PROFIT-MAXIMIZATION – TWO-STEP APPROACH

For each level of output Q, produce it at minimum cost:

$$\min \{ \, w\,L + r\,K \mid F(L,K) \geq Q) \, \}$$

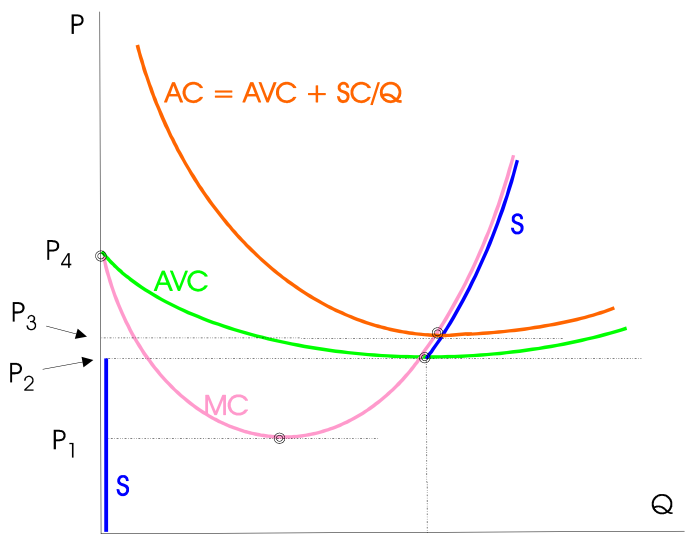
Result: conditional input demands $L^*(w,r,Q)$, $K^*(w,r,Q)$
and the "dual" or minimized cost function $C^*(w,r,Q)$

Then choose Q to $\max \; p\,Q - C^*(w,r,Q)$

FONC: $p = \partial C^* / \partial Q$ (price $=$ marginal cost)

SOSC: $\partial^2 C^* / \partial Q^2 > 0$ (rising marginal cost)

If fixed cost, need to compare against $Q = 0$



$P < P_1$: no critical point; $Q = 0$ optimum

$P_1 < P < P_2$: $Q = 0$ global optimum, local along MC

$P_2 < P < P_3$: global optimum along MC but lossmaking

$P_3 < P$: global optimum along MC and profitmaking

$P_1 < P < P_4$: local min on decreasing portion of MC

PROFIT MAXIMIZATION – SINGLE-STEP APPROACH

$$\max \ \Pi = p \ F(K, L) - w\, L - r\, K$$

FONCs – price of each input = value of its marginal product

$$p \ \partial F / \partial L = w, \qquad p \ \partial F / \partial K = r$$

SOSCs – (1) diminishing marginal returns to each input,
(2) diminishing returns to scale (this is not fully rigorous)

Result - (unconditional) input demand functions

$L^*(p, w, r)$, $K^*(p, w, r)$, yielding $Q^* = F(K^*, L^*)$

Substitute in profit expression to get "dual" profit function

$$\Pi^*(p, w, r) = p \ Q^* - w\, L^* - r\, K^*$$

Properties of dual profit function:
(1) Homogeneous degree 1, and (2) convex in (p, w, r)
(3) Hotelling's lemma:

$$Q^* = \partial \Pi^* / \partial p, \quad L^* = - \ \partial \Pi^* / \partial w, \quad K^* = - \ \partial \Pi^* / \partial r$$

Proof of these follows same lines as those of concavity of expenditure functions - take initial (p^a, w^a, r^a) and initially optimum L^a, K^a, Q^a. Could go on using these when prices change, so new optimum choices should yield no less profit.

EMPIRICAL ESTIMATION

U.S. MANUFACTURING (Ernst Berndt, 1991)

$$\begin{aligned} \ln C &= \ln(\alpha_0) + \sum_i \alpha_i \ \ln(P_i) \\ &+ \tfrac{1}{2} \sum_i \sum_j \gamma_{ij} \ln(P_i) \ln(P_j) \\ &+ \alpha_Y \ \ln Y + \tfrac{1}{2} \gamma_{YY} \ (\ln Y)^2 \\ &+ \sum_i \gamma_{iY} \ln(P_i) \ln Y \end{aligned}$$

i, j = inputs K, L, E, and M

$$\sum_i \alpha_i = 1, \qquad \gamma_{ij} = \gamma_{ji}, \qquad \sum_i \gamma_{ij} = 0$$

Find factor cost share functions and estimate, e.g.

$$\frac{P_L L}{C} = \frac{P_L}{C} \frac{\partial C}{\partial P_L} = \frac{d \ln C}{d \ln P_L} \,.$$

Results: Elasticities of substitution

$$\sigma_{KL} = 0.97, \ \sigma_{KE} = -3.60, \ \sigma_{KM} = 0.35,$$

$$\sigma_{LM} = 0.61, \ \sigma_{EM} = 0.83, \ \sigma_{LE} = 0.68$$

Own price elasticities of factor demands

$$\epsilon_K = -0.34, \ \epsilon_L = -0.45, \ \epsilon_E = -0.53, \ \epsilon_M = -0.24 \,.$$

CREDIT UNIONS (Moeller, Princeton Sr Thesis 1999)

$$\begin{aligned} \ln C &= a + b_1 \ \ln Q + b_2 \ (\ln Q)^2 \\ &\quad + \sum_i c_i \ \ln W_i + \sum_j \ d_j \ \ln F_j + \mu \,, \end{aligned}$$

where $Q =$ size (output) of the credit union
W_i factor prices, F_j other structural variables
μ is stochastic error term.

Results

$$\begin{aligned} b_1 &= 0.6537 \qquad \text{with standard error 0.0231,} \\ b_2 &= 0.0204 \qquad \text{with standard error 0.0015.} \end{aligned}$$

$b_1 < 1$, $b_2 > 0$: initial economies of scale
and eventual diseconomies

Averge cost is minimized when

$$\ln Q = (1 - b_1)/(2 \ b_2) = 8.48, \quad \text{or} \quad Q = 4764$$

85% of U.S. credit unions were to the left of this.
Median $Q = 705$, AC penalty 7.8 %.