} \qquad \text{or} \qquad \left.\frac{dx_2}{dx_1}\right|_{\text{firm 2's RF}} > \left.\frac{dx_2}{dx_1}\right|_{\text{firm 1's RF}}$$

Using expressions for the slopes of the reaction functions of the two firms, this becomes

$$R_2'(x_1) > 1\,/(\partial R_1/\partial x_2)$$

That is, firm 1's reaction function is more negatively sloped in (x_1, x_2) space (is steeper).

Similar interpretations can be found in other cases of strategic complements.

Comparative statics

Totally differentiate the first order conditions:

$$\begin{bmatrix} F^1_{11} & F^1_{12} \\ F^2_{21} & F^2_{22} \end{bmatrix} \begin{bmatrix} dx_1 \\ dx_2 \end{bmatrix} = -\begin{bmatrix} F^1_{1K} \\ 0 \end{bmatrix} dK.$$

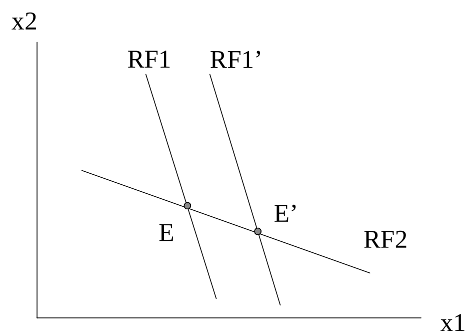
The solution is

$$\begin{bmatrix} dx_1 \\ dx_2 \end{bmatrix} = \frac{-1}{\Delta} \begin{bmatrix} F^2_{22} & -F^1_{12} \\ -F^2_{21} & F^1_{11} \end{bmatrix} \begin{bmatrix} F^1_{1K} \\ 0 \end{bmatrix} dK$$

$$= \frac{-1}{\Delta} \begin{bmatrix} F^2_{22}\, F^1_{1K} \\ -F^2_{21}\, F^1_{1K} \end{bmatrix} dK.$$

Figure 1 shows this for a particular case. The two players' actions are strategic substitutes, therefore both reaction functions $RF1$ and $RF2$ slope downward. The stability condition tells us that $RF1$ is steeper (more negatively sloped). The Nash equilibrium is at E. Then K increases to K'. The figure shows the case where investment (increase in K) is a commitment by player 1 to be more aggressive, that is, $F^1_K > 0$, so his reaction function shifts rightward to $RF1'$. The equilibrium moves to the right and down along player 2's reaction function $RF2$, to the new point E'. There x_1 is higher and x_2 is lower than at E. Verify these algebraically from the equations above.

Figure 1: Reaction Function and Equilibrium Shift as K Increases



Strategic choice of K

Player 1 chooses K to maximize F^1, bearing in mind the effects that operate through the second-stage equilibrium values of x_1 and x_2. Therefore he calculates a total derivative

$$\frac{dF^1}{dK} = F^1_1 \, \frac{dx_1}{dK} + F^1_2 \, \frac{dx_2}{dK} + F^1_K .$$

The first term is zero because of player 1's second-stage first-order condition; this is an "envelope property". The third term is the direct effect of K, and would be the only one considered if there were no strategic effect, as for example if the game were played simultaneously with player 1 choosing (K, x_1) and player 2 choosing x_2. But the second term is the additional strategic effect that arises because of the sequencing: player 1's stage-1 choice of K constitutes a commitment that affects player 2's stage-2 choice and player 1 can manipulate this to his own advantage.

Using the comparative statics results, the strategic effect is

$$\frac{1}{\Delta} \, F^1_2 \, F^2_{21} \, F^1_{1K} .$$

Of the four factors in this, the first is positive by the stability condition and the second negative by the choice of actions as aggressive. The remaining two can have either sign, so four cases arise. If the overall strategic effect is positive, that induces player 1 to choose the level of investment too high (relative to its direct or non-strategic level); if negative, too low. We classify the cases in Table 1 following the terminology of Fudenberg and Tirole (American Economic Review Papers and Proceedings, May 1984).

Table 1: The Fudenberg-Tirole Zoo

		Investment makes player 1	
		aggressive	weak
Slope of Player 2's reaction function	Down (strategic substitutes)	Top Dog: Overinvest to become more aggressive	Lean and Hungry: Underinvest to become more aggressive
(Relationship between the two strategies)	Up (strategic complements)	Puppy Dog: Underinvest to become less aggressive	Fat Cat: Overinvest to become less aggressive

Many concepts in industrial organization and international trade (for example "strategic trade policy") are applications of this general theory of two-stage games. Related references: Bulow, Geanakoplos and Klemperer (Journal of Political Economy 1985), Eaton and Grossman (Quarterly Journal of Economics 1986).