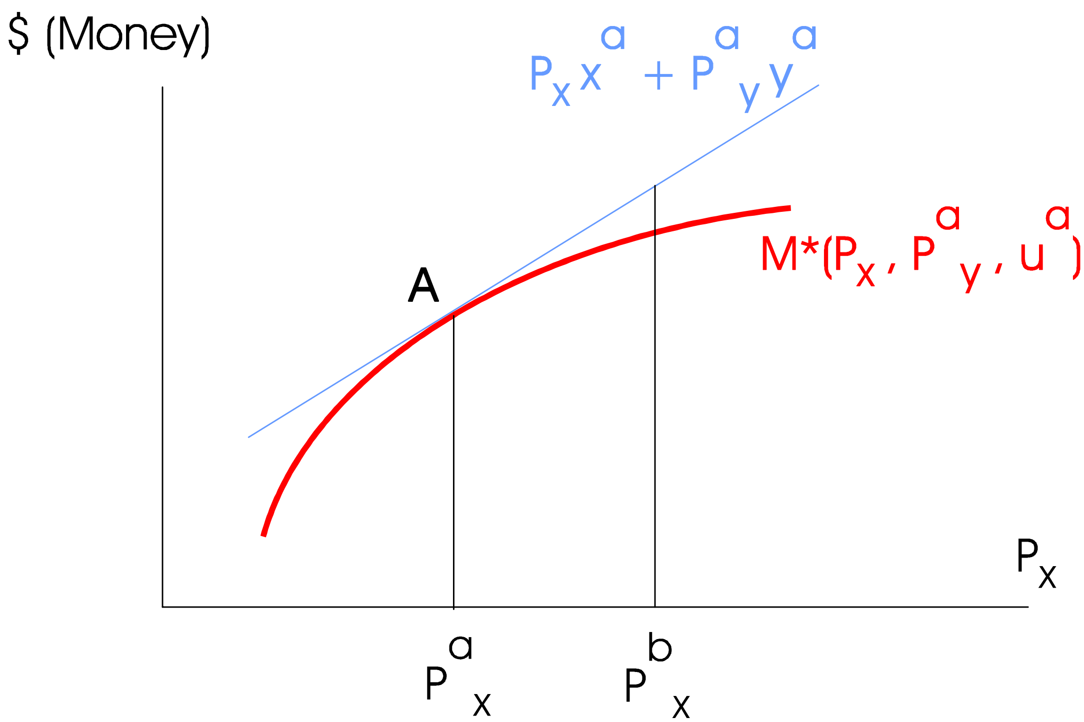
APPLICATIONS OF EXPENDITURE FUNCTION

1. EXACT COMPENSATION FOR PRICE CHANGES

Compare two situations: Old A:

prices (P_x^a, P_y^a), quantities (x^a, y^a), utility u^a

New B: Change P_x only, to P_x^b



Could get old utility by consuming old quantities:

That would cost $P_x^b\, x^a + P_y^a\, y^a$

New expenditure-minimizing bundle cannot cost more:

$M^*(P_x^b, P_y^a, u^a) \leq P_x^b\, x^a + P_y^a\, y^a$

Graphing each of the two sides against P_x,
the line is tangent to graph of M^* at A

The graph of M^* lies everywhere below the tangent

Its slope at A is

$$x^a = \left. \frac{\partial M^*}{\partial P_x} \right|_{\text{at } a}$$

Since A could be any point, this proves concavity
and Hotelling's Lemma in one step

If no substitution: (x^a, y^a) only way to achieve u^a,
$M^*(P_x, P_y, u^a) = P_x\ x^a + P_y\ y^a$ linear in prices
coincides with tangent

Thus possibility of substitution of quantities
makes expenditure function concave

Substitution occurs when *relative* prices change

If all prices change in same proportion
as with pure inflation
then $(P_x^b, P_y^b) = k\ (P_x^a, P_y^a)$, no subst'n
and $M^*(P_x^b, P_y^b, u^a) = k\ M^*(P_x^a, P_y^a, u^a)$

Initial budget line A, optimal choice X

After increase in price of x,

If no compensation, budget line B.

If (Hicks) compensation to maintain old utility:

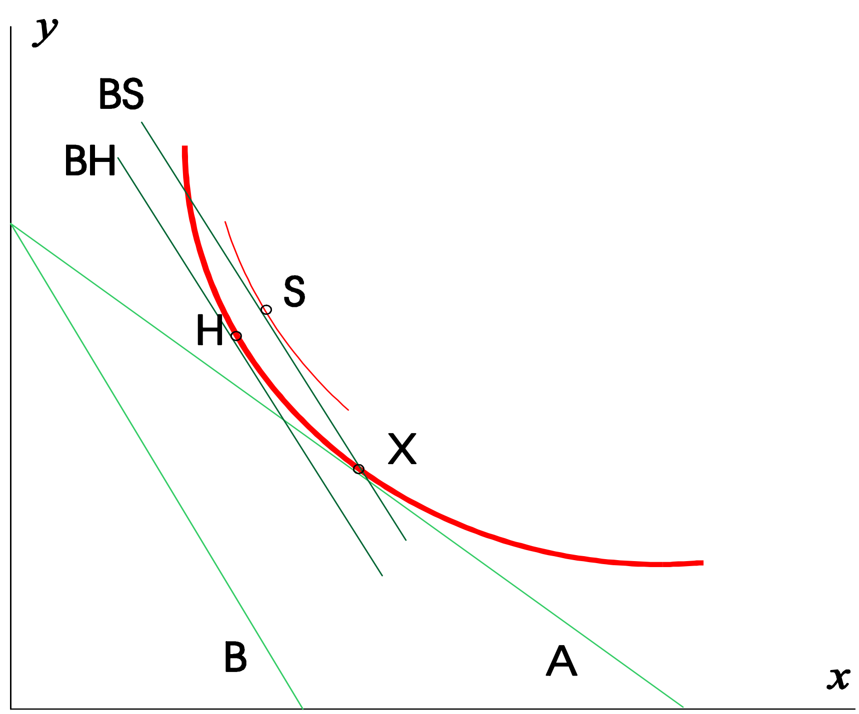
Budget line BH, choice H

If (Slutsky) compensation to allow purchase of old X:

Budget line BS

Slutsky compensation $= (P_x^b - P_x^a)\ x^a$ (too much: consumer "cheats," consumes S for higher utility)

Hicks comp'n $= M^*(P_x^b, P_y^a, u^a) - M^*(P_x^a, P_y^a, u^a)$



H, S coincide with X if L-shaped indiff. curves

2. COST OF LIVING INDEXES

Initial point A; prices $\mathbf{P}^a$, income I^a
quantities $\mathbf{x}^a$, utility $u^a = U^*(\mathbf{P}^a, I^a)$

New prices $\mathbf{P}^b$. To preserve old utility, need income
$M^*(\mathbf{P}^b, u^a) = M^*(\mathbf{P}^b, U^*(\mathbf{P}^a, I^a)\,)$

Note: combination of M^* and U^* cancels effect of
special cardinal choice of utility ("anchoring")

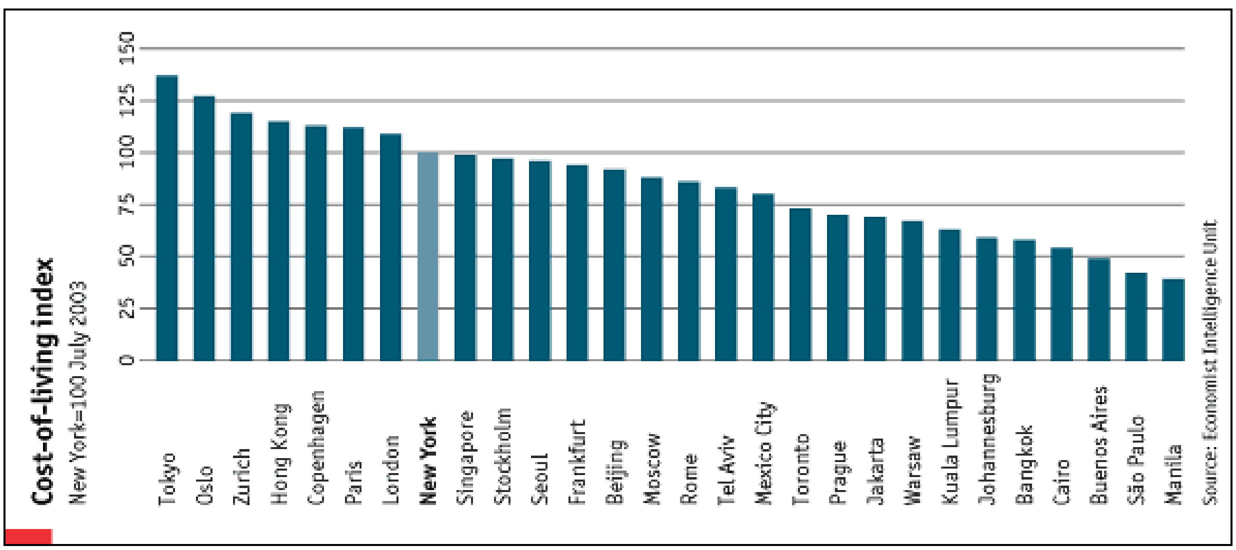
True cost of living index: $M^*(\mathbf{P}^b, u^a) \,/\, \mathbf{P}^a \cdot \mathbf{x}^a$

Usual initial quantity-based index: $\mathbf{P}^b \cdot \mathbf{x}^a \,/\, \mathbf{P}^a \cdot \mathbf{x}^a$

True index $\leq$ Conventional index, with equality
only if no substitution or all prices change in proportion

Examples: (1) Bias in the consumer price index over time
Relative prices change (teleconferencing vs. meeting)

(2) *The Economist*'s cost of living index for cities:



3. DEAD-WEIGHT BURDEN OF TAX ON GOODS

Lump-sum tax T: Given M, P_x, P_y,
consumer achieves u defined by $M - T = M^*(P_x, P_y, u)$
OR: Tax t per unit of good x keeping utility same:

$$M = M^*(P_x + t, P_y, u), \qquad x = \left.\frac{\partial M^*}{\partial P_x}\right|_{(P_x+t, P_y, u)}$$

Revenue raised by this tax: $R = t\,x$.
How do the two compare? Take Taylor expansion of M^*
around the "base point" $(P_x + t, P_y, u)$:

$$M^*(P_x, P_y, u) \approx M^*(P_x + t, P_y, u) + (-t)\,\frac{\partial M^*}{\partial P_x} + \tfrac{1}{2}\,(-t)^2\,\frac{\partial^2 M^*}{\partial P_x^2}$$

Write

$$\Delta x = -t \left.\frac{\partial x}{\partial P_x}\right|_{u = \text{constant}} = -t\,\frac{\partial^2 M^*}{\partial P_x^2}\,,$$

pure substitution part of the reduction in x due to tax.
Then

$$M - T \approx M - t\,x - \tfrac{1}{2}\,t\,\Delta x$$

$$R \approx T - \tfrac{1}{2}\,t\,\Delta x\,.$$

So lump-sum tax better. Reason: substitution.
Difference is the dead-weight loss.

This fits with the usual “consumer surplus” idea:
Tax $t$raises the price from OA to OD,

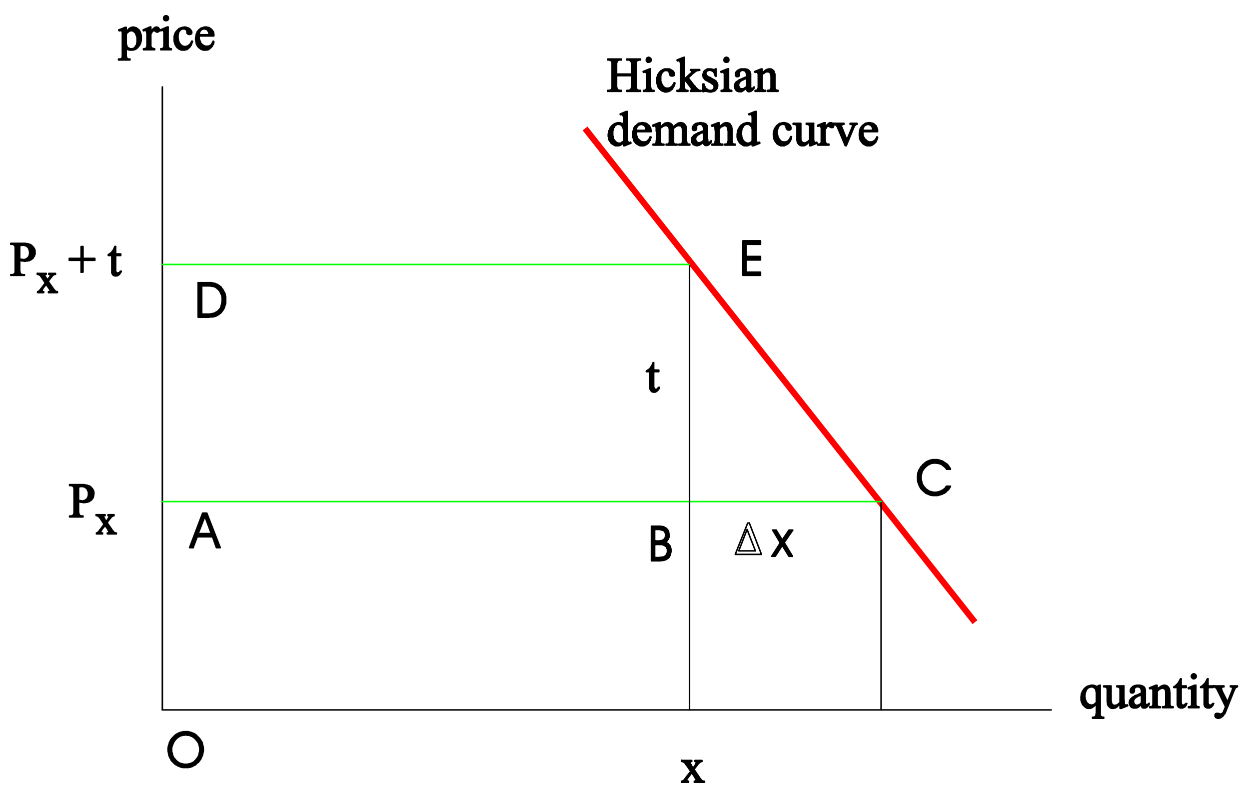
and quantity falls from AC to DE = AB

Tax revenue = rectangle ABED

Dead-weight loss or Excess burden of tax = triangle BCE

These sum to trapezoid ACED

= Loss of consumer surplus



New feature, different from ECO 102:

Consumer surplus better measured as the area
to the left of the Hicksian demand curve,
not the Marshallian demand curve