Location as an Asset*

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Abstract

The location of individuals determines their job and schooling opportunities, amenities, and housing costs. We conceptualize the location choice of individuals as a decision to invest in a 'location asset'. This asset has a current cost equal to the location's rent, and a future payoff through better job and schooling opportunities. As with any asset, savers in the location asset transfer resources into the future by going to expensive locations with high future returns. In contrast, borrowers transfer resources to the present by going to cheap locations that offer few other advantages. Holdings of the location asset depend on its comparison to other assets, with the distinction that the location asset is not subject to borrowing constraints. We propose a dynamic location model and derive an agent's mobility choices after experiencing income shocks. We document the investment dimension of location and confirm the core predictions of our theory using French individual panel data from tax returns.

1 Introduction

Few decisions shape an individual's life more than the location decision. Spatial differences in job and schooling opportunities, as well as living amenities and the corresponding rents and housing prices are enormous. Life prospects for a kid growing up in Palo Alto are staggeringly different than those for someone growing in central Detroit, even if they come

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from similar backgrounds and both go to local public schools. The obvious question that arises is then, why do people fail to move to the locations that seem to offer the best prospects for them and for their families?

A potential answer to this question is that agents, or their families, optimized their location at some point, but regions deteriorated and large migration costs make moving to better locations not worth the cost.[1] Similarly, agents might fail to locate in what seem as better locations due to idiosyncratic attachments to the places where they, or their families, grew up. The problem with these explanations is that it is hard to imagine that moving costs or idiosyncratic preferences are sufficiently large to bridge the gap between the best and worse neighborhoods in virtually all regions of the world. These largely unobserved costs seem to be just a stand-in for another mechanism. Something is missing from this basic notion of static spatial equilibrium where similar marginal movers equalize utility across locations adjusted for moving costs.

In this paper we propose a different way of conceptualizing the location decision of agents, one that puts at the forefront the intertemporal trade-off involved in location decisions. We argue that the location decision can be understood as an asset investment decision. Buying more of the asset involves moving to better locations that cost more today but give better returns tomorrow, while selling the asset implies moving to cheaper locations with little opportunities.[2] The 'location as an asset' view can explain why agents prefer locations that seem undesirable from a static spatial equilibrium perspective even in the absence of moving costs. It can also explain why local living costs compensate the benefits from desirable locations for some agents but not for others, even in the absence of non-homotheticities or differences in preferences. The 'location asset' should not be confused with 'an asset at a location', like a house. The location asset is used by all agents, including renters and owners, when they make location choices.[3]

The 'location asset' has some specific features that make it different from other assets and determine its use. As any other asset, unconstrained agents use it to determine their

[1] Kennan and Walker (2011) estimate that moving costs as large as $380 thousand 2010 dollars (for young movers, 312 thousand for average ones) are needed to account for observed migration flows using a state-of-the-art model of location decisions. Diamond et al. (2019) using a policy that implements rent-controls in the San Francisco area find a smaller but still large fixed cost of around $40 thousand.

[2] We can think of at least two channels through which returns to location would accrue over time. First, different places may provide different labor market prospects during one's work-life, as documented by De La Roca and Puga (2017). Second, different locations may offer different rates of intergenerational human capital accumulation. For example, by offering different schooling options, as shown in Chetty and Hendren (2018).

[3] We view housing wealth as financial wealth that is perhaps less liquid. Hence, borrowers will run it down completely before they become financially constrained and start using the location asset to transfer resources intertemporally.

consumption-savings profile only to the extent that the return from doing so dominates that of other assets, in particular, risk free bonds or bank borrowing and lending. The unique characteristic of the location asset is that it is not subject to borrowing constraints. Agents can always borrow, namely, transfer resources from the future to the present, by going to a cheaper neighborhood or city with worse opportunities. If an agent is not in the worst possible neighborhood already, she can keep transferring resources from the future to the present by 'selling' the location asset. The other unique characteristic of this asset is that the amount of the asset that an agent can hold is limited by the housing needs, labor supply, fertility decisions and other choices that determine the current cost and the future benefits of living in a particular location. As such, the asset has heterogeneous returns depending on the holder of the asset.

Conceptualizing location decisions as buying and selling a 'location asset' is particularly useful to understand where people move as a result of shocks, like unemployment, that affect their income or wealth shocks. Consider an agent with little or no wealth that receives a front-loaded income shock. For example, a blue-collar worker in the automobile industry in Detroit that gets fired. Where will she go? A good neighborhood with excellent schools for her children and plenty of job opportunities or a run-down neighborhood in Saint Louis? Think first about the consumption-savings decision of this agent. The front-loaded shock makes her want to transfer consumption from the future to the present. In the absence of accumulated wealth, smoothing consumption requires borrowing. The absence of collateral, however, implies that she will be constrained to borrow using standard financial assets. What is left is to borrow using the location asset and downgrade to a cheaper location with worse opportunities. Hence, constrained agents that receive bad shocks will have a higher demand for locations that offer few opportunities at minimal cost. Similarly, front-loaded positive shocks will make constrained individuals upgrade location so as to save using the location asset.

Some additional aggregate implications follow. For example, changes in the rewards for particular occupations will result in front-loaded shocks for dynastic families, since heads-of-households have already invested in an occupation while their descendants have yet to choose. Hence, these changes in rewards will lead to spatial segregation as auto workers who borrow locate in Detroit and computer programmers and Yoga teachers who save locate in Palo Alto.[4] Furthermore, our view underscores that place-based policies may hurt the currently poor as they reduce the supply of cheap locations where those individuals may

[4] Our view also implies that low rates of return in financial market (e.g. low interest rates) result in low rates of return of the 'location asset' and, therefore, larger price differentials across locations, reminiscent of the low interest rate period after the 2008 financial crisis where some of the differentials in house prices increased.

prefer to locate.

To make precise our conceptualization of the location decision as an asset that does not face borrowing constraints, we start by proposing a simple two period economy where agents have heterogeneous asset holdings, incomes, and levels of skills. Agents have access to a risk free bond but face a standard borrowing constraint that prevents them from borrowing beyond an exogenous amount. Individuals choose a consumption profile and a location, which in turn determines their current rent and income next period as a function of their skill. There is a continuum of locations that differ in the marginal return of a unit of skill. In equilibrium, wealthy agents locate in their ideal city conditional on their skill, while constrained individuals, either because they have low assets levels or back-loaded incomes, locate in cities that pay less but where rents are lower. Namely, they borrow using the location asset. Back and front-loaded shocks have the effects described above.

We then present a fully-fledged infinite horizon dynamic model where agents face an idiosyncratic income process and where wealth is now endogenous. As a result of an idiosyncratic temporary income shock, unconstrained individuals first run down their financial assets until they are at the borrowing constraint. Once there, they start borrowing using the location asset and so downgrade their location in order to minimize fluctuations in their level of consumption. This downgrading of location continues until individuals reach the worse location they are willing to go to, or the income shock reverts to the high value. Once the temporary shock has reverted, individuals move progressively back to the initial location.[5]

We evaluate empirically the implications of our theory, as well as those of alternative views, using detailed individual panel data from France. We use a longitudinal 8% panel of workers which allows us to track the same individual over several years. By merging tax return data from both households and employers, our dataset constitutes one of the first large-scale administrative datasets with detailed information on financial assets, high-resolution location, and matched employer-employee labor market characteristics for a large economy like France.

We first document that moving to locations with a higher rank pays off gradually over time. Building on De La Roca and Puga (2017), we focus on the dynamic gains in the labor market across finely disaggregated neighborhoods.[6] We find that the returns to moving to

[5]Our infinite horizon model shares many features with dynamic portfolio problems with investors who face a credit constraint on risk-free bonds. Thus, we build a Huggett (1993) economy with a second asset: the 'location asset'. Related work includes, but is not limited to, Angeletos (2007) and Moll (2014). Importantly, individuals in our model always wish to hold a convex combination of both assets, due to the endogenously nonlinear returns of the 'location asset'.

[6]Lack of data on school quality in France prevents us from estimating intergenerational rates of human capital accumulation across locations due to heterogeneous schooling options. Therefore, our results should be interpreted as a lower bound on the dynamic gains of location.

the best location relative to moving to the worst double after 10 years for observationally equivalent individuals. Comparing similar individuals, movers to the best location relative to movers to the worst location receive wages that are 10% higher upon moving, but that gap widens to 20% after 10 years. These findings support the view that moving to better locations pays off gradually over time, thus underscoring the importance of the investment aspect of location.[7]

We then empirically investigate the location decisions of individuals that receive negative income shocks. We use an event-study design to track how the rank of an individual's location changes over time. The results are stark. Conditional on municipality, income, occupation, age, and home ownership, after a negative income shock of at least 25%, individuals that move and start at the bottom quintile of the financial wealth distribution (and so are presumably financially constrained) downgrade their location by about 2 percentile points relative to movers at the top quintile. We show that these results are robust to controlling for a wide array of observable characteristics. In addition, since we find that only individuals at the bottom quintile downgrade differentially, our results falsify theories based on a static decision to consume amenities and that do not rely on dynamic optimization and financial constraints.

The predictions of the theory can be verified in relative differences across wealth groups, as discussed, as well as in levels. We show that following the negative income shock, low-wealth individuals on average move to lower ranked locations, while high-wealth individuals do not adjust their location much. As predicted by our mechanism, the location decision of individuals should be mirrored by their holdings of financial assets, since unconstrained individuals smooth consumption with the asset that has the lowest return at the margin. We find that low-wealth individuals who receive the shock downgrade their location but do not adjust their holdings of financial assets, consistent with them being close to the credit constraint. In contrast, wealthy individuals who receive the shock do not downgrade their location but reduce their holdings of financial assets. Together, our empirical findings provide clear evidence of the use of the 'location asset' to intertemporally smooth the consumption of income shocks.

There is a large literature documenting the large variation in income levels and other outcomes across locations.[8] Kennan and Walker (2011) argue forcefully that inter-state migration decisions are made based on income prospects, but are also influenced importantly

[7] Glaeser and Mare (2001) also find some evidence in the U.S. for dynamic gains from migrating to larger cities. Baum-Snow and Pavan (2012) show, in a structural model, that differences in initial wages are important drivers of pay differentials between small cities, while differences in wage growth explain a large part of pay differentials between larger cities in the U.S.

[8] See Wilson (1987), Denton and Massey (1998), Cutler and Glaeser (1997), Desmet and Rossi-Hansberg (2013), Altonji and Mansfield (2011), and Hsieh and Moretti (2019) among many others.

by geographic differences. In fact, Diamond (2016) and Giannone (2017) show that the U.S. has experienced increasing skill segregation, indicating that spatial gaps are not diminishing. Bilal (2020) emphasizes that spatial unemployment differentials are large and persistent, and lead to substantial human capital gaps as workers in high-unemployment areas are repeatedly scarred by unemployment.[9] Kaplan and Schulhofer-Wohl (2017) show that mobility in the U.S. is declining.[10] Going one step further, Fogli and Guerrieri (2018) argue that spatial segregation is related to income inequality because it affects the returns to human capital and therefore offsprings' education.

Most equilibrium analysis of individual location choices is either cast in partial equilibrium and so does not consider the valuation side of the 'location as an asset' view (like Kennan and Walker 2011, or Diamond 2016) or static and based on a simple spatial equilibrium condition that does not include the investment aspect of location decisions (like Desmet and Rossi-Hansberg 2013, Allen and Arkolakis 2014, or Redding 2016). Giannone (2017) and Desmet et al. (2018) do provide dynamic general equilibrium setups with costly migration, but migration decisions only provide static gains or losses. In Caliendo et al. (2019) and Bilal (2020), agents solve forward looking problems in deciding their location but they simply consume their income and so do not solve a consumption-savings decision or accumulate wealth.

The view of investment as an asset was hinted at initially by Sjaastad (1962). Lucas (2004), Morten (2019) and Cavalcanti Ferreira et al. (2018) also present evidence and arguments to view migration as a stepping-stone or a form of self-insurance.[11] Some of the most detailed studies of mobility for low income, and likely constrained individuals, are consistent with the 'location as an asset' view. For example, in the "Moving to Opportunity" randomized experiment, conditioning aid on upgrading location reduced the use of housing vouchers by about a third (21 percentage points). Furthermore, while the literature using this experiment initially found that economic outcomes were not affected by an upgrade in location (Duncan et al. 2013), the most recent studies have found strong evidence that the outcomes for children that moved when young are positive (Chetty et al. 2016, and Davis et al. 2015), consistent with our emphasis on the investment dimension of location decisions rather than on the current benefits. Using tax records, Chetty and Hendren (2018) found a trade-off between child future earnings and rents. They estimate that a 1% increase in a

[9] Qualitatively, this channel provides one explanation for the differential returns to mobility that we document. Quantitatively, scarring effects from unemployment can account for about half of the differential returns to mobility.

[10] Kaplan and Schulhofer-Wohl (2017) link the decline in U.S. mobility to falling wage differentials within occupations.

[11] Fernandez and Rogerson (1998) and Fogli and Guerrieri (2018) discuss the trade-off between location and children education.

child's future earnings can be achieved by moving to a location with a median rent that is \$176 higher. The 'location as an asset' view argues that constrained agents might not want to take what seems like a good bargain, since they are constrained and want to borrow not invest further.

The rest of the paper is organized as follows. The next section, Section 2, introduces the simplest model necessary to make precise our notion of location decisions as investment decisions. It also contrast the mobility implications of the 'location as an asset' view and those of alternative theories. This simple two period model is then extended to an infinite horizon model in Section 3. In that section we present examples of the implied dynamic consumption, asset, and location paths of individuals. Section 4 presents our empirical analysis using the French individual level panel to show that agent's location decision respond to income shocks as our theory predicts. Section 5 concludes. An Appendix includes the technical proofs, additional robustness tests, and detailed data descriptions (Bilal and Rossi-Hansberg 2021).

2 A Simple Model

We aim to provide the simplest setup in which our 'location as an asset' view can be made precise. Because we need location to be an investment, we need a model with at least two periods. The economy consists of a unit mass of individuals that differ in their skill, $s \in [\underline{s}, \bar{s}]$, and their income in period 0 and 1, $\{y_t\}_{t=0}^{1} \in [\underline{y}_t, \overline{y}_t]$. The income of the individual in period 0 includes her labor income plus any wealth she is initially endowed with. Thus, an individual is characterized by a triplet (y_0, y_1, s). We denote the joint probability density function over these outcomes by f and the cumulative distribution by F. We require that f has compact support, is bounded and $\underline{s} > 0$.

There is a continuum of locations or 'cities'. We denote locations by an index $z \in [\underline{z}, \overline{z}]$ with $\underline{z} \geq 0$. The density of cities with characteristic z is given by h with cumulative density H. The skill of an individual determines the benefits from locating in cities. We assume that the returns for an individual of skill s to living in city z are given by zs. The supermodularity of this function will lead to positive assortative matching conditional on other individual characteristics, as we describe below.

Agents can move freely across locations. Hence, the population density, $L(z)$, of individuals living in cities of type z, as well as land rents, $q(z)$, in those cities are determined endogenously. We assume that the cost of supplying housing increases with population size due to some form of decreasing returns. Hence, $q(z) = Q(L(z))$ for $z \in [\underline{z}, \overline{z}]$, where $Q(0) = 0$ and Q is strictly increasing. That is, housing is free in locations without population and rents are strictly increasing in city size.

Individuals have access to a risk free bond with gross interest $R > 1$. We assume that this world interest rate is exogenous and determined in world markets.[12] Agents are subject to a standard borrowing constraint that limits their asset holdings between period 0 and 1, a, to be above some level $\underline{a}$. Hence, if, for example $\underline{a} = 0$, agents can only save but not borrow with the financial asset.

2.1 Asset and Location Choices

Households maximize lifetime utility with a discount factor given by $\beta \leq 1$. For simplicity we specify the period utility function as $u(c) = \log c$ but virtually all our results go through for any concave utility function that satisfies Inada conditions. The problem of a household is then to choose consumption in each period, purchases of the risk free bond, and location in period 1, to solve

$$\begin{aligned} & V(y_0, y_1, s) = \max_{c_0, c_1, a, z} \log c_0 + \beta \log c_1 \qquad (1) \\ s.t. \quad & c_0 + a + q(z) = y_0, \\ & c_1 = zs + y_1 + Ra, \\ & a \geq \underline{a}. \end{aligned}$$

That is, individuals maximize utility subject to budget constraints each period, as well as the borrowing constraint. In period zero, an agent's income includes anything he earns today and all of his wealth. Note that we have abstracted from any returns from the complementarity between an agent's skill and the city where she starts (say, $z_0 s$). We think of this term as also being embedded in y_0. Not explicitly recognizing this term explicitly avoids carrying z_0 as a state variable in the consumer problem. This is without loss of generality given that free mobility implies that current location only affects an agent's decisions through current income.

Note also that we make the agent pay rent one period in advance. So land rent for their chosen z location, $q(z)$, enters the left-hand-side of the period 0 budget constraint only. Rent paid for living in location z_0 in period 0 is not modeled and would simply be included in the resulting period 0 income. Making household pay rent one period in advance underscores the investment nature of the location choice. Namely, it recognizes that the good jobs,

[12] Technically, we only need $R > 0$, which we can allow without loss of generality. In addition, it would be simple to endogenize the interest rate R through an asset market clearing condition without changing any of our core results.

amenities, or education associated with living in a good location are enjoyed over time and not necessarily immediately after arriving there.[13]

The problem above abstracts from a flexible housing demand choice since it makes anyone living in location z pay the same cost $q\left(z\right)$. We decided not to incorporate this margin explicitly because, absent adjustment costs, the housing demand choice is a static choice that does not eliminate or prevent the use of the location asset (although it can affect its return). The problem in (1) also abstracts from income risk.[14]

The first-order conditions of the problem in (1) imply the standard 'Financial Euler equation'

$$\frac{c_1^*(y_0,y_1,s)}{\beta c_0^*(y_0,y_1,s)} \geq R \text{ for all } (y_0,y_1,s), \tag{2}$$

with equality if and only if the borrowing constraint is not binding, namely $a^*(y_0,y_1,s) > \underline{a}$. We denote all individual optimal choices with an asterisk (*).

Absent borrowing constraints, the desired asset holdings of an individual (y_0,y_1,s), denoted by $\tilde{a}(y_0,y_1,s)$, are given by their income net of rents in period zero $(y_0 - q(z))$ minus permanent consumption, which is given by $\left(y_0 + \frac{y_1+z^*s}{R} - q(z^*)\right)/\left(1+\beta\right)$.[15] Namely,

$$\tilde{a}(y_0,y_1,s) = y_0 - q(z^*(y_0,y_1,s)) - \frac{y_0 + \frac{y_1+z^*(y_0,y_1,s)s}{R} - q(z^*(y_0,y_1,s))}{1+\beta}.$$

Thus, actual savings in the financial asset are given by $a^*(y_0,y_1,s) = \max\left\{\tilde{a}(y_0,y_1,s)\ ,\ \underline{a}\right\}$.

Free mobility implies that individuals are never constrained in the 'location asset'. Hence, for all (y_0,y_1,s), the location decision yields a 'Mobility Euler equation' given by

$$\frac{c_1^*(y_0,y_1,s)}{\beta c_0^*(y_0,y_1,s)} = \frac{s}{q'(z^*(y_0,y_1,s))}. \tag{3}$$

Hence agents can optimize their intertemporal consumption path by choosing their holdings of financial assets and what we have dubbed the 'location asset'. To make the analogy

[13] Note that, although we do not incorporate other current urban costs as commuting, congestion, or crime, or urban benefits as amenities, all of them can be thought of as included in the net current price of location $q\left(z\right)$.

[14] In Online Appendix C.2 we develop an extension where agents can choose the size of their house and pay a prize $q\left(z\right) + p\ell$ for renting ℓ units of housing in location z. In Section 3 we write a multi-period extension with uncertainty about the realization of the income process. However, in this simple model without uncertainty, the location asset is used to transfer consumption across time, but not across states of nature or for precautionary purposes. Of course, in a richer environment the location asset could also be used for these alternative purposes.

[15] Whenever it is clear by the context we abbreviate optimal choices and do not write the dependence on the agent's type. Namely, we might write z^* instead of $z^*\left(y_0,y_1,s\right)$.

with a standard asset more precise, we can propose two interpretations. First, one in which each location z constitutes an asset, and agents moving to location z buy the asset, and the ones moving out sell it. How much of it they buy is limited by their housing demand and labor supply. Here, for simplicity, we have limited labor supply and housing demand to be equal to one. The return of the asset depends on the skill of the individual, s, and is given by the right-hand-side of equation (3), namely, $s/q'(z^*)$.

An alternative interpretation is to consider only a single asset with unit cost. The quantity purchased of the asset is equal to the housing costs, q, and returns of the asset depend both on the quantity purchased and the skill of the individual. Again, those returns are given by $s/q'(z^*)$. Under both these interpretations, the individual's problem (1) can be seen as a standard portfolio choice problem in which the risk-free bond is subject to a borrowing constraint, and the return to the 'location asset' is endogenously nonlinear and specific the individual's skill.

We are ready to define a competitive equilibrium in our economy.

Definition 1 *Given a distribution F of triplets $(y_0, y_1, s) \in [\underline{y}_0, \overline{y}_0] \times [\underline{y}_1, \overline{y}_1] \times [\underline{s}, \overline{s}]$ and an interest rate R, an equilibrium is a set of individual decision functions $c_0^*, c_1^*, a^* : [\underline{y}_0, \overline{y}_0] \times [\underline{y}_1, \overline{y}_1] \times [\underline{s}, \overline{s}] \to \mathbb{R}_+$ and $z^* : [\underline{y}_0, \overline{y}_0] \times [\underline{y}_1, \overline{y}_1] \times [\underline{s}, \overline{s}] \to [\underline{z}, \overline{z}]$, and rent and population functions $q, L : [\underline{z}, \overline{z}] \to \mathbb{R}_+$ such that*

- *individuals solve the problem in (1) and*
- *land rents are such that $q(z) = Q(L(z))$ for $z \in [\underline{z}, \overline{z}]$ where, if $\mathbf{1}$ denotes the indicator function, city population $L(z)$ satisfies, for all $z \in [\underline{z}, \overline{z}]$,*

$$\int_{\underline{z}}^{z} L(z) H(dz) = \int_{\underline{y}_0}^{\overline{y}_0} \int_{\underline{y}_1}^{\overline{y}_1} \int_{\underline{s}}^{\overline{s}} \mathbf{1}\left[z^*(y_0, y_1, s) \leq z\right] F(dy_0, dy_1, ds). \tag{4}$$

Condition (4) guarantees that the number of people in locations worse that z (the left-hand-side of the condition) is equal to the number of people that choose to live in those locations (the right-hand-side of the condition). Note that Condition (4) has to hold for all $z \in [\underline{z}, \overline{z}]$ and so it implicitly determines the population density function $L(z)$.

2.2 Equilibrium Allocation and House Rents

In order to understand agent's location choices, consider a city z in which an unconstrained individual (y_0, y_1, s) lives. Because $a^*(y_0, y_1, s) > \underline{a}$, equation (2) holds with equality and so the returns she faces on the financial and the location asset need to be equal. That is, $R = s/q'(z^*(y_0, y_1, s))$. This implies that unconstrained individuals sort into cities on the basis of their skill component s only. Then, if $q(\cdot)$ is a strictly increasing function (something we show below), there exists a matching function $\mathcal{Z}^U(s) = z^*(y_0, y_1, s)$ for unconstrained individuals, such that $R = sq'(\mathcal{Z}^U(s))$. Furthermore, when $q(\cdot)$ is convex (which we also show below), $\mathcal{Z}^U(s)$ is strictly increasing. Of course, whether individuals are constrained on the financial asset depends on their income path and skill, and the resulting location choice. For example, a flat income path with y_0 high relative to the values of future income, y_1, and skill, s, implies that the individual is not constrained.

Now consider an individual with the same y_1 and s but low enough $y_0' < y_0$ such that she is constrained. This individual has a larger marginal rate of substitution than the interest rate, so the Financial Euler equation (2) holds with strict inequality. Since the agent can still use the location asset, and so (3) holds, this implies that $s/q'(\mathcal{Z}^U(s)) = R < s/q'(\mathcal{Z}^C(y_0', y_1, s))$ where $\mathcal{Z}^C(y_0', y_1, s)$ is the constrained agent's location choice. Note that the constrained agent's location choice depends on all the individual characteristics, not just s. Hence, for $q(\cdot)$ strictly increasing, $\mathcal{Z}^U(s) > \mathcal{Z}^C(y_0', y_1, s)$. Constrained individuals locate in cities with lower land rents and lower returns to skill than unconstrained individuals with the same skills. The reason is that they use the location asset rather than the financial asset to adjust their intertemporal consumption path. More specifically, they borrow using the location asset to transfer resources to the present, something financial markets do not allow them to do.

$\mathcal{Z}^C(y_0, y_1, s)$ is increasing in y_0 and in fact will converge to $\mathcal{Z}^U(s)$ as we increase y_0. In contrast, it is decreasing in y_1, since larger future income results in larger need to borrow from the future and therefore more use of the location asset to do so. Finally, more skilled individuals locate in better cities, whether constrained or unconstrained, due to the skill complementary we introduce in individual earnings. Note that the reason the individual location choice is always uniquely determined is our setup is the supermodular income in z and s. In contrast, if agents had identical skills, they would be indifferent about where to locate when unconstrained, but their use of the location asset to transfer consumption to the present would still determine their location choice when constrained. We formalize this discussion in the following lemma that characterizes the location decision of agents.

Lemma 1 *There exists a pair of matching functions* $\mathcal{Z}^U(\underset{+}{s})$ *and* $\mathcal{Z}^C(\underset{+}{y_0}, \underset{-}{y_1}, \underset{+}{s})$ *such that individual* (y_0, y_1, s) *chooses city*

(i) $z^*(y_0, y_1, s) = \mathcal{Z}^U(s)$ *if* $y_0 \geq Y_0(y_1, s)$, *so she is unconstrained, and*

(ii) $z^*(y_0, y_1, s) = \mathcal{Z}^C(y_0, y_1, s) < \mathcal{Z}^U(s)$ *if* $y_0 < Y_0(y_1, s)$*, so she is constrained,*

where $\underset{+\ +}{Y_0(y_1, s)} = \{y_0 \,|a^*(y_0, y_1, s) > \underline{a}\}$ *and* $\mathcal{Z}^U$ *and* $\mathcal{Z}^C$ *are determined by a system of ordinary differential equations described in Appendix A.1.*

Proof. See Appendix A.1. ■

Lemma 1 characterizes the threshold for current income y_0 that determines whether an individual is constrained using the function $Y_0(y_1, s)$. Because the rent function is increasing in z as we show below, and since $\mathcal{Z}^U(s)$ is increasing in s, this threshold is increasing in both arguments. More future income makes unconstrained individuals want to consume more in the present and therefore makes the constraint on borrowing more binding. Similarly, more skilled individuals will earn more in the future and will live in more expensive cities, making the constraint more binding.

Of course, given the monotonicity of $\mathcal{Z}^U(s)$ and $\mathcal{Z}^C(y_0, y_1, s)$ in s, we can define the inverse as $\mathcal{S}^U(z) = \mathcal{Z}^{U-1}(z)$ and $\mathcal{S}^C(y_0, y_1, z) = \mathcal{Z}^{C-1}(y_0, y_1, z)$. These functions then tell us the skill of the set of constrained and unconstrained individuals that live in a given city z. In equilibrium, unconstrained individuals always locate in better cities than constrained ones, hence there exists a threshold $\hat{z}$ such that for $z < \hat{z}$ all individuals in the city are constrained and above that we have a mixed of constrained and unconstrained individuals. The best city, $\bar{z}$, is an exception and has no constrained agents. The following corollary states these results formally.

Corollary 2 *There exists a threshold* $\hat{z}$ *such that individuals in city* $z \geq \hat{z}$ *are either*

(i) unconstrained with skill $s = \mathcal{S}^U(\underset{+}{z})$ *and* $y_0 \geq Y_0(y_1, \mathcal{S}^U(z))$*, or*

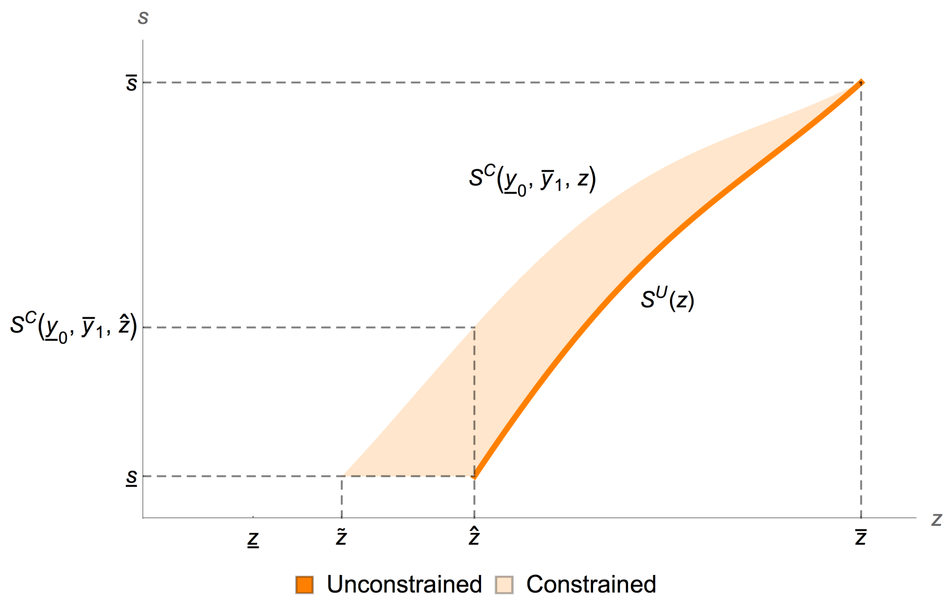
(ii) constrained with skill $s = \mathcal{S}^C(\underset{-}{y_0}, \underset{+}{y_1}, \underset{+}{z}) = \frac{\mathcal{S}^U(z)(y_1 + R\underline{a})}{\beta R(y_0 - \underline{a} - q(z)) - z\mathcal{S}^U(z)} > \mathcal{S}^U(z)$ *and* $Y_0(y_1, \mathcal{S}^U(z)) > y_0$.

In cities $z < \hat{z}$*, all individuals are constrained, and* $\mathcal{S}^C(y_0, y_1, z) = \frac{q'(z)(y_1 + R\underline{a})}{\beta(y_0 - \underline{a} - q(z)) - zq'(z)}$.

Proof. Direct corollary of Lemma 1. ■

Figure 1 represents these results graphically. We have discussed all the elements in the figure except for $\tilde{z}$ that represents the lowest city that has non-negative housing rents. Namely, $\tilde{z}$ is implicitly defined by $q(\tilde{z}) = 0$. If $q(z)$ is strictly increasing in z, any city with $z < \tilde{z}$ is not feasible. Note that the upper bound of the correspondence of skills that live in the city is given by $\mathcal{S}^C(y_0, y_1, z)$ evaluated at the lowest current income (denoted by $\underline{y_0}$) and highest future income ($\overline{y_1}$). Namely, the most constrained individual in the city, which is the highest skilled individual using the location asset the most. Note that below $\hat{z}$ the city has only constrained individuals, and only the lowest skilled individuals locate in the worst city $\tilde{z}$ (as long as $\underline{z}$ is low enough).

Figure 1: Allocation of skills to cities

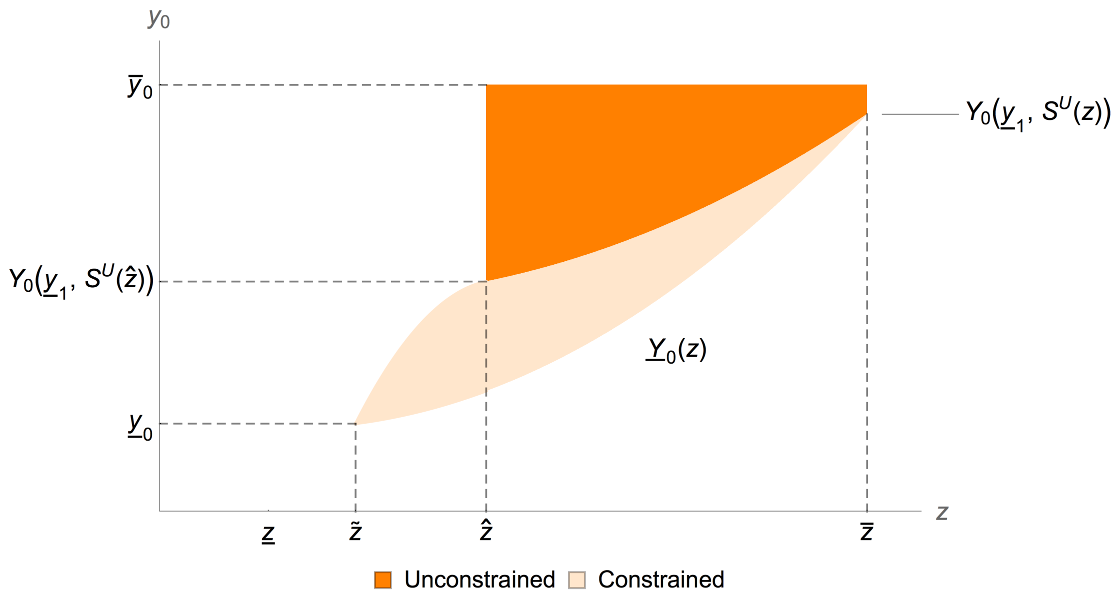


We can also represent graphically the set of current income levels, y_0, of individuals that locate in a given city. Of course, current income and initial wealth are indistinguishable in our two-period setup. We do so in Figure 2. In city z, all individuals with incomes $y_0 \geq Y_0(y_1, \mathcal{S}^U(z))$ are unconstrained and locate according to their skill level only. Other individuals that locate in those cities are constrained and have low income, and either high skills, high future income or both. Because lower current income leads individuals to choose worse cities, it must be that the lowest income present in a given city z is the income of the individual with the highest incentives to save in the location asset. Namely, the highest skill agent with the lowest future income present in the city. This lower bound, denoted by $\underline{Y}_0(z)$ in the figure, can be found by evaluating the expression for $\mathcal{S}^C$ in Corollary 2 at $\bar{s}$ and the lowest future income $\underline{y}_1$.

We finish the discussion of an equilibrium in our simple two-period economy with a characterization of the house rent schedule. As we alluded already above, land rents are increasing in z since higher z cities yield higher income for individuals of all skills. Furthermore, the complementarity between z and s implies that the highest skilled unconstrained individuals locate there, which implies that rents grow more than proportionally with city type, as does the income of its unconstrained residents. Hence, rents are convex.

In cities with unconstrained individuals the slope of the rent function is given by the

Figure 2: Allocation of income groups to cities



$\mathcal{S}^U(z)/R$. Namely, the slope of the rent function is determined by the skill of unconstrained individuals in the city and is inversely proportional to the interest rate. Thus, a low interest rate implies that the house rent schedule is steeper. Since the return of the location asset for an unconstrained individual with skill s is $s/q'\left(\mathcal{Z}^U(s)\right)$, this naturally also implies a lower return of the competing location asset by no-arbitrage. That is, lower returns in the financial market result in steeper rents that reduce the return of the location asset. Furthermore, a lower interest rate R implies that more agents wish to borrow and hence are constrained. This implies more downgrading and segregation. So the model predicts that periods of low interest rates should be periods of increasing rent differentials across cities and more segregation, reminiscent of the pre- and post-2008 crisis housing markets around the world. We formalize these results in the following lemma.

Lemma 3 *The equilibrium house rent function $q(z)$ is increasing and convex. For, $z \geq \hat{z}$, $q'(z) = \mathcal{S}^U(z)/R$ and $\frac{\partial q'(z)}{\partial R} < 0$ if $\overline{s} - \underline{s}$ is sufficiently small.*

Proof. See Appendix A.2. ■

Note that our results on the use of the location asset do not require either skill heterogeneity or the assumed supermodularity between s and z. Consider the case in which there is only one skill, say s_0, and so everyone obtains the same benefit from living in a given city. In this case, there is complete spatial segregation between constrained and unconstrained individuals. Namely, there is a threshold location $\hat{z}$, so that all unconstrained individuals are indifferent between locations $z \geq \hat{z}$, and constrained individuals locate in places $z < \hat{z}$, where the return to the location asset is higher than the interest rate.

2.3 Optimal Allocation

The equilibrium allocation of the model described above is inefficient due to the presence of borrowing constraints. The inefficiency is reflected in the use of the location asset by constrained individuals. Their use of the location asset ameliorates the effect of the financial constraint. However, because it reduces total output in the second period by driving agents to locations where they earn less, the resulting allocation is still inefficient relative to an economy without financial constraints.

We focus on efficient allocations that maximize discounted second period output net of housing costs.[16] Given the assumed supermodularity between s and z, we show in Lemma 4 that the efficient allocation involves a one-to-one increasing matching function. Hence, in contrast to the equilibrium allocation, only one type of agent locates in a given city.

Lemma 4 *In an efficient allocation without credit constraints*

(i) individual locations are determined by an increasing matching function $\mathcal{Z}^{SP}(s)$*,*

(ii) and the decentralized allocation yield strictly less present value of output and less present value of output net of housing costs.

Proof. See Appendix A.3. ■

2.4 Placed-Based Policies

The equilibrium described above determines the distribution of population across cities, $L(z)$, for all $z \in [\underline{z}, \overline{z}]$ with $L(z) > 0$ for $z \in [\tilde{z}, \overline{z}]$. In the equilibrium allocation, agents with low values of s that are constrained decide to locate in the lower range of cities because they use the location asset to borrow. We now want to consider the effect that place based-policies have on welfare for the different types of agents. Place-based policies aim to improve the characteristics of some of the worse locations in the economy. This is naturally costly, and implies taxing other locations. Therefore, as a stylized representation of these policies, consider policies that shrink the range of characteristics of equilibrium cities $[\tilde{z}, \overline{z}]$ to a singleton $\{z_0\}$, keeping the mass of cities constant. We choose z_0 to guarantee that the average income that individuals derive from cities stays unchanged, namely, $E[sz] = z_0 E[s]$.[17] Thus, this

[16] The optimal allocation of consumption in both periods across individuals of different types is increasing in discounted second period output net of housing cost but depends on a postulated social welfare function.

[17] The expectation on the left-hand-side is taken with respect to the equilibrium allocation in space in the competitive equilibrium with different cities, before the policy change. The expectation on the right-hand-side involves only the exogenous marginal distribution of skill.

policy captures the essential elements of place-based policies if they are implemented without generating any aggregate loss of resources. Note also that positive sorting between skills and city types implies that $z_0 = E[zs/E[s]] > E[z]$. That is, the targeted city type is better than the average.

To explain our general result below it is useful to start with an example where $\underline{s} = 0$. Namely, the lowest skilled individuals in the economy have zero skill and, therefore, derive no benefits from living in better cities. These individuals in equilibrium locate in the worst cities in the economy, $\tilde{z}$, and pay zero rent $q(\tilde{z}) = 0$. Naturally, such individuals will be worse off if we implement our place-based policy. In the equilibrium with place-based policies rents are positive and identical in all cities, but for the lowest skilled individuals the benefits of locating in the improved cities are still zero. By continuity, there is a range of individuals with $s > 0$ that are also worse off after the policy. In other words, the policy prevents them from borrowing with the location asset. Something they would like to do. As long as $\underline{s} = 0$, the logic above applies for any policy that reduces the range of cities at the bottom of the distribution.

The logic described above for the case of $\underline{s} = 0$ can be extended to a more general setting with $\underline{s} > 0$, when $Q(L(z)) = L^{\eta}$ with $\eta < 1$.[18] In this case we can characterize the set of individuals that lose using the matching functions. The individuals that are guaranteed to lose are the ones between the lowest skill, and the skill of the individuals that locate, in the original equilibrium, in the average city. The reason is, again, that up to that point the convexity of housing prices implies that the increase in rents associated with the policy does not compensate the future gain in income for these agents. That is, these agents get low returns for the location asset, so they like to use it to borrow, not to save. This is particularly true for constrained individuals, so the set of skills of constrained individuals that lose is larger than the set of unconstrained individuals that lose from the policy. The next lemma presents the formal result.

Lemma 5 *Suppose house rents are concave in population, i.e.* $Q(L(z)) = L^{\eta}$ *with* $\eta < 1$. *Then a place-based policy that makes all cities have characteristic* z_0 *makes*

(i) all unconstrained agents with $s \in \left[\underline{s}, S^U\left(E[z]\right)\right]$ *worse off and*

(ii) all constrained agents (y_0, y_1, s) *with* $s \in \left[\underline{s}, S^C\left(y_0, y_1, E[z]\right)\right]$ *worse off.*

[18] This is a natural assumption that holds, for example, in the standard circular monocentric city model with a central business district and commuting (as in Desmet and Rossi-Hansberg, 2013). In that case, $\eta = 1/2$.

Since $S^C(y_0, y_1, E[z]) > S^U(E[z])$, the set of skills of constrained individuals that are worse off is larger.

Proof. See Appendix A.4. ■

2.5 The Location Effect of Front and Back-Loaded Shocks

The results above can also be used to describe how agents react to shocks of different types. We are particularly interested in income shocks that affect the relative slope of an individual's income path. Namely, shocks that affect income today, y_0, relative to income tomorrow, $y_1 + sz$. These shocks will induce agents to adjust their savings using the financial and location assets. In Section 4 we study how workers in France reallocate across regions as a result of such an income shock. These shocks are front-loaded since, by design, they reduce income today but not necessarily income tomorrow.

Consider an individual (y_0, y_1, s) that experiences an idiosyncratic negative front-loaded shock that decreases y_0 to $y_0' < y_0$ but increases y_1 to $y_1' \geq y_1$. Because the shock is idiosyncratic, it does not affect the equilibrium matching function or rent schedule. The results in Lemma 1 imply that agents that are constrained will use the location asset more and will downgrade their location, since $\mathcal{Z}^C(y_0', y_1', s) < \mathcal{Z}^C(y_0, y_1, s)$. Unconstrained individuals that become constrained due to the shock also downgrade their location, since $\mathcal{Z}^C(y_0', y_1', s) < \mathcal{Z}^U(s)$. In contrast, unconstrained individuals that remain unconstrained (individuals such that $y_0 > y_0' > Y_0(y_1', s) \geq Y_0(y_1, s)$) stay where they are, since $\mathcal{Z}^U(s)$ is independent of the income path. Hence, constrained individuals, or those that become constrained, borrow more using the location asset, while unconstrained individuals use the financial asset to transfer consumption to the present. Of course, since what matters for the argument is the slope of the income path, a positive back-loaded shock has a similar effect on location choices and the use of the location asset.[19] We contrast these exact implications with French data in Section 4.

Note that permanent adverse (or positive) shocks can also imply a change in the slope of the income profile. For example a permanent shock that changes both y_0 and y_1 induces borrowing if $y_0' - y_1'/\beta R < y_0 - y_1/\beta R$. Such a shock then generates the same qualitative effects on the use of the location asset as a front-loaded negative shock. In contrast, if $y_0' - y_1'/\beta R > y_0 - y_1/\beta R$, the shock induces extra savings and so has a similar qualitative effect than a back-loaded negative shock. Of course, if $y_0' - y_1'/\beta R = y_0 - y_1/\beta R$, location decisions remain unchanged.

[19] A positive front-loaded shock or a negative back-loaded shock have exactly the reverse effect.

As a last possibility consider an individual that acquires more skill, namely, an increase in s. Because s increases income in the future, some of the implications of the increase in s are similar to those of a back-loaded positive shock. On top of this, an increase in s increases the return of the location asset relative to the financial asset which implies that agents want to save more using the location asset. Hence, they want to upgrade their city. Lemma 1 tells us that the the second effect always dominates, given that both $\mathcal{Z}^C(y_0, y_1, s)$ and $\mathcal{Z}^U(s)$ are increasing in s.

2.6 Amenities

The model we have proposed so far emphasizes the investment dimension of location choices. Agents choose a better location to obtain the future benefits zs. Of course, part of the location choice might also be related to the quality of living, or amenities, particular locations offer. We show here that a view of location choices centered on amenities has distinct implication to the 'location as an asset' view. These implications cannot rationalize the empirical findings we present in Section 4. We start this section by characterizing analytically location choices following a negative income shock when we add to our model amenities that are ranked identically by all individuals. We then discuss the effect of income shocks on mobility in models with common or idiosyncratic amenities for locations but without borrowing constraints.

Our first claim is that the presence of amenities that all individuals value identically implies that unconstrained, wealthy individuals should downgrade *more* than constrained, low-wealth individuals after a comparable income shock, conditional on both individuals being in the same location to begin with. The reason is straightforward: a low-wealth constrained individual locates in the same place as a high-wealth unconstrained individual only if the constrained individual has a higher return to location s. As a result, the low-wealth, constrained, individual is *less location-elastic* to income shocks than the wealthy individual. Thus, the presence of amenities yields the *opposite* relative location implications from an income shock than the 'location as an asset' view (explained in Section 2.5).

To make this intuition precise, we extend our basic model to incorporate amenities. Suppose now that individuals solve

$$\begin{aligned} & V(y_0, y_1, s) = \max_{c_0, c_1, a, z} \log c_0 + \beta \log c_1 + Az \\ s.t.\quad & c_0 + a + q(z) = y_0, \\ & c_1 = zs + y_1 + Ra, \\ & a \geq \underline{a}. \end{aligned}$$

This specification enriches our baseline model with an extra utility term Az, which implies that locations with better income prospects z also provide more residential amenities.[20] The parameter A governs the relative value of amenities to non-durable consumption.

The only, but critical, change to the individual's optimality conditions is that now their 'Mobility Euler equation' is given by

$$\frac{c_1^*(y_0, y_1, s)}{\beta c_0^*(y_0, y_1, s)} \left[1 - \frac{A/q'(z^*(y_0, y_1, s))}{1/c_0^*(y_0, y_1, s)}\right] = \frac{s}{q'(z^*(y_0, y_1, s))}.$$

If agents enjoy the amenities conveyed by the city ($A > 0$), going to a city with a larger z, gives an extra utility benefit of $A/q'(z^*(y_0, y_1, s))$ per unit of extra rent. Dividing by the marginal utility of current consumption ($1/c_0^*(y_0, y_1, s)$) determines the price discount in terms of current rent implied by amenities. For every marginal dollar paid in rent, agents effectively face only a fraction of the cost, since they obtain the additional benefits from amenities. Hence, the term in brackets, which is equal to 1 when $A = 0$ and is smaller than 1 when $A > 0$, multiplied by the marginal rate of substitution between non-durable consumption today and tomorrow must now equal the future monetary return to the location asset. Conditional on a rent function $q(\cdot)$, amenities result in a lower effective price for the location asset, and since future benefits are not affected, a larger rate of return.[21]

To formalize our theory's predictions when locations also provide residential amenities, we consider two individuals P and R with initial income $y_0^P < y_0^R$, and $y_1^P = \tau y_0^P$, $y_1^R = \tau y_0^R$. Individual P has lower initial income and/or wealth than individual R, but both individuals expect the same income growth over time. We are interested in the change in the location decision of individual $j \in \{P, R\}$ after a proportional income shock $y_0'^j = \nu y_0^j$, where $\nu \leq 1$, conditional on both individuals choosing to locate in z in the absence of the shock. Formally, we consider the derivative with respect to ν, evaluated at $\nu = 1$, $D^j(z, A) = y_0 \frac{\partial z^*}{\partial y_0}(y_0^j, y_1^j, \mathcal{S}(y_0^j, y_1^j, z))$, where $\mathcal{S}$ is equal to either $\mathcal{S}^C$ or $\mathcal{S}^U$ depending on the constrained status of individuals.[22] The following lemma states our results.

Lemma 6 *If $A > 0$, and given a rent function $q(z)$,*

(i) in the absence of credit constraints ($\underline{a} = -\infty$), $D^R(z, A) - D^P(z, A) > D^R(z, 0) - D^P(z, 0) = 0$;

[20] Our specification assumes a perfect correlation between income prospects and amenities to make the amenity channel as stark as possible.

[21] In equilibrium, the rent function $q(\cdot)$ adjust since rents are increasing in population density.

[22] We omit the dependence of individuals' optimal choices on A for notational brevity.

(ii) in the presence of credit constraints that bind for P but not for R, when A is not too large, $D^R(z, A) - D^P(z, A) < 0$ is increasing in A.

Proof. See Online Appendix C.1. ■

The first result in Lemma 6 states that, when individuals are not constrained, the presence of amenities makes wealthy individuals more location-elastic to income shocks than low-wealth individuals. Therefore, the basic amenities channel works against the 'location as an asset' channel. The second result reveals that this conclusion carries through to a mixed model with credit constraints and amenities. As an individuals' amenity valuation increases from 0, the difference between a constrained individuals' location response and unconstrained individuals' location response diminishes.[23] Lemma 6 is particularly useful to interpret the empirical results in Section 4. There, we use controls for local amenities when we estimate the differential effect of income shocks on individuals' location. Of course, one can always argue that individuals value amenities in ways that we are not able to fully control for. Reassuringly, however, Lemma 6 implies that any estimated differential effect of income shocks on individuals' location will have to be a *lower bound* on the true effect of the 'location as an asset' mechanism.

Of course, the result in Lemma 6 that observing two individuals with different wealth in the same location implies that the wealthy one is more location-elastic can be relaxed by introducing idiosyncratic preferences for locations with high enough variance, as is popular in the literature. In Online Appendix C.4 we add them to a version of our model without borrowing constraints and no dynamic location payoff. This additional layer of heterogeneity requires us to solve the model numerically. We show there that idiosyncratic preference shocks can rationalize the relative downgrading of wealth-poor individuals relative to wealthy individuals when idiosyncratic preferences exhibit enough variance. However, as we also show in Online Appendix C.4, such a model has two additional core empirical implications that are distinct from those of the 'location as an asset' view. First, we should observe a relative downgrading of poorer individuals vis-à-vis richer individuals at all points of the wealth distribution. Instead, the 'location as an asset' mechanism implies that this relative downgrading should only happen for credit-constrained individuals at the very bottom of the wealth distribution. Second, wealthy individuals should also downgrade their location in response to a negative income shock relative to wealthy individuals who do not receive a shock. Instead, the 'location as an asset' mechanism implies that wealthy individuals should remain at their preferred location regardless of receiving a shock. Our empirical results in Section 4 are consistent with the 'location as an asset' view along both implications, and

[23] In Appendix C, we show that these conclusions continue to hold for non-proportional income shocks.

falsify the amenity-based hypothesis.

In order to illustrate further the exact predictions of the 'location as an asset' view that we will bring to the data, we now present a fully dynamic quantitative model with asset accumulation, location decisions, and credit constraints.

3 The Infinite Horizon Model

In this section we extend our model to an infinite horizon economy. The key differences with the model presented in the previous section is that now agents live forever and receive an idiosyncratic income stream y_t. Depending on their skill, location, asset holdings, and income, they make consumption and savings decisions. To do so they use the financial market subject to a borrowing constraint and the location asset by choosing where to live. As before, cities differ in their return to skill and their rent. Also as before, one can view individuals as solving a two-asset portfolio choice problem subject to a borrowing constraint on the risk-free bond. In contrast to the two period model, the infinite horizon version determines the invariant distribution of wealth in the population and therefore the wealth composition of cities as well.

3.1 Model Setup

In any period t, infinitely-lived individuals receive an idiosyncratic income shock y_t, which follows a first-order Markov chain with states $y_1, ..., y_N$ and a given transition matrix, Λ. Throughout we assume that individuals have a permanent skill s.[24] In period t, an individual in location z_t with an asset level a_t, chooses how much to consume, c_t, how much to save, a_{t+1}, in a one period risk-free bond with interest rate R, and where to live next period, z_{t+1}. Agents can move freely across locations. Their income in period t is $y_t + sz_t$. To go to location z_{t+1}, they need to pay the rent $q(z_{t+1})$ one period in advance, i.e. in period t. Finally, we assume that the risk-free bond is subject to a standard credit constraint $a_{t+1} \geq \underline{a}$.

Given an increasing and concave flow utility function u satisfying Inada conditions, and a discount factor $\beta < 1$, individuals maximize

$$V(a_t, z_t, y_t, s) = \max_{\{a_{t+1}, z_{t+1}\}_{t=0}^{\infty}} E_0 \left[\sum_{t=0}^{\infty} \beta^t u(c_t) \right]$$

[24] It is feasible to relax this restriction and introduce idiosyncratic skill shocks, although at some computational cost.

$$
\begin{aligned}
s.t.\quad & c_t + a_{t+1} + q(z_{t+1}) = y_t + s z_t + R a_t, \\
& a_{t+1} \geq \underline{a}.
\end{aligned}
$$

As before, the financial and mobility Euler equations imply that

$$
\frac{s}{q'(z^*_{t+1}(a_t, z_t, y_t, s))} \geq R,
$$

with equality if and only if $a^*_{t+1}(a_t, z_t, y_t, s) > \underline{a}$. Note that, as before, for non-constrained individuals city choice $z^*_{t+1}(a_t, z_t, y_t, s)$ only depends on skill s.

Denote by F_t the joint distribution of the four-tuple (a_t, z_t, y_t, s) in period t. Then the distribution of people across cities, $L_t(z)$ is given by

$$
\int_{\underline{z}}^{z} L_t(z)\, H(dz) = \sum_{i=1}^{N} \int_{\underline{a}}^{\infty} \int_{\underline{z}}^{\bar{z}} \int_{\underline{s}}^{\overline{s}} \mathbf{1}\left[z^*(a, z, y_i, s) \leq z\right] F_t(da, dz, y_i, ds) \ \text{ for all } z \in [\underline{z}, \overline{z}]
$$

and rents are given by $q(z) = Q(L(z))$. This economy converges to a steady state where the distribution F_t is constant over time.

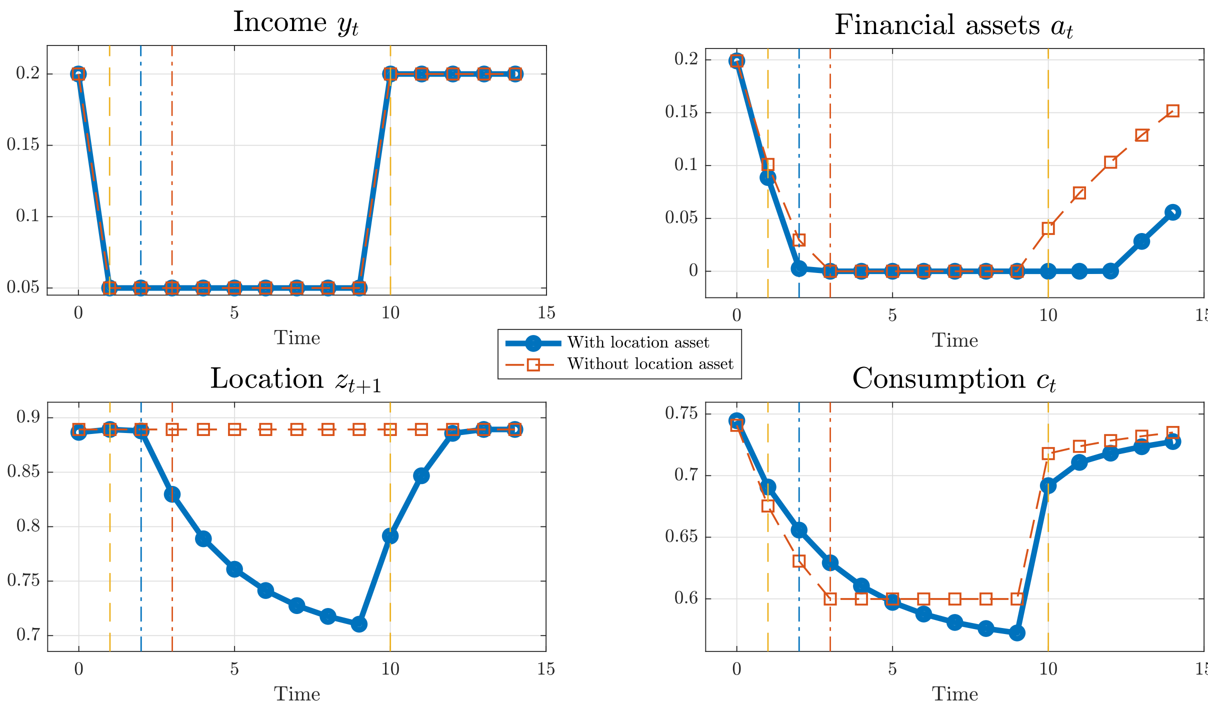
An equilibrium of the model above can be computed numerically. We do so for a CRRA utility function, for a uniform distribution of cities, and for a particular house rent schedule.[25] We choose reasonable parameters values that allow us to illustrate the main forces at work. The exact values, specifications, and solution method are described in Online Appendix D.

3.2 A Quantitative Illustration of the Use of the Location Asset

Figure 3 presents the results of a simulation of this model. We focus on the reaction of a particular individual to a transitory income shock. The figure presents five panels, each of them displaying a different variable. For comparison purposes we present the behavior of an individual that can move (solid dark lines), and therefore use the location asset, and the behavior of an individual that cannot move from her preferred location when unconstrained, $\mathcal{Z}^U(s_0)$ (dashed light lines). The difference between these two cases represents the way in which the location asset helps the individual deal with the transitory income shock. We plot the effects for a particular individual with a fixed skill level.

[25]In principle, specifying a given house rent schedule is without loss of generality, because we can find a skill distribution that would lead to this particular house rent schedule as an equilibrium outcome. Of course, endogenizing the house rent schedule is essential to perform aggregate counterfactual simulations. In the exercises below, we only consider counterfactuals that change the state of a particular individual and therefore do not affect the aggregate equilibrium allocation and prices.

Figure 3: Dynamic reaction to a temporary income shock



The top-left panel in Figure 3 simply plots the income shock over time. The agent can be in two income states: high, $y_H = 0.2$, and low, $y_L = 0.05$.[26] In period one, the agent transitions from the high to the low income level. It stays there until the ninth period when he transitions back to the high income. This income process is identical for both scenarios, with and without mobility.

The top-right plots the level of financial assets. We start the individual at assets that are equal to the transitory income level in the high state. The individual also receives an income proportional to her skill and the city where she lived, $z_t s_0$. This additional income represents most of the individual's income. The transitory path represents between 5 and 15% of the agent's total income. As a result of the shock, the agent consumes part of her financial assets and therefore the asset balance declines until it hits zero, which is the level of the financial constraint. That is, individuals cannot borrow at all in financial markets. This decline in financial assets happens a bit faster when individuals can use the location asset, since in that case they know that when they hit the financial constraint they will be able to smooth consumption by moving. In period 3, the agent that cannot move hits the borrowing constraint and stays there for several periods. The agent that can use the location asset hits the borrowing constraint one period earlier. When the income shock reverses in

[26] We could think of the low state as unemployment, and the high state as non-employment. Our calibration of the transition matrix Λ then implies a steady-state non-employment rate of about 14%, in line with employment rates for prime-age males in France.

period 10, without the location asset, the agent immediately starts saving and building a financial asset stock. In contrast, because at that point the location asset pays a higher return than the financial asset, the individual that can use the location asset, uses it to save. Such an individual stays stuck at the constraint for an extra two periods while it moves to better locations. Eventually, she reaches her desired location, the return she perceives on the location asset goes down, and she starts saving with the financial asset. Note that the presence of the location asset makes the individual stay longer at the financial constraint!

The bottom-right panel plots the location of the agent over time. The ideal unconstrained location of the agent is at city $\mathcal{Z}^U(s_0) = 0.88$. The agent that cannot move simply stays there throughout. The one that can move stays there until financial assets hit the financial constraint. Once she runs down financial assets to zero, she starts borrowing using the location asset. That is, she starts downgrading her location progressively. In this case the total downgrade is about 40%. This downgrading continues until the agent either reaches the minimum location she is willing to live in, or the shock reverses. In the plot, it continues until the 9th period, the last period the agent obtains the low income. After the shock reverses to the high income state, the agent starts upgrading her location progressively. The last period where she is financially constrained, she reaches her unconstrained preferred city and starts saving with the financial asset only.[27] Note that location downgrading happens smoothly over time since there is no fixed cost of migration. In the presence of fixed costs, if income shocks are large enough, agents would still use the location asset, although only sporadically.

Finally, the bottom-left panel in the figure shows the agent consumption path with and without mobility. As we have underscored, the use of the location asset allows the agent to smooth consumption better since it can borrow even when she is at the financial constraint. The result is a consumption path that declines more slowly and smoothly than without the location asset. Because borrowing with the location asset involves sacrificing future income, given that this is an unexpectedly long shock, the total fall in consumption is also eventually somewhat larger. Once the shock reverses, the path out of the consumption slump is also a bit smoother for agents that can use the location asset. Overall, these dynamic behavior patterns vary substantially with and without the location asset.

The ability to use the location asset results in expected welfare gains for the agent.[28] The

[27] Throughout the transition, the change in the city component of income due to the use of the location asset reaches 0.18, which is of the same order of magnitude than the high idiosyncratic income state. The share of housing expenditure in total income lies between 20% and 40%, similar to the data (Davis and Ortalo-Magne 2011)

[28] The gains from using the location asset for one particular path of realizations can be either negative or positive. For example, in Figure 3, the negative shock lasts for nine periods. This increases the set of periods where agents that use the location asset obtain less consumption. However, this particular path is

presence of some gains is obvious given that the location asset provides a way of relaxing the friction imposed by the financial constraint and the agent can always decide not to move. In our calibration the gains can be as large as 10% in flow consumption for low-income individuals close to the constraint that live in their preferred location. Welfare gains for low income individuals close to the constraint can be as large as 0.5%, but fade away quickly for individuals with more wealth (see Figure 11 in Online Appendix D.2).

The empirical exercises in Section 4 presents evidence on how individuals use the 'location asset' using an event study design that follows the location and asset holdings of agents that experience income shocks and start with different levels of wealth at the same location. Figure 3 shows that, in our theory when hit by a negative income shock, individuals first dis-save in their financial assets. Once they hit the credit constraint, they start dis-saving in the 'location asset' by downgrading their location. Of course, this pattern also manifests itself when comparing individuals who reside in a given location, but have different wealth levels. Our next exercise presents this event study in our modelled economy. The implied qualitative patterns are the implications that will be looking for in the parallel exercises in the data. We select individuals who satisfy the following criteria, as we will also do in the data. First, they must be in either the bottom quintile, or the top quintile of the invariant asset distribution in the economy. Second, they must reside in the same given location z_0, and currently be in the high income state.[29] We then hit all these individuals with a front-loaded income shock: they remain in the low state for two periods, before reverting to the high state.

Figure 4: Disposable income, asset, and location response to a front-loaded idiosyncratic income shock conditional on initial location, bottom and top quintiles.

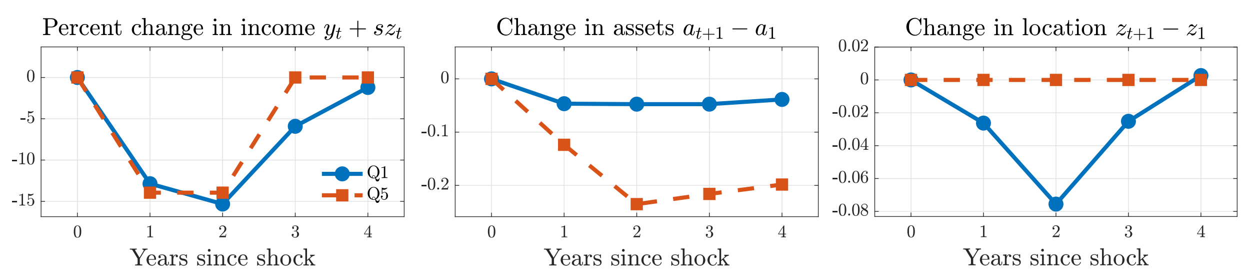


Figure 4 shows these individuals' asset and location over time and illustrates the main implications we will look for in the empirical exercise. First, conditional on residing in the same location, and following a front-loaded income shock, wealth-poor individuals downgrade their location but keep close to constant and negligible amounts of financial assets. Once the

relatively unlikely. Other paths with shorter duration of the negative transitory shock yield larger benefits from the use of the location asset, and are more likely. In expectation, there are gains since the agent has a larger, more flexible choice set.

[29]We use $z_0 = \mathcal{Z}^U(s_0)$ with $s_0 = 1$.

shock reverts, they upgrade their location progressively.[30] Wealth-rich individuals dis-save financial assets while the shock lasts and remain in the same location throughout.[31] We have also performed similar exercises where we conditioning on the type s, or on whether poor individuals are at the constraint, or simply close to it, and find very similar results (see Figure 12 in Online Appendix D.2). We now explore these implications of our view of location and savings choices using French data on individual income, asset, and location paths.

4 Location and Moving Choices in France

We have discussed in detail several implications of our view of location decisions as investing in a location asset. In particular, constrained individuals will downgrade their location as a result of a negative front-loaded income shock while their assets remain minimal and unchanged. In contrast, unconstrained individuals will not react to these shocks by moving, but instead by reducing their wealth. In this section we contrast both these predictions with individual level data.[32]

We use tax return data from a longitudinal panel representative of all households in the French economy from 2008 to 2015. We have both household and individual identifiers. Crucially for our analysis, we observe the households' annual financial asset income. It is broken down into various categories that include bank accounts, financial vehicles such as mutual funds and stocks, as well as housing (rent payments minus outstanding mortgage payments).[33]

We match this household tax income dataset to employer tax return data using individual identifiers. Using employer tax returns allows us to obtain precise information about individuals wage income at the employment year-spell level. Since the address reported in the household income tax data is often inaccurate, it is critical for us to use the residence

[30] Note that agents in the bottom quintile eventually move to a location that is slightly above the one where they started. The reason is that many of them were initially constrained and, therefore, were borrowing with the location asset.

[31] Of course, because we have assumed that there is no cost of mobility at all, in our model agents optimize their location every period. Small moving costs would make adjustments to the agents' location, and therefore borrowing and saving with the location asset, more infrequent (although still beneficial).

[32] Our theory also has implications on *who* decides to move at all. Decisions to move, however, are also directly impacted by fixed moving costs and municipality-level shocks that change job opportunities and prices in an agent's origin municipality. These confounding factors, however, do not affect the decision of *where* to move, and the implications on wealth dynamics for constrained and unconstrained individuals conditional on moving. Hence, we focus on these latter implications in our empirical exercise.

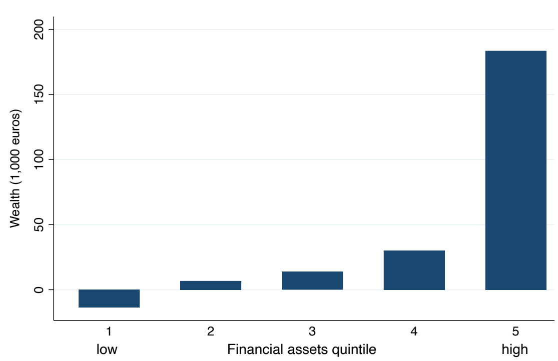
[33] These measures do not include the flow value of a owner-occupied house with no outstanding mortgage. Thus we instead simply control for home-ownership in our analysis. We also note that France's pension system is pay-as-you-go, and therefore pensions are not relevant to our analysis.

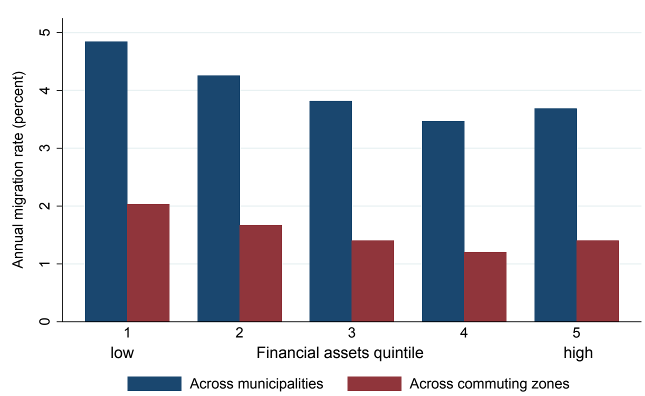
and workplace municipality reported by the current employer of individuals. Municipalities in France compare to ZIP codes in the US: there are 36569 municipalities in France, with an average area of 15 squared kilometers and 435 inhabitants. The employer tax return data allows us to also observe a number of worker characteristics like age, gender, occupation, birthplace, and home ownership status.

Put together, these data sources constitute one of the first large-scale administrative datasets with information on financial assets, high-resolution location, and matched employer-employee labor market characteristics for a large economy like France. Nevertheless, contrasting our data with our theoretical predictions involves several choices. First, since the data does not have direct information on the stock of assets of households, we simply assume that the income flow from financial assets is increasing in the value of assets. We then bin households into five quintiles of our measure of financial income in every year, which under the assumption is equivalent to grouping them by financial assets, and study outcomes across these quintiles. For interpretation purposes, sometimes it is convenient to have a measure of the level of financial assets and not simply the wealth rank of individuals. Hence, we divide the flow income from all financial and housing assets by a common interest rate of 5% to back out an implied stock of assets. Consistent with our theory, our interpretation is that households at the bottom quintiles of the financial income distribution are more likely to be constrained. Figure 5 shows that assets are slightly negative for the bottom quintile.

Second, we need to determine which locations are more complementary with skill, or more attractive. To address this challenges we use our theoretical model. In our theory there is positive assortative matching between a worker's skill and her earnings, which we observe. Furthermore, as implied by the model in Section 2, residents of cities with higher z have higher average incomes. Hence, we can determine the $z-$rank of cities using the rank of their average income (see Figure 1).

Figure 5: Financial assets and fraction of movers in a year by financial assets quintile.





4.1 The Impact of Location on Wages

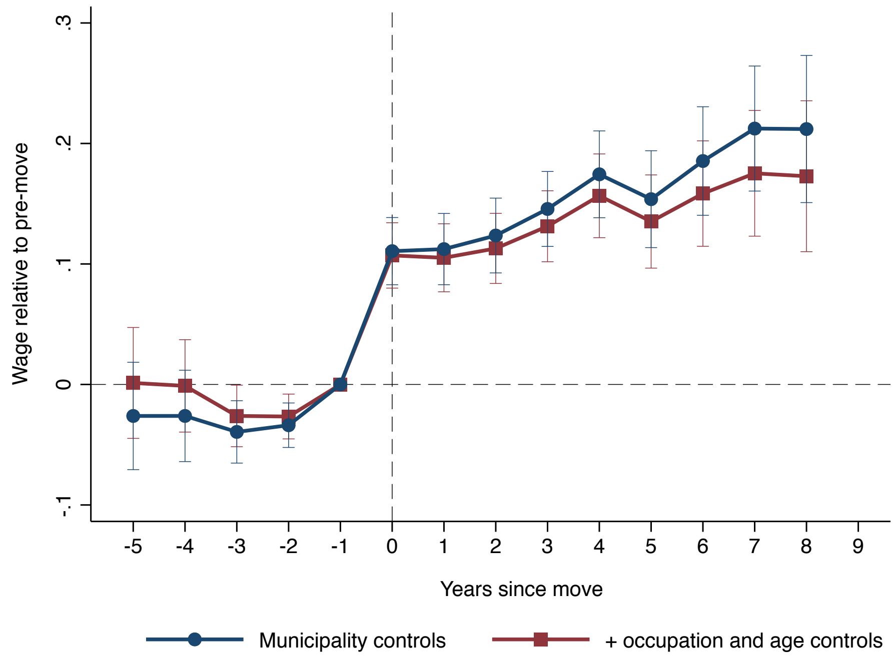
In order for location to resemble an investment decision, it is essential that some of the benefits (or costs) of living in a given location accrue over time. The ideal experiment to test if the returns of moving to a better municipality increase over time would randomly allocate identical workers across different locations, and would compare wages over time of workers who were allocated to good locations relative to those who were allocated to bad locations. In practice, however, finding instruments that achieve such a random spatial allocation is difficult. Therefore, we turn to an event study specification in which we control for as many observable characteristics as possible. We isolate male movers between 25 and 62 years old, and estimate $\log \frac{w_{i,t}}{w_{i,-1}} = \alpha_{G(i,t)} + \beta_t P(z_{i0}) + \varepsilon_{i,t}$, pooled over all individuals i and years t. $P(z_{i0})$ is the percentile of the municipality where individuals migrated in $t = 0$. The dependent variable $\log \frac{w_{it}}{w_{i,-1}}$ denotes wage growth between the period just before the move (period -1) and period t. The difference specification controls for any time-invariant worker characteristic (a worker fixed effect). $\alpha_{G(i,t)}$ controls non-parametrically for age, year, 2-digit origin occupation, and origin municipality fixed effects interacted with linear time trends and with a post-move dummy. Occupation and municipality are measured before the move at $t = -1$. Finally, estimating this equation on movers only avoids picking up unobserved heterogeneity between movers and stayers. For this exercise, we use data for an 8% representative panel of French workers between 2002 and 2015. Online Appendix E.1 provides a description of our dataset.

The investment dimension of mobility is captured by the difference $\beta_t - \beta_0$.[34] The identifying assumption that lends a causal interpretation to this estimate is that there are no (a) worker-specific trends that are systematically correlated with the location decision at $t = 0$ and subsequent wage growth, and (b) no unobserved shocks that are systematically correlated with mobility decisions and wage growth between 0 and t, conditional on our controls. In Appendix B.1, we show that our results are robust to directly controlling for (a) by including worker-specific time trends in the estimation. However, if individuals receive an idiosyncratic worker-level shock at $t = -1$ that makes their wage grow systematically faster in better municipalities, in a way that is orthogonal to worker fixed effects, the trend controls, and pre-move wages, then we would not be able to interpret $\beta_t - \beta_0$ in a causal way.

Figure 6 shows our baseline estimation results. It displays the point estimates relative to period -1. The estimate for $t = 0$ reveals that moving to the best location in France conditional on our controls leads to about 11% higher wages than moving to the worst

[34] The initial effect, $\beta_0 - \beta_{-1}$, could capture, on top of the immediate effect from moving, a short term investment component that is not realized immediately but takes less than 2 years to be reflected in wages.

Figure 6: Effect of migration on wages over time.



Note: Plot of the $\beta_t - \beta_{-1}$ coefficients, for $t = -5...8$, and observed daily real wages. $t = 0$ is the first move of a worker and is the instantaneous effect of location. Standard errors clustered by commuting zone and 2-digit occupation. Vertical bars depict 95% confidence intervals. Depending on the specification, the set of controls includes: fixed effects for the time-0 municipality, interacted with a post-move dummy and with a linear slope; fixed effects for the time-0 2-digit occupation, interacted with a post-move dummy and with a linear slope; and 5-year age bin fixed effects, interacted with a post-move dummy and with a linear slope.

location. Comparing the estimate at $t = 8$ to the estimate at $t = 0$ shows that the return to migration almost doubles after 8 years: wages are then 21% higher. This increase represents the dynamic gains from location.

Note that we focus on dynamic benefits from location in wages because we can measure this effect with our data. Of course, high-z locations convey other dynamic benefits like better schools and learning from more knowledgeable or able peers and neighbors. Therefore, the dynamic location effect we have documented, probably understates the actual dynamic benefits from living in a high-z location. We conclude that location in fact has a payment structure that resembles an intertemporal asset.

4.2 Location Decisions after Income Shocks

The previous subsection showed that location can be viewed as an asset. We now turn to exploring if agents actively use this asset. To do so, we study individual changes in residential locations as a result of an income shock. Figure 5 above provides some basic statistics for our dataset. The left panel plots the average financial wealth of individuals by wealth quintile. As is common in empirical wealth distributions, it is heavily skewed. Individuals in the bottom quintile have negative wealth, while individuals in the top quintile own over 200,000 euros on average. The right panel shows the annual migration rate of the different quintiles across municipalities and commuting zones. Perhaps surprisingly, but consistent with our theory, individuals in the bottom wealth quintile move *more* frequently than their wealthier counterparts. Almost 5% of them move across municipalities every year. Maintaining the view that individuals face very large moving costs, or idiosyncratic preferences for specific locations, seems hard in light of this evidence.

As described in Section 2.5 and illustrated quantitatively in Figure 4, the main implication of our model is that, upon receiving a front-loaded income shock, low-wealth constrained individuals should downgrade their location relative to individuals in the same location who are at the top of the wealth distribution and, therefore, financially unconstrained. We construct annual wage income growth for each individual, and call a negative income shock a decline in annual wage income that is no less than 25%. In Figure 13 in Online Appendix E.2, we show that income initially falls, and then mean-reverts over time following the shock. Thus, these income shocks are indeed front-loaded income shocks.

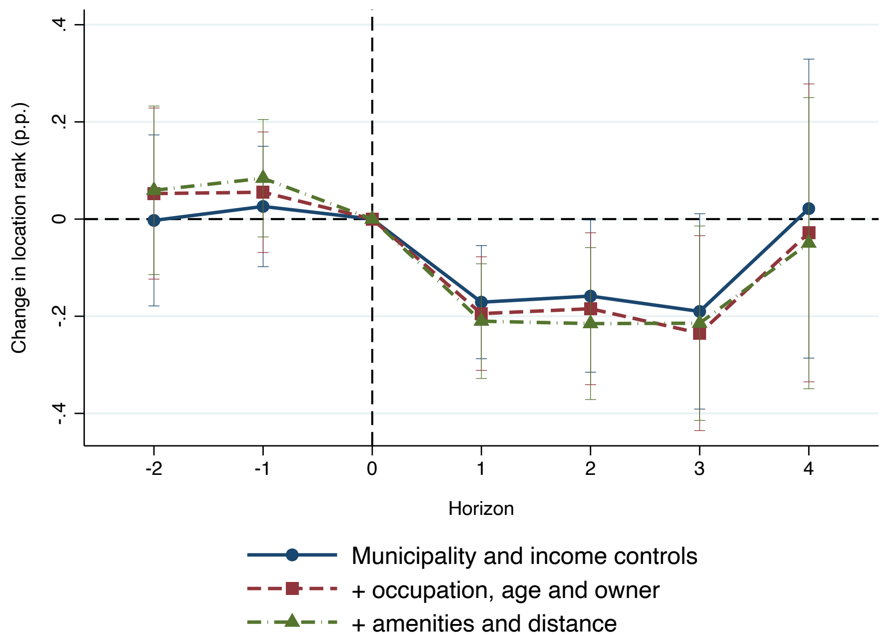
We use an event-study design. Parallel to the quantitative exercise in Section 3.2, we compare the location of individuals who receive the shock in the bottom wealth quintile to individuals who also receive the shock but their assets put them in the top quintile. Our control group then consists of individuals in the same wealth quintile, but who did not receive the shock. We start by estimating the following regression:

$$P(z_{it}) - P(z_{i0}) = \sum_{q=1}^{5}\sum_{n=0}^{1}\sum_{t=-2}^{4} \alpha_{q,n,t} \cdot I_{Q(a_{i0})=q} \cdot I_{N(i)=n} \cdot I_t + \beta_X \cdot \mathbf{X}_{it} + \varepsilon_{it}, \tag{5}$$

where i indexes individuals in the sample and t is the current year. $P(z_{it})$ is the percentile of i's location at time t, $I_{Q(a_{i0})=q}$, is a set of asset quintiles indicators. $I_{N(i)=n}$ is a set of indicators for the negative income shock at time 0. I_t is a set of time indicators, $\mathbf{X}_{it}$ is a vector of pre- and post-move worker controls and fixed effects. It includes year, asset quintile, time-0 wage income, destination amenities, commuting distance, origin municipality, occupation, age, and home-ownership fixed effects. Finally, ε_{it} is a mean zero error term which we assume has the standard mean independence properties.

We are particularly interested in the difference between the location of low-wealth individuals who receive the shock and the location of high-wealth individuals who also receive the shock: $\alpha_{1,1,t} - \alpha_{5,1,t}$. Since some of the agents in the bottom quintile are financially

Figure 7: Differential location effect of a negative income shock (Q1 - Q5).



Note: Difference between location of individuals with low financial assets (Q1) and individuals with high financial assets (Q5) $\alpha_{1,1,t} - \alpha_{5,1,t}$ following a negative income shock relative to individuals who do not receive the shock. $t = 0$ is the year before the income shock. Standard errors clustered by commuting zone and 2-digit occupation. Vertical bars depict 95% confidence intervals. Depending on the specification, the set of controls includes: fixed effects for the time-0 municipality, log wage income at period 0, fixed effects for the time-0 2-digit occupation, 5-year age bin fixed effects, a home-ownership (HO) fixed effect, our measure of amenities for the current location, and log commuting distance at the current residence and workplace.

constrained, if not all, the theory predicts that agents that are in the lowest quintile of the wealth distribution should downgrade relative to those in the top quintile as a result of the income shock. So our 'location as an asset' view implies that $\alpha_{1,1,t} - \alpha_{5,1,t} < 0$ for $t \geq 0$.

Figure 7 presents the results for $\alpha_{1,1,t} - \alpha_{5,1,t}$ for all individuals. All specifications include income controls and over 36,000 municipality fixed effects to control for the location of individuals. As implied by the 'location as an asset' theory, the estimated difference is negative and significant. The magnitude of the difference is close to 0.2 percentage point in the first year, and remains similar for the next two years, it drops to zero in the fourth year. The estimated differences in location choices are not very sensitive to adding fixed effects for 2-digit occupation (64), age bin (6), and home-ownership status (2). All standard errors are clustered at the commuting zone by 2-digit occupation level.

The modest magnitude of the effects we detect masks two attenuating forces. First, individuals may anticipate the negative income shock. In that case, they may move preemptively, mechanically reducing the effects we estimate. Second, only a fraction of individuals move. As a result, all the stayers pull our estimates towards zero. To check whether our estimates are sensitive to those two forces, we run equation (5) on restricted samples of individuals.

First, we restrict attention on individuals who receive the shock as part of a mass layoff event, in which their employer's employment shrunk by at least 25%. Our assumption is that an income loss that happens concurrently to a mass layoff event is more likely to be unexpected and is therefore less likely to be preceded by preemptive moves. Second, we also restrict attention to movers. Table I shows the average post-shock effect across all specifications. In particular, column (5) reveals that restricting attention to movers during mass layoff events increases our coefficient of interest to 3.4 percentage points. Column (6) shows that there is no significant evidence of negative differential pre-trends. Consistent with Figures 7, Table IV in Appendix B.2 shows that the results are virtually unchanged after controlling for destination amenities and commuting distance. The coefficient on Shock in Table I estimates the location response of individuals in the fifth wealth quintile who receive the shock relative to those individuals who do not receive the shock. The 'location as an asset' view implies that these wealthy individuals do not move to systematically lower-ranked locations as a result of the shock, since they prefer to use the financial asset to smooth consumption. Consistently, we find quantitatively small and statistically insignificant location effects on wealthy individuals.

4.2.1 Location Decisions and Amenities

One possible concern with our interpretation is that, as a result of the negative income shock, poor individuals could have decided to consume less amenities relative to wealthy residents, and therefore move to relatively lower-z locations, which likely rank lower in terms of amenities as well. As we argued in Section 2.6, this reasoning is faulty since we condition on initial location and the shock is at least as large for wealthy individuals (as shown in Figure 13 in Online Appendix E.2). If common amenities consumption choices determine location decisions, wealthy individuals must necessarily be more elastic than low-wealth individuals, since otherwise we would not observe them in the same location in equilibrium. Hence, wealthy individuals would downgrade more, not less as observed in Figure 7. If amenities matter in addition to the use of the 'location asset', Figure 7 reflects the balance of these two forces, and shows that the net effect is close to the one predicted by our theory. Nevertheless, to ensure that our baseline difference-in-difference results are not the consequence of a decision by the poorest agents to consume less amenities, we include two measures of local amenities for the destination municipality as controls in the estimation.[35] Finally, to guarantee that our results are also not driven by an increase in

[35]Online Appendix E.1 describes the construction of our first amenity measure using the number and identity of local establishments. Our second measure uses the amenities recovered through the structural model in Bilal (2020).

Table I: Effect of an income shock on location rank (p.p.) by financial assets quintiles.

	Post shock					Pre shock
	(1)	(2)	(3) Mass layoffs	(4) Movers	(5) Mass & movers	(6)
Q1 × Shock	-0.23***	-0.17***	-0.61***	-0.87***	-3.57***	0.04
	(0.06)	(0.06)	(0.19)	(0.32)	(1.07)	(0.06)
Q2 × Shock	-0.08	-0.03	-0.20	-0.15	-0.93	-0.01
	(0.06)	(0.05)	(0.16)	(0.31)	(1.03)	(0.05)
Q3 × Shock	-0.01	-0.05	-0.25	-0.20	-0.66	0.09*
	(0.06)	(0.05)	(0.17)	(0.31)	(1.10)	(0.05)
Q4 × Shock	-0.00	-0.01	-0.14	0.05	-0.16	-0.01
	(0.06)	(0.05)	(0.17)	(0.30)	(1.24)	(0.05)
Shock	0.00	-0.05	0.14	-0.29	0.59	-0.02
	(0.04)	(0.03)	(0.11)	(0.21)	(0.71)	(0.03)
Controls and FEs						
Year, Q1-Q5, Q2-Q4 × Shock	✓	✓	✓	✓	✓	✓
Inc., Mun., Occ., Age, HO		✓	✓	✓	✓	✓
Obs.	5139677	5138559	3062074	675975	378282	2957728
R^2	0.001	0.151	0.131	0.381	0.375	0.096

Note: S.E.s in parenthesis, clustered at commuting zone by occupation level. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. Average difference between location of individuals with financial assets in first four quintiles (Q1 to Q4) and high financial assets (Q5) $\alpha_{q,1} - \alpha_{5,1}, q = 1..4$, as well as location of individuals with high financial assets (Q5) $\alpha_{5,1}$, following a negative income shock, relative to individuals who did not receive the income shock. We pool all years in the sample. The baseline set of controls includes: year fixed effects, dummies for each financial asset quintile. Additional controls include log wage income at time 0, time-0 municipality fixed effects, time-0 2-digit occupation fixed effects, 5-year age bin fixed effects, and a home-ownership fixed effect (HO).

worker's commuting time in response to income shocks, we control for commuting distance after the income shock.[36] Thus, we are comparing mobility patterns of individuals holding constant commuting distance. Figure 7 reveals that explicitly controlling for amenities and commuting distance barely affects our estimates.

If amenities are idiosyncratic with enough variance, conditioning on location is not sufficient to guarantee that our baseline results are not rationalized by the relative choice to consume less amenities. However, two additional results invalidate this view. First, as discussed in Section 2.6, such a view would imply that even wealthy individuals should downgrade as a result of the negative income shock. Table I finds no evidence thereof. Second, the static trade-off view also implies that individuals with a medium wealth level should downgrade relative to wealthy individuals, while the 'location as an asset' view implies that the relative

[36]To construct a measure of commuting distance, we simply compute the geodesic distance between centroids of residence and workplace municipalities.

effects should be present only at the bottom of the wealth distribution. Table I shows our coefficient estimates for all asset quintiles, and reveals that our relative effects are significant and large only for the bottom wealth quintile.[37]

4.2.2 Location Decisions after Positive Wealth Shocks

Our analysis so far has emphasized the effects of negative income shock. Of course, our theory also predicts that low-wealth individuals should upgrade their location relatively more than wealthy individuals as a result of a *positive* shock. Hence, we can use a similar difference-in-difference strategy to study the location effect of positive shock. We illustrate the effect of a positive shock using positive wealth shocks since they are frequent due to the prevalence of donations and inheritance.[38] Table V in Appendix B.2 reveals that the data supports the 'location as an asset' view also for positive wealth shocks larger than 30 thousand Euros.

4.3 Financial Wealth Dynamics after Income Shocks

So far we have shown that low-wealth individuals use location in a way that is consistent with our 'location as an asset' view, i.e. that they smooth consumption when they receive a negative front-loaded income shock by downgrading their location. We have also shown that wealthy individuals do not change their location as a result of a similar shock. We interpreted those results as evidence that low-wealth individuals cannot borrow in financial markets and therefore use location as an asset. Similarly, we argued that wealthy individuals smooth consumption by withdrawing from their financial assets instead of downgrading their location. We now test directly the second implication of our 'location as an asset' view; namely, that low-wealth individuals do not adjust their financial wealth as a result of the negative shock, while wealthy individuals do reduce their holdings of financial assets. To do so, we run the same event study as in equation (5), but we replace the dependent variable

[37]The p-values of tests $\hat{\alpha}_{5,1} \equiv \hat{\alpha}_{4,1}$, $\hat{\alpha}_{4,1} \equiv \hat{\alpha}_{3,1}$, $\hat{\alpha}_{3,1} \equiv \hat{\alpha}_{2,1}$, $\hat{\alpha}_{2,1} \equiv \hat{\alpha}_{1,1}$ are $0.027, 0.299, 0.844, 0.969$, respectively. We reject that Q1-individuals react similarly to Q2-individuals, but fail to reject that Q2, Q3, Q4 or Q5 individuals exhibit differential behavior. One more subtle alternative arises if individuals receive idiosyncratic amenity shocks for locations each period. In a given period, we could observe wealthy individuals with a particularly high realization of the amenity shock in the same location as a wealth-poor individual with an average realization of the amenity shock. Wealthy individuals would then mean-revert to their average preferred location over time. This type of mean-reversion cannot account for our results because it would arise irrespectively of the income shock. However, column 6 of Table I shows no evidence of differential location decisions in the periods prior to the shock.

[38]An additional reason to use positive wealth shocks rather than positive income shocks is that positive income shocks are likely to be the result of a job promotion that is tied to an individual's particular location, which would confound our interpretation of the results. Note, in contrast, that a negative income shock such as job loss severs the link a worker has with a particular workplace.

Table II: Effect of an income shock on financial assets (1,000 euros) by financial assets quintiles.

	Post shock					Pre shock
	(1)	(2)	(3) Mass layoffs	(4) Movers	(5) Mass & movers	(6)
Q1 × Shock	18.60***	20.55***	23.23***	14.54**	6.43	-4.61
	(4.83)	(4.88)	(8.01)	(6.53)	(10.82)	(6.33)
Shock	-20.85***	-20.35***	-20.56***	-17.67***	-11.22	7.97
	(4.60)	(4.76)	(6.46)	(6.35)	(10.16)	(5.15)
Controls and FEs						
Year, Q1-Q5, Q2-Q4 × Shock	✓	✓	✓	✓	✓	✓
Inc., Mun., Occ., Age., HO		✓	✓	✓	✓	✓
Obs.	5139270	5138156	3061919	675851	378228	2957584
R^2	0.001	0.009	0.011	0.029	0.108	0.012

Note: S.E.s in parenthesis, clustered at commuting zone by occupation level. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. Average difference between financial assets of individuals with low financial assets (Q1) and high financial assets (Q5) $\alpha_{1,1} - \alpha_{5,1}$, as well as location of individuals with high financial assets (Q5) $\alpha_{5,1}$, following a negative income shock, relative to individuals who did not receive the income shock. We pool all years in the sample. The baseline set of controls includes: year fixed effects, dummies for each financial asset quintile, and dummies for financial asset quintiles interacted with the 'Shock' dummy. Additional controls include log wage income at time 0, time-0 municipality fixed effects, time-0 2-digit occupation fixed effects, 5-year age bin fixed effects, and a home-ownership fixed effect (HO).

with financial wealth.

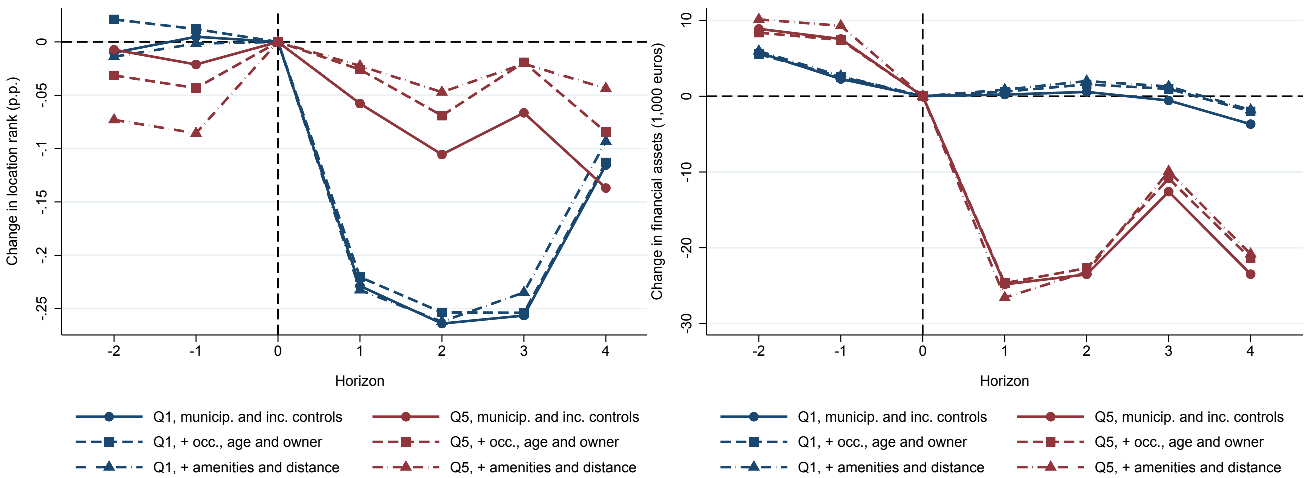
Table II presents the results. Individuals in the top wealth quintile who experience the shock reduce their holdings of financial wealth by about 20 thousand euros relative to individuals in the top wealth quintile who do not experience and income shock (i.e. the coefficient on the 'Shock' variable is -20.85). In contrast, individuals in the bottom quintile of the distribution do not change their financial wealth in a statistically significant way.[39] Table VII in Appendix B.3 shows that our results remain very similar after controlling for amenities and commuting distance. As with location, we find no evidence of pre-trends. We find somewhat smaller and more noisy effects when we condition on a sample of movers, particularly when we do not control for amenities and commuting. This might be the result of changes in the quality and cost of the house that serves as primary residence after a move, which is not accounted for in our measure of financial income. It might also be simply the result of a significantly smaller sample size in a specification with a large number of fixed effects.

To summarize, Figure 8 shows the response of location and financial assets to an income

[39]To obtain the change in the wealth of individuals in the bottom quintiles, sum the coefficients on Shock and on Q1 × Shock. For instance, in column (1), it is -2,150 euros.

Figure 8: Location and wealth effect of a negative income shock by financial assets quintile.

(a) Location (b) Financial assets



Note: Location (left panel (a)) and financial assets (right panel (b)) of individuals with low financial assets (Q1) and individuals with high financial assets (Q5), relative to individuals who did not receive the income shock. We plot the estimated effects $\alpha_{1,1,t}$ and $\alpha_{5,1,t}$ for both location and wealth, for three different sets of controls. $t = 0$ is the year before the income shock. Standard errors omitted for readability. Depending on the specification, the set of controls includes: fixed effects for the time-0 municipality, log wage income at period 0, fixed effects for the time-0 2-digit occupation, 5-year age bin fixed effects, a home-ownership (HO) fixed effect, our measure of amenities for the current location, and log commuting distance at the current residence and workplace.

shock in levels and estimated year by year.[40] These results can be directly compared to those in the quantitative exercise in Figure 4. The similarity is uncanny. Not only is the behaviour of location and assets exactly as predicted, but controlling for an agent's type does not seem to matter much once we condition on initial location. We conclude that the joint changes in location and wealth after a front-loaded negative income shock support our 'location as an asset' view.

4.4 Changing Location Within and Between Commuting Zones

The geographic unit of analysis we have used so far is a municipality. These municipalities are small, and so they allow us to compare workers that in fact live in the same location (e.g. housing prices vary substantially across municipalities within a commuting zone). Furthermore, since our measure of z is based on the average income of *residents* in a municipality, and many of these residents work in neighboring municipalities, it already captures the relevant commuting zone-level variation in earnings. In addition, when analyzing the mobility decisions of agents across municipalities, we control for commuting distance and amenities in the new destination (see Table IV in Appendix B.2). Therefore, our results are not driven

[40]We omit confidence intervals for readability, but the significance of the effect is established in Tables I and II.

by low-wealth individuals deferentially moving to locations with longer commutes or lower amenities conditional on having similar labor market opportunities in a commuting zone.[41] Hence, if municipalities are smaller than the relevant local labor markets, we expect our effects to be mostly driven by moves between, rather than within, commuting zones, which are formed by collections of municipalities.

We investigate whether our results are driven by individuals who move between or within commuting zones. We split movers based on their destination commuting zone and report results for each group in Table VI in Appendix B.2. Consistent with the view that commuting zones are the appropriate notion of a local labor market and that individuals' response to income shocks is based on local labor market opportunities, we find that our effects are driven by individuals who move *between* commuting zones, not *within* them.

5 Conclusions

This paper provides an novel view of individual location decisions. One that puts at the forefront the dynamic consumption-savings decision of agents. Individuals that are constrained to borrow in financial markets use the location asset to transfer consumption to the present by living in locations with low housing costs but relatively poor work and educational opportunities. Our view can rationalize the differential patterns of moving choices across wealth quintiles observed in France as a result of income shocks. In contrast, these patterns falsify views based on amenities or moving costs. The main reason is that agents move significantly across municipalities, but only those that are presumably constrained downgrade their location significantly as a result of negative income shocks. We also find corroborating evidence in the evolution of individual wealth holdings. The change in perspective is relevant for policy. We have shown that using place-based policies to improve some of the worse locations can harm some of the less skilled agents in the economy.

Of course, the 'location as an asset' view is more general than the particular model we put forward in this paper. For example, modelling location choices in an overlapping generations model with multiple locations could help us understand the implications of our view for life-cycle patterns and investment in the skills of descendants. Modelling location choices as changing the properties of an agents income process (by, for example, affecting the likelihood of becoming unemployed) would allow us to study the value of the location asset to manage risk. In addition, in general equilibrium, the agents that decide to locate in a particular

[41] As we argue in Section 2.6 the differential implication across constrained and unconstrained individuals in a model with amenities would be the opposite. Wealthy individuals would downgrade more as a result of a front-loaded negative income shock.

region determine, at least partly, the characteristic of the region. Incorporating this form of external effects could lead to interesting insights for policy. Finally, embedding this type of consumption-savings decision with borrowing constraints in a fully-fledged quantitative spatial model with skill complementarities, factor price determination, as well as mobility and trade costs, could help decompose the role of the location asset in determining net mobility patterns relative to other forces. It could also help us understand how the use of location as an asset affects the evaluation of global phenomena that affect factor rewards in particular locations, occupations, and industries.

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A Appendix: Proofs for the model in Section 2

A.1 Proof of Lemma 1

Location decisions. Supermodularity of problem (1) in (s,z) implies that location decisions of unconstrained and constrained individuals are given by functions $\mathcal{Z}^U(s,y_0,y_1)$, $\mathcal{Z}^C(s,y_0,y_1)$ increasing in s. Equations (2)-(3) imply that $\mathcal{Z}^U(s)$ is independent from (y_0,y_1): $q'(z)=R\mathcal{S}^U(z)$.

Re-arrange (3) to obtain the skill $\mathcal{S}^C(y_0,y_1,z)$ of constrained individuals in z:

$$\mathcal{S}^C(y_0,y_1,z)=\frac{q'(z)(y_1+R\underline{a})}{\beta[y_0-\underline{a}-q(z)]-zq'(z)}, \tag{6}$$

where $q'(z)=\mathcal{S}^U(z)R$ in cities z with at least one unconstrained individual of skill $\mathcal{S}^U(z)$. Using (2), we obtain $\mathcal{S}^C(y_0,y_1,z)>\mathcal{S}^U(z)$ and so $\mathcal{Z}^C(s,y_0,y_1)<\mathcal{Z}^U(s)$.

Re-arrange (6) to obtain $y_0\geq \underline{Y}_0(y_1,z)=\underline{a}+q(z)+\frac{1}{\beta R}\left(z\mathcal{S}^U(z)+(y_1+R\underline{a})\frac{\mathcal{S}^U(z)}{\overline{s}}\right)$ in city z with at least an unconstrained individual. A similar bound involving $q'(z)$ holds for cities in which only unconstrained individuals live.

Equilibrium in cities with at least one unconstrained individual. For any s, $\mathcal{Z}^C(s,y_0,y_1)<\mathcal{Z}^U(s)$, and so cities with unconstrained individuals have higher z than those with only constrained individuals. Thus, there exists a cutoff $\hat{z}$ such that a city has at least one unconstrained individual iff $z\geq\hat{z}$.

Denote by $\tilde{A}(y_0,y_1,s,\mathcal{Z}^U(s),q(\mathcal{Z}^U(s)))=y_0-q(\mathcal{Z}^U(s))-\frac{y_0-q(\mathcal{Z}^U(s))+\frac{y_1+s\mathcal{Z}^U(s)}{R}}{1+\beta}$ desired savings as a function of individual characteristics and the matching function. Denote also $\Sigma(y_0,y_1,s,\mathcal{Z}^U(s),q(\mathcal{Z}^U(s)))=\frac{s(y_1+R\underline{a})}{\beta R(y_0-\underline{a}-q(\mathcal{Z}^U(s)))-\mathcal{Z}^U(s)s}$ the skill of a constrained individual in location $\mathcal{Z}^U(s)$.

Population that locates in cities $[\mathcal{Z}^U(s),\mathcal{Z}^U(s)+\mathcal{Z}^U_s(s)ds)$ per ds is the sum of the unconstrained individuals of the same skill and constrained individuals of higher skill:

$$G(s,\mathcal{Z}^U(s),q(\mathcal{Z}^U(s)),\mathcal{Z}^U_s(s))=\iint f(y_0,y_1,s)\mathbf{1}\left[\tilde{A}(y_0,y_1,s,\mathcal{Z}^U(s),q(\mathcal{Z}^U(s)))>\underline{a}\right]dy_0dy_1$$
$$+\iint \mathbf{1}\left[\tilde{A}(y_0,y_1,s,\mathcal{Z}^U(s),q(\mathcal{Z}^U(s)))\leq\underline{a}\right]f(y_0,y_1,\Sigma(y_0,y_1,s,\mathcal{Z}^U(s),q(\mathcal{Z}^U(s))))\times\frac{d[\Sigma(y_0,y_1,s,\mathcal{Z}^U(s),q(\mathcal{Z}^U(s)))]}{ds}dy_0dy_1.$$

We then compute explicitly the total derivative in the second term, use again (3), collect terms and use land market clearing $h(\mathcal{Z}^U(s))L(\mathcal{Z}^U(s))\mathcal{Z}^U_s(s)=G(s,\mathcal{Z}^U(s),q(\mathcal{Z}^U(s)),\mathcal{Z}^U_s(s))$ to arrive at $(\mathcal{Z}^U)'(s)=\frac{A(s,q(\mathcal{Z}^U(s)),\mathcal{Z}^U(s))}{h(\mathcal{Z}^U(s))L(\mathcal{Z}^U(s))-B(s,q(\mathcal{Z}^U(s)),\mathcal{Z}^U(s))}$ for functions A,B that depend on integrals over (y_0,y_1) and are uniformly Lipschitz continuous when $\underline{s}>0$ and f is bounded. Using the inverse matching function for $z\geq\hat{z}$, we finally obtain a nonlinear system of coupled

Ordinary Differential Equations (ODEs): $(\mathcal{S}^U)'(z) = \frac{h(z)L(z)-B(\mathcal{S}^U(z),Q(L(z)),z)}{A(\mathcal{S}^U(z),Q(L(z)),z)}$ and $L'(z) = \frac{R}{\mathcal{S}^U(z)Q'(L(z))}$. Boundary conditions are $\mathcal{S}^U(\overline{z}) = \overline{s}$, and $\mathcal{S}^U(\hat{z})$, determined by total population supply as defined below. Diverging $\mathcal{S}^U$ or $L(z)$ cannot arise because f has compact support and house prices cannot exceed income which is bounded above. Thus, the ODE system is uniformly Lipschitz continuous. Thus, conditional on boundary conditions, standard results ensure existence and uniqueness of a bounded, global solution, that is continuous in boundary conditions.

Equilibrium in cities with only constrained individuals. We apply the exact same logic as in the case for cities with at least one unconstrained individuals. Near-identical steps imply that q solves the second-order ODE: $q''(z) = \frac{h(z)Q^{-1}(q(z))-D(z,q(z),q'(z))}{E(z,q(z),q'(z))}$ (*), $z_{min} \leq z \leq \hat{z}$. Boundary conditions are $q(\hat{z}^-) = q(\hat{z}^+) = R/\mathcal{S}^U(\hat{z})$ due to no-arbitrage at $\hat{z}$, and $q(z_{min}) = 0$ that pins down z_{min} when land supply is large enough. D, E are again uniformly Lipschitz continuous functions. As before, the ODE has a unique global solution that is continuous in boundary conditions.

Finally, $\hat{z}, \mathcal{S}^U(\hat{z})$ are determined by the requirement that total population sums up to 1, $\int h(z)L(z)dz = 1$, and that $\underline{y}_0 = \underline{Y}_0(\overline{y}_1, \hat{z})$. These are compact maps in $(\hat{z}, \mathcal{S}^U(\hat{z}))$, and so Schauder's fixed point theorem ensures that an equilibrium exists.

A.2 Proof of Lemma 3

Convexity. For $z \geq \hat{z}$, $q'(z) = R\mathcal{S}^U(z)$ implies that q is convex. Optimality implies $q'(z_{min}) = 0$, and since $q'(z) > 0$, we obtain $q''(z_{min}) \geq 0$. Since q is locally convex at both ends of $[z_{min}, \hat{z}]$, inspection of (*) in the proof of Lemma 1 implies $q''(z) \geq 0$ for all $z_{min} \leq \hat{z}$.

Derivative w.r.t. *R*. Suppose that $\underline{s} = \overline{s} = s$. Unconstrained individuals are indifferent between all locations $z \geq \hat{z}$. Constrained individuals all locate below $\hat{z}$. Then, $\frac{d[q'(z)]}{dR} = -\frac{s}{R^2} < 0$. By continuity, this result holds also when $\overline{s} - \underline{s}$ is positive but small.

A.3 Proof of Lemma 4

We first specify the production technology of housing. Suppose housing is produced using the final good k, according to $H = xk^\theta$, where $\eta = \frac{1-\theta}{\theta}$ and $q_0 = \frac{1}{\theta x^{1/\theta}}$. The iso-elastic assumption is without loss of generality and for notational simplicity. Under perfect competition in the housing sector, this results in house rents $Q(L) = q_0 L^\eta$ in the decentralized equilibrium. The planner's problem is split into two stages: (1) allocate individuals over space to maximize second period output net of discounted housing creation, and (2) redistribute output for consumption. The planner chooses the joint distribution of (s, z), $g(s, z)$, to solve

$$\max_{g,k} \int szg(s,z)dsdz - R\int k(z)dz \text{ s.t. } \int g(s,z)dz = f(s), \int g(s,z)ds = xk(z)^\theta, \int g(s,z)dsdz = 1,$$

where f is the given marginal skill distribution.

Taking the F.O.C. for $k(z)$, the shadow price of land $q(z)$ solves $k(z) = (\theta x)^{\frac{1}{1-\theta}} q(z)^{\frac{1}{1-\theta}}$. So the planner's problem is equivalent to: $\max_{g,q} \int szg(s,z)dzdz - R(\theta x)^{\frac{1}{1-\theta}} \int q(z)^{1+\frac{1}{\eta}}dz$ s.t. $\int g(s,z)dz = f(s)$, $\int g(s,z)ds = q_0^{-1/\eta} q(z)^{1/\eta}$. Conditional on q, this is a standard optimal transport problem with supermodular surplus. From Theorem 4.7 p. 39 in Galichon (2016), there exists a unique solution, and it features is perfect Positive Assortative Matching (PAM): there exists an increasing matching function $S(z)$ such that $\int_{S(z)}^{\bar{s}} f(x)dx = q_0^{-1/\eta} \int_z^{\bar{z}} q(z')^{1/\eta} dz'$.

The house rent schedule from the competitive equilibrium is in the planner's choice set for q, and the planner's solution must yield weakly higher output than the equilibrium. Since the equilibrium delivers imperfect PAM, the planner's solution must yield strictly higher gross and net output than the competitive equilibrium.

A.4 Proof of Lemma 5

Net income as sufficient statistic for welfare losses. Indirect utility of unconstrained individuals is $V^U = \log \frac{(\beta R)^\beta}{(1+\beta)^{1+\beta}} + (1+\beta)\log\left(y_0 + \frac{y_1}{R} + \frac{sz^*}{R} - q^*\right)$. For constrained individuals, it is $V^C = \log(y_0 - q^* - \underline{a}) + \beta\log(y_1 + z^*s + R\underline{a})$. Consider now a small individual-level change in q (dq) and zs $(d(zs))$. Then $c_0 dV^U = d\left[\frac{z^*s}{R} - q^*\right]$ and $c_0 dV^C < d\left[\frac{z^*s}{R} - q^*\right]$. Integrating over a continuum of small changes, it is enough to show that net income $I = \frac{z^*s}{R} - q^*$ declines with the policy to conclude to a decline in indirect utility.

Net income change for unconstrained individuals. For unconstrained individuals of skill s, net income satisfies $I^U(s) = \frac{s\mathcal{Z}^U(s)}{R} - q(\mathcal{Z}^U(s))$. It is equal to $\bar{I}^U(s) = \frac{z_0 s}{R} - \bar{q}_0$ after the policy, where $\bar{q}_0$ is unique the rent after the policy change. Finally, note that $(I^U)'(s) = \frac{\mathcal{Z}^U(s)}{R}$. Using Jensen's inequality twice, $\bar{q}_0 = q_0 L_0^\eta = q_0 E[L]^\eta > q_0 E[L^\eta] = E[q] > q(E[z])$.

Define $s_1 < s_0$ such that $\mathcal{Z}^U(s_1) = E[z] < z_0 = \mathcal{Z}^U(s_0)$. For unconstrained individuals with $s \in [s_1, s_0]$, integrate $E[z] \leq (I^U)'(s) \leq z_0$ to obtain $\frac{E[z](s_0 - s_1)}{R} < I^U(s_0) - I^U(s_1) < \frac{z_0(s_0-s_1)}{R}$. Therefore, $I^U(s_1) > \frac{s_1 z_0}{R} - \bar{q}_0 = \bar{I}(s_1)$. Convexity of $I^U(s)$ implies that $I^U(s) > \bar{I}(s)$, $\forall s \leq s_1$. Thus, unconstrained individuals with $s \leq S^U(E[z])$ lose from the policy.

Net income change for constrained individuals. We repeat exactly the same argument as for unconstrained individuals, simply conditioning on (y_0, y_1).

B Appendix: Empirical Robustness Exercises

B.1 The Impact of Location on Wages:

Table III: Wage growth before and after move.

	Basic		Worker slopes	
	(1)	(2)	(3)	(4)
Pre-move slope	0.006	-0.002		
	(0.005)	(0.006)		
Post-move slope	0.014***	0.010***	0.008	0.011
	(0.003)	(0.003)	(0.012)	(0.013)
Pre- & Post-Move Dummy	✓	✓	✓	✓
FEs levels and slopes				
Year, Municipality	✓	✓	✓	✓
Age, 2-digit occ.		✓		✓
Obs.	409109	409109	239393	239393
R^2	0.228	0.280	0.330	0.401
P-value test pre slope = post slope	0.18	0.07*		

Note: S.E.s in parenthesis, clustered at commuting zone by occupation level. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. Column (1) and (3) include as controls: fixed effects for the time-0 municipality, interacted with a post-move dummy and with a linear slope. Columns (2) and (4) add fixed effects for the time-0 2-digit occupation, interacted with a post-move dummy and with a linear slope; and 5-year age bin fixed effects, interacted with a post-move dummy and with a linear slope.

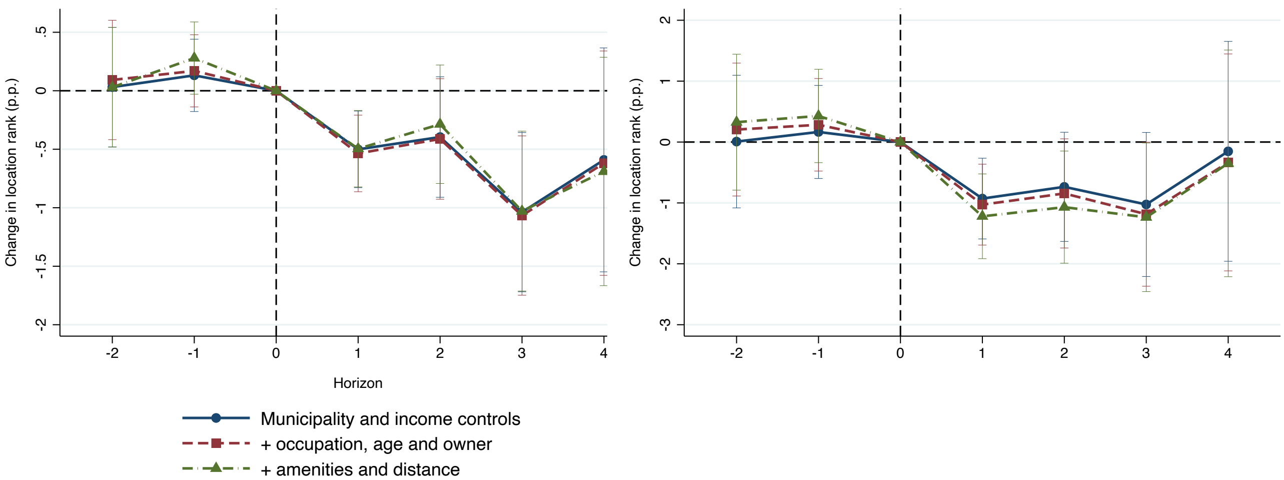
Table III shows our results for equation (4.1) with linear individual trend controls, estimated on the pre-period $t \leq -1$ only.[42] Consistent with our baseline estimates, columns (1) and (2) show that wages grow 1 to 1.4% faster per year in $z = 1$ locations relative to $z = 0$ locations. The point estimate of this slope effect remains between 0.8 and 1.1% per year after netting out worker-specific trends. Our estimates with worker trends are more noisy because we estimate workers' pre-move slope on 3 to 5 observations per worker. Consistent with mean-zero measurement error in the dependent variable, the point estimate remains similar although standard errors increase substantially.

[42]We first project $\log \frac{w_{i,t}}{w_{i,-1}} = \gamma_i + \delta_i \times t + u_{i,t}$, for $t \leq -1$, and re-run equation (4.1) with $\log \frac{w_{i,t}}{w_{i,-1}} - \hat{\gamma}_i - \hat{\delta}_i \times t$ as our dependent variable. This exercise is therefore equivalent to comparing the average post-move slope to the average pre-move slope in Figure 6.

B.2 Location Decisions

Figure 9: Differential location effect of a negative income shock for mass layoffs and movers (Q1-Q5).

(a) Mass layoffs (b) Movers



Note: Difference between location of individuals with low financial assets (Q1) and individuals with high financial assets (Q5) $\alpha_{1,1,t} - \alpha_{5,1,t}$ following a negative income shock for (a) individuals part of a mass layoff, and (b) movers, relative to individuals who did not receive the income shock. $t = 0$ is the year before the income shock. Standard errors clustered by commuting zone and 2-digit occupation. Vertical bars depict 95% confidence intervals. Depending on the specification, the set of controls includes: fixed effects for the time-0 municipality, log wage income at period 0, fixed effects for the time-0 2-digit occupation, 5-year age bin fixed effects, a home-ownership fixed effect, our measure of amenities for the current location, and log commuting distance at the current residence and workplace.

Table IV: Effect of a negative shock on location rank by financial asset quintile.

	Post shock				Pre shock
	(1)	(2) Mass layoffs	(3) Movers	(4) Mass & movers	(5)
Q1 × Shock	-0.14**	-0.49***	-0.72**	-2.70***	0.03
	(0.06)	(0.17)	(0.31)	(1.00)	(0.05)
Shock	-0.04	0.11	-0.20	0.29	-0.03
	(0.03)	(0.10)	(0.20)	(0.67)	(0.03)
Controls and FEs					
Year, Q1-Q5, Q2-Q4 × Shock	✓	✓	✓	✓	✓
Inc., Mun., Occ., Age., HO	✓	✓	✓	✓	✓
Distance, Amenities	✓	✓	✓	✓	✓
Obs.	4782989	2845891	600097	336134	2743274
R^2	0.233	0.215	0.445	0.441	0.159

Note: S.E.s in parenthesis, clustered at commuting zone by occupation level. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. Average difference between location of individuals with low financial assets (Q1) and high financial assets (Q5) $\alpha_{1,1} - \alpha_{5,1}$, as well as location of individuals with high financial assets (Q5) $\alpha_{5,1}$, following a negative income shock, relative to individuals who did not receive the income shock. We pool all years in the sample. The baseline set of controls includes: year fixed effects, dummies for each financial asset quintile, and dummies for financial asset quintiles interacted with the 'Shock' dummy. Additional controls include log wage income at time 0, time-0 municipality fixed effects, time-0 2-digit occupation fixed effects, 5-year age bin fixed effects, a home-ownership fixed effect (HO), log current commuting distance, and two measures of amenities of the current location.

Table V: Effect of a positive wealth shock (30k) on location rank (p.p.) by financial assets quintiles.

	Post shock			Pre shock
	(1)	(2)	(3) Movers	(4)
Q1 × Shock	0.29***	0.14***	1.06***	-0.02
	(0.05)	(0.04)	(0.28)	(0.05)
Shock	-0.12***	0.09***	0.56***	-0.04
	(0.02)	(0.02)	(0.14)	(0.02)
Controls and FEs				
Year, Q1-Q5, Q2-Q4 × Shock	✓	✓	✓	✓
Inc., Mun., Occ., Age., HO		✓	✓	✓
Obs.	5139677	5138559	675975	2957728
R^2	0.001	0.151	0.381	0.096

Note: S.E.s in parenthesis, clustered at commuting zone by occupation level. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. Average difference between location of individuals with low financial assets (Q1) and high financial assets (Q5) $\alpha_{1,1} - \alpha_{5,1}$, as well as location of individuals with high financial assets (Q5) $\alpha_{5,1}$, following a positive wealth shock, relative to individuals who did not receive the wealth shock. We pool all years in the sample. The baseline set of controls includes: year fixed effects, dummies for each financial asset quintile, and dummies for financial asset quintiles interacted with the 'Shock' dummy. Additional controls include log wage income at time 0, time-0 municipality fixed effects, time-0 2-digit occupation fixed effects, 5-year age bin fixed effects, and a home-ownership fixed effect (HO).

Table VI: Effect of a shock on location and wealth by financial assets quintiles.

	Location rank (p.p.)		Wealth (1,000 euros)	
	(1) W/in-CZ movers	(2) Cross-CZ movers	(3) W/in-CZ movers	(4) Cross-CZ movers
Q1 × Shock	-0.18	-1.62**	10.61	21.18***
	(0.33)	(0.66)	(9.00)	(7.68)
Shock	-0.09	-0.78	-16.40*	-21.49***
	(0.21)	(0.47)	(8.86)	(6.84)
Controls and FEs				
Year, Q1-Q5, Q2-Q4 × Shock	✓	✓	✓	✓
Inc., Mun., Occ., Age., HO	✓	✓	✓	✓
Obs.	485560	188494	485517	188413
R^2	0.470	0.426	0.026	0.167

Note: S.E.s in parenthesis, clustered at commuting zone by occupation level. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. Average difference between location and financial assets of individuals with low financial assets (Q1) and high financial assets (Q5) $\alpha_{1,1} - \alpha_{5,1}$, as well as location of individuals with high financial assets (Q5) $\alpha_{5,1}$, following a negative income shock, relative to individuals who did not receive the income shock. We pool all years in the sample. The baseline set of controls includes: year fixed effects, dummies for each financial asset quintile, and dummies for financial asset quintiles interacted with the 'Shock' dummy. Additional controls include time-0 municipality fixed effects, time-0 2-digit occupation fixed effects, 5-year age bin fixed effects.

B.3 Financial Assets

Table VII: Effect of an income shock on financial assets (1,000 euros) by financial assets quintiles.

	Post shock				Pre shock
	(1)	(2) Mass layoffs	(3) Movers	(4) Mass & movers	(5)
Q1 × Shock	21.79***	25.77***	16.48***	12.52	-6.10
	(5.30)	(8.67)	(5.14)	(11.38)	(6.79)
Shock	-21.26***	-22.80***	-19.80***	-17.40	9.76*
	(5.17)	(7.12)	(4.78)	(10.65)	(5.56)
Controls and FEs					
Year, Q1-Q5, Q2-Q4 × Shock	✓	✓	✓	✓	✓
Inc., Mun., Occ., Age., HO	✓	✓	✓	✓	✓
Distance, Amenities	✓	✓	✓	✓	✓
Obs.	4782623	2845746	599997	336090	2743136
R^2	0.008	0.010	0.031	0.106	0.009

Note: S.E.s in parenthesis, clustered at commuting zone by occupation level. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. Average difference between financial assets of individuals with low financial assets (Q1) and high financial assets (Q5) $\alpha_{1,1} - \alpha_{5,1}$, as well as location of individuals with high financial assets (Q5) $\alpha_{5,1}$, following a negative income shock, relative to individuals who did not receive the income shock. We pool all years in the sample. The baseline set of controls includes: year fixed effects, dummies for each financial asset quintile, and dummies for financial asset quintiles interacted with the 'Shock' dummy. Additional controls include log wage income at time 0, time-0 municipality fixed effects, time-0 2-digit occupation fixed effects, 5-year age bin fixed effects, a home-ownership fixed effect (HO), log current commuting distance, and amenities of the current location.

SUPPLEMENT TO "LOCATION AS AN ASSET"

ONLINE APPENDIX

C Extensions with Amenities and Variable Housing

C.1 Proof of Lemma 6

The optimality conditions from the extended model of Section 2.6 are

$$\frac{c_1}{\beta c_0} \geq R \text{ with equality iff } a^* > \underline{a} \ , \ \frac{c_1}{\beta c_0} = \frac{s}{q'(z)} + \frac{Ac_1}{\beta q'(z)}$$

We impose $\beta = R = 1$ to simplify the exposition, but the results generalize to $\beta < 1 < R$

Unconstrained individuals. Combine both budget constraints to obtain $c_1 = y_1 + a + sz = y_1 + sz + y_0 - c_0 - q(z)$. Using the Euler equation $c_1 = c_0$, obtain $c_0 = c_1 = \frac{1}{2}I(s, y_0, y_1, z)$, where $I(s, y_0, y_1, z) \equiv y_0 - q(z) + y_1 + sz \equiv \bar{I}(y_0, y_1) + sz - q(z)$. The location decision then writes $q'(z) = s + Ac_1$. Re-arrange to $s = \frac{2q'(z)+Aq(z)-A\bar{I}(y_0,y_1)}{2+Az}$, implying $q'(z) - s = \frac{A}{2+Az}\Big[zq'(z) - q(z) + \bar{I}(y_0, y_1)\Big]$.

The location response of an unconstrained individual to a y_0 shock, $D_U = \partial_{y_0} z^*$, is then $2q''(z)D_U + Aq'(z)D_U - A = AsD_U$. Re-arranging, $D_U = \frac{A}{2q''(z)+\frac{A^2}{2+Az}\Big[zq'(z)-q(z)+\bar{I}(y_0,y_1)\Big]}$.

Proportional income shock. We now compare the location response of two unconstrained individuals P, R who would locate in the same location z absent the shock. For any individual $j \in \{P, R\}$, where $y_0^R > y_0^P$, $\frac{(1+\tau)A}{2+Az}y_0 D_U \equiv \frac{\tilde{y}_0}{X(A,z)+\tilde{y}_0}$, where $\tilde{y}_0 = \frac{(1+\tau)A^2 y_0}{2+Az}$ and $X(A, z) = 2q''(z) + \frac{A^2}{2+Az}\Big[zq'(z) - q(z)\Big]$. $\frac{\tilde{y}_0}{X(A,z)+\tilde{y}_0}$ is an increasing function of $\tilde{y}_0$ as long as $X(A, z) > 0$, which we show below. Therefore, $D^R = y_0^R D_U^R > y_0^P D_U^P = D^P$.

We now prove that that $X(A, z) > 0$. Suppose first that q is convex. When $\underline{a} = -\infty$, every populated location has an unconstrained individual, and so the last populated location has $q(z) = 0$. Convexity of $q(z)$ then implies that $q(z)/z \equiv p(z)$ is an increasing function. Then $q'(z) = p(z) + zp'(z)$ and so $zq'(z)/q(z) = 1 + z^2p/(z)/q(z) \geq 1$, and $X(A, z) > 0$.

We now prove that that q is convex in equilibrium. We know that q must be convex when $A = 0$. Increasing A continuously keeps q convex because market clearing conditions are continuous in A. As we increase A, q can become locally concave only if q' becomes constant some z^*. As in the baseline case, this implies only unconstrained individuals that satisfy $s + Ac_1 = q'(z^*)$ optimally locate in a neighbourhood of z^* of infinite Radon-Nikodym derivative relative to the Lebesgue measure dz. In contrast, individuals in a comparable slice of the distribution of (y_0, y_1, s) locates in a neighborhood of any other z with a finite Radon-Nikodym derivative relative to the Lebesgue measure. As long as the population distribution of (y_0, y_1, s) is absolutely continuous, the infinite Radon-Nikodym derivative violates land market clearing as it implies zero housing prices, a contradiction. Thus, q must be convex for any $A > 0$.

Constrained individuals. Constrained individuals have $a^* = \underline{a}$. Impose $\underline{a} = 0$ for notational simplicity but without loss of generality. Their location choice is given by $s + Ac_1 = q'(z)\mathcal{R}(s, y_0, y_1, z)$ where $\mathcal{R}(s, y_0, y_1, z) \equiv \frac{y_1 + sz}{y_0 - q(z)}$. Express s as $s = \frac{\frac{q'(z)}{y_0 - q(z)} - A}{1 + Az - \frac{zq'(z)}{y_0 - q(z)}} y_1$. As a result, $c_1 = y_1 + sz = \frac{y_1}{1 + Az - \frac{zq'(z)}{y_0 - q(z)}}$ and $c_0 = y_0 - q(z)$, where $\mathcal{R} = \frac{y_1}{(y_0 - q(z))(1 + Az) - zq'(z)}$ Differentiate the location FOC: $\Big(\mathcal{R}q''(z) + q'(z)\mathcal{R}_z\Big)D_C + q'(z)\mathcal{R}_{y_0} = AsD_C$. Re-arranging and after some algebra: $D_C = \frac{q'(z)}{q''(z)(y_0 - q(z)) + 2q'(z)^2 - 2q'(z)[A(y_0 - q(z))] + [A(y_0 - q(z))]^2}$. Now, D_C is increasing in A iff $\partial_A\Big(- 2q'(z)[A(y_0 - q(z))] + [A(y_0 - q(z))]^2\Big) < 0$, i.e. $2(y_0 - q(z))^2 A < 2q'(z)(y_0 - q(z))$, i.e. $A < \frac{q'(z)}{y_0 - q(z)}$. But from the location FOC, we know that $0 \leq \frac{s}{c_1} = \frac{q'(z)}{y_0 - q(z)} - A$, and so the inequality above is always satisfied. Therefore, D_C is always strictly increasing in A: a constrained individual also downgrade more when receiving a negative income shock when amenities are valued.

Downgrading of constrained relative to unconstrained individuals. When $A \to 0$, $y_0^R D_U^R \approx y_0^R \frac{A}{2q''(z)}$. Similarly, $y_0^P D_C^P \approx d(y_0^P, z) + \frac{q'(z)}{q''(z)(y_0^P - q(z)) + 2(q'(z))^2} \cdot \frac{2q'(z)(y_0^P - q(z))}{q''(z)(y_0^P - q(z)) + 2q'(z)^2} A$. Therefore, for a level income shock, $D_U^R - D_C^R \approx d(y_0^P, z) + \frac{1}{2q''(z)} \cdot \left[\frac{XY}{(X+Y)^2} - 1\right] A$, where $X = q''(z)(y_0^P - q(z))$ and $Y = 2q'(z)^2$. Now, $(X+Y)^2 = X^2 + Y^2 + 2XY = (X-Y)^2 + 4XY \geq 4XY > XY$. Therefore, $D_U^R - D_C^R \approx d(y_0^P, z) - f(y_0^P, z)A$ which is a decreasing function of A since $f(y_0^P, z) > 0$.

For a proportional income shock, $y_0^R D_U^R - y_0^P D_C^R \approx y_0^P d(y_0^P, z) + \frac{1}{2q''(z)} \cdot \left[\frac{XY}{(X+Y)^2} y_0^P - y_0^R\right] A$ and the term that multiplies A is still negative. Therefore, $y_0^R D_U^R - y_0^P D_C^R$ is decreasing in A to a first order.

C.2 Generalized two-period model

We enrich the extended model of Section 2.6 with variable housing choice, amenities, and city income in both periods. Suppose that individuals indexed by (y, s) solve the following problem:

$$V(y_0, y_1, s) = \max_{c_0, c_1, h_0, h_1, a, z} \log(A(z) \cdot h_0^\alpha c_0^{1-\alpha}) + \beta \log(A(z) \cdot h_1^\alpha c_1^{1-\alpha}) \tag{7}$$

$$\begin{aligned} \text{s.t. } c_0 + a + q(z) + p(z)\ell_0 &= y_0 + \tau\Phi(s,z) \\ c_1 + \theta[q(z) + p(z)\ell_1] &= y_1 + Ra + \Phi(s,z) \\ a &\geq \underline{a} \end{aligned}$$

where, relative to the model in the main text, τ governs how much of the mobility returns individual receive immediately, and θ governs how much housing costs must be paid in the second period. When $\Phi(s,z) = sz$, $\tau = \theta = p(z) = 0$, $A(z) = 1$, and $\alpha = 0$, we obtain the model in the main text.

Maximizing out the housing choice, we obtain

$$\begin{aligned} V(y_0, y_1, s) &= \max_{c_0, c_1, a, z} \log(B(z) \cdot c_0) + \beta \log(B(z) \cdot c_1) \\ \text{s.t. } \frac{c_0}{1-\alpha} + a + q(z) &= y_0 + \tau\Phi(s,z) \\ \frac{c_1}{1-\alpha} + \theta q(z) &= y_1 + Ra + \Phi(s,z) \\ a &\geq \underline{a} \end{aligned}$$

where $B(z) = \frac{A(z)}{p(z)^\alpha}$ are perceived amenities after variable housing consumption has been internalized. The financial Euler equation is unchanged. The mobility Euler equation becomes

$$\frac{c_1}{\beta c_0} = \frac{\Phi_z(s,z) - \theta q'(z)}{q'(z) - \tau\Phi_z(s,z)}.$$

Lemma 7 *Suppose the following assumptions hold: (i) $A(z)$ is continuously differentiable and nondecreasing in z; (ii) $p(z) = p_0$ is constant across locations (materials for construction); (iii) the housing production technology results in land prices $q(z) = Q(L(z))$ where $L(z)$ is total population in z, and Q is an increasing function such that $Q(0) = 0$ and $\lim_{L\to+\infty} Q(L) = +\infty$; (iv) the supply of land exceeds population: $\int h(z)dz \geq 1$, where $h(z)$ is the density of land of quality $z \geq 0$; (v) individual income is of the form $I(y,s,z) = y + \Phi(s,z)$, where Φ is continuously differentiable and $\Phi(s,z) > 0$, $\Phi_z(s,z) > 0$, $\Phi_{sz}(s,z) > 0$; (vi) there are no credit constraints: $\underline{a} = -\infty$.*

Consider two individuals A and B who solve Problem (7), with the same future income and location choice. Namely, they have: (a) the same period-1 income: $y_1^A = y_1^B$; (b) different period-0 incomes: $y_0^A < y_0^B$. A is initially lower-income than B; (c) he same location choice: $z^A = z^B = z^*$.

Suppose that they both receive a negative income shock in period 0, such that both individuals loose income down to $y_0'^A = y_0'^B < y_0^A$. Then the initially high-income individual (B) downgrades location more than the initially low-income (A):

$$z'^B < z'^A < z^*$$

This result holds under the less restrictive, single assumption of positive sorting between individuals and locations, which is implied by assumptions (i-v).

Corollary 8 *Replace assumptions (i-v) in Lemma 7 by: primitives are such that individuals choose location according to a matching function $Z(\mathbf{y}, s)$, where $\mathbf{y}$ is permanent income, and such that $Z_y(y, s) > 0$ and $Z_s(y, s) > 0$. Then the implications of Lemma 7 continue to hold.*

We now prove Lemma 7 and Corollary 8.

Equivalence with a static problem. Using the Euler equation and combining both budget constraints into the intertemporal budget constraint, we obtain: $V(y_0, y_1, s) = \max_z B(z)[\mathbf{y} + \mathbf{\Phi}(s, z) - \mathbf{q}(z)]$, where $\mathbf{\Phi}(s, z) = (\tau + R^{-1})\,\Phi(s, z)$, $\mathbf{y} = y_0 + R^{-1}y_1$, and $\mathbf{q}(z) = (1 + \theta R^{-1})\,q(z)$. The FOC is $\nu(z) + \frac{\mathbf{\Phi}(s,z) - zq'(z)}{y + \mathbf{\Phi}(s,z) - \mathbf{q}(z)} = 0$, where $\nu(z) = \frac{zB'(z)}{B(z)}$ is the elasticity of perceived amenities. Re-arrange it as $y = \frac{(\nu(z)+\eta(z))\mathbf{q}(z) - (\phi(s,z)+\nu(z))\Phi(s,z)}{\nu(z)}$, where $\phi(s, z) = \frac{z\mathbf{\Phi}(s,z)_z(s,z)}{\mathbf{\Phi}(s,z)(s,z)}$. Define $G(z, s) = \frac{(\nu(z)+\eta(z))\mathbf{q}(z) - (\phi(s,z)+\nu(z))\Phi(s,z)}{\nu(z)}$.

Proof of Corollary 8. Suppose that there is positive sorting, i.e. that there exists a unique solution $Z(y, s)$ to $y = G(Z(y, s), s)$, with $Z_y(y, s), Z_s(y, s) > 0$. The implicit function theorem implies that $G_z > 0$ and $G_s < 0$. In particular, $\nu(z) > 0$. Now consider individuals A and B, before and after the shock. Then $G(z'^A, s^A) = G(z'^B, s^B)$. Because $G_s < 0 < G_z$, it must be that $z'^A > z'^B$. Thus, the initially high-income individual downgrades more.

Proof of Lemma 7. We show that positive sorting obtains in equilibrium under Assumptions (i-vi). That $A(z)$ is increasing and $p(z) = p_0$ ensure that $\nu(z) = \frac{zB'(z)}{B(z)} \geq 0$. Denote $u(z; y, s) = B(z)[y + \Phi(s, z) - q(z)]$. Then $\frac{zu_z(z;y,s)}{u} \cdot [y + \Phi(s, z) - q(z)] = \nu(z)y + \nu(z)\Phi(s, z) + z\Phi_z(s, z) - \nu(z)q(z) - zq'(z)$. The assumption of excess land together with $Q(0) = 0$ ensures that there is always a worst city which is empty with zero land price. Thus, in equilibrium, $y + \mathbf{\Phi}(s, z) - \mathbf{q}(z) \geq 0$. Finally, denote again the optimal location choice $Z(y, s)$. If $u_z(Z(y, s); y, s) = 0$, then $u_z(Z(y, s); y, s') > 0$ for $s' > s$. Therefore, it must be that $Z(y, s)$ is weakly increasing in s: $Z_s(y, s) \geq 0$. If the matching function is locally flat (i.e. the derivative is zero), then the assumption that $\lim_{L\to+\infty} Q(L) = +\infty$

implies that prices are locally infinite. This cannot be an equilibrium, and hence the matching function is strictly increasing. The same logic applies for $Z_y(y, s) > 0$. The conclusion follows from the proof of Corollary 8.

C.3 Infinite-horizon extension without credit constraints

We now extend the previous results in our two-period model to an infinite-horizon model without credit constraints. Individuals solve

$$
\begin{aligned}
V(a_0, y_0, z_{-1}, s) \quad &= \quad \max_{c,a,h,z} \sum_{t=0}^{\infty} \beta^t \log\Big(A(z_t) c_t^{1-\alpha} h_t^{\alpha}\Big) \qquad (8) \\
&\quad s.t. \quad c_t + q(z_t) + p(z_t) h_t + a_{t+1} = R a_t + s\Big(z_{t-1} + \tau z_t\Big) + y_t
\end{aligned}
$$

where a_t are assets, z_t is location, c_t is consumption of a perishable good, h_t is housing consumption. s is a permanent skill that governs returns to location. τ governs the fraction of location-specific income that accrues upon arrival in a location. y_t is an exogenous income stream. $R \geq 1$ is an exogenous interest rate on financial assets. $A(z)$ are amenities, $p(z)$ is the price of variable housing, and $q(z)$ is the price of the fixed component of housing. Lemma 7 extends as follows. Assume perfect foresight for simplicity.

Corollary 9 *Impose either assumptions (i-vi) of Lemma 7, or assumptions of Corollary 8 for period-0 location choice. Consider two individuals A and B solving Problem (8) in a stationary equilibrium, with the same initial assets, past location and future income and location choice. Namely, they have (a) the same income after period 1: $y_t^A = y_t^B$ for all $t \geq 1$; (b) the same asset holdings $a_0^A < a_0^B$; (c) the same past location $z_{-1}^A = z_{-1}^B$, (d) different period-0 income $y_0^A < y_0^B$: A is initially lower-income than B; (e) the same location choice in period 0: $z_0^A = z_0^B = z_0^*$.*

Suppose that they both receive a negative income shock in period 0, such that both individuals loose $y_0'^A = y_0'^B < y_0^A$.

Then the initially high-income individual (B) downgrades location more than the initially low-income (A): $z_0'^B < z_0'^A < z_0^*$

Proof. As in the two-period model, (i) maximize out variable housing choice, (ii) use the Euler equation to link consumption across time periods, (iii) iterate forward on the budget constraints and use the transversality condition to re-write Problem (8) in present-value terms:

$$V(a_0, \{y_t\}_t, z_{-1}, s) = \max_{\{z_t\}_{t\geq 0}} B(\{z_t\}_t)[Y(a_0, \{y_t\}_t) + \Psi(s, \{z_t\}_t) - Q(\{z_t\}_t)]$$

where

$$B(\{z_t\}_t) = \exp\left[\sum_{t=0}^{\infty} \beta^t \log \frac{A(z_t)}{p(z_t)^\alpha}\right], \qquad Y(a_0, \{y_t\}_t) = R\left[a_0 + \sum_{t=0}^{\infty} R^{-t} y_t + \Phi(s, z_{-1})\right],$$
$$\Psi(s, \{z_t\}_t) = \sum_{t=0}^{\infty} R^{-t}(\Phi(s, z_t) + \tau\Phi(s, z_{t+1})), \qquad Q(\{z_t\}_t) = \sum_{t=0}^{\infty} R^{-t} q(z_t).$$

$\beta R = 1$ must hold in a stationary equilibrium. Then the utility value of amenities decays at the same rate as income. Taking the FOC with respect to z_t, we obtain $\nu(z_t) + \frac{(\tau+R^{-1})\Phi_z(s,z_t)-q'(z_t)}{Y_0+\Psi(s,\{z_t\}_t)-Q(\{z_t\}_t)} = 0$ where ν is the elasticity of $\frac{A(z)}{p(z)^\alpha}$. Further denote

$$B(z) = \frac{A(z)}{p(z)^\alpha}, \quad \mathbf{\Phi}(s, z) = (\tau + R^{-1})\Phi(s, z) \;\; z_0^* = Z(Y_0, \{y_t\}_{t\geq 1}, s)$$
$$\mathbf{Y}(a_0, \{y_t\}_{t\geq 1,} z_{-1}) = R\left[a_0 + \sum_{t-1}^{\infty} R^{-t} y_t + \Phi(s, z_{-1})\right] - \sum_{t-1}^{\infty} R^{-t} q(Z(Y_t, \{y_r\}_{r\geq t}, s))$$
$$+ \sum_{t-1}^{\infty} R^{-t}(\Phi(s, Z(Y_t, \{y_r\}_{r\geq t}, s)) + \tau\Phi(s, Z(Y_t, \{y_r\}_{r\geq t+1}, s)))$$

The problem for solving for z_0 as a function of (y_0, s) given $(a_0, \{y_t\}_{t\geq 1}, z_{-1})$ is now equivalent to solving the static problem

$$V(y_0, s; a_0, \{y_t\}_{t\geq 1}, z_{-1}) = \max_z B(z)[y_0 + \mathbf{Y}(a_0, \{y_t\}_{t\geq 1,} z_{-1}) + \mathbf{\Phi}(s, z) - q(z)]$$

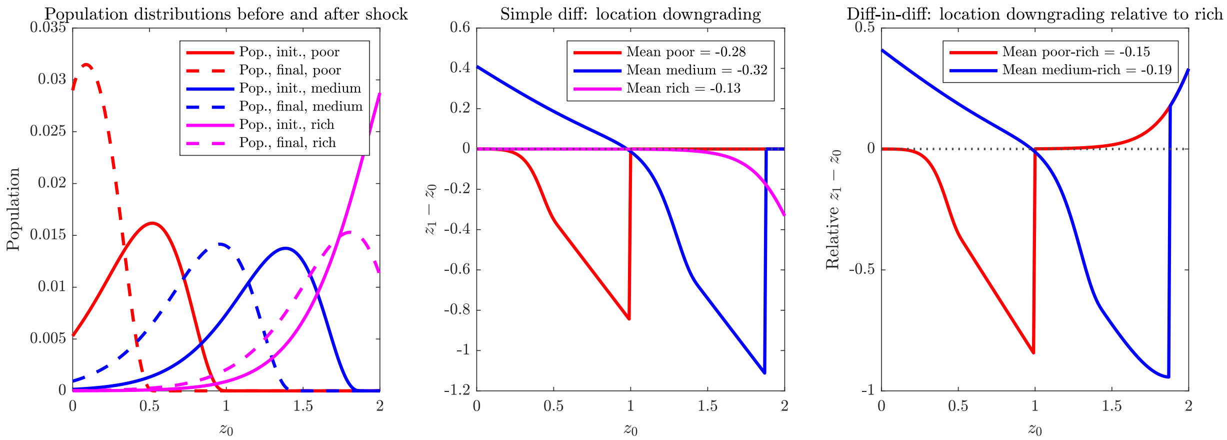
The result then follows from the proof of Lemma 7.

C.4 Two-period model with idiosyncratic preferences for locations

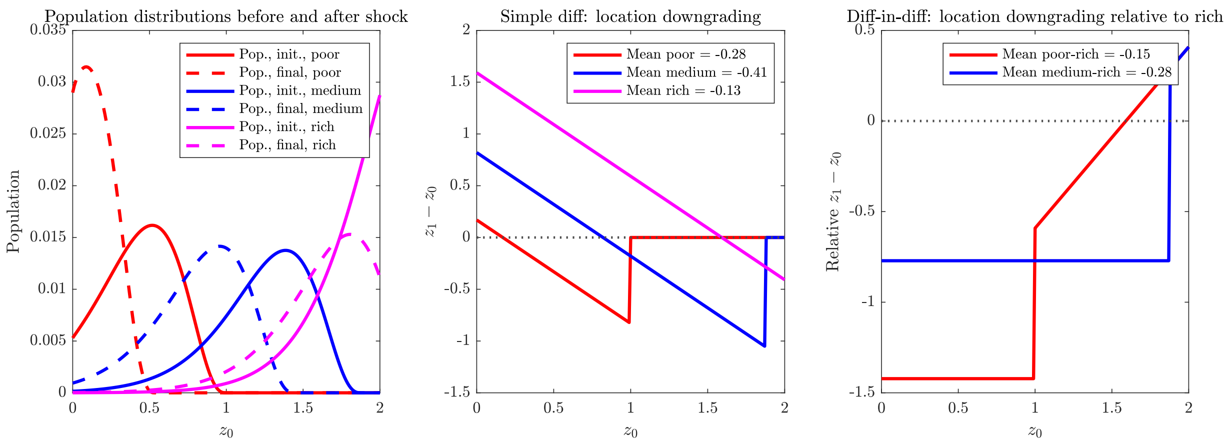
We now present an extension of our two-period model that features idiosyncratic preferences for locations. Since the goal of this section is to contrast the predictions of a model in which spatial sorting arises solely due to taste for amenities, we shut down any borrowing and

Figure 10: Location decisions following an income shock with amenities only.

(a) Location changes: permanent preference shocks.



(b) Location changes: i.i.d. preference shocks.



saving between time periods. Individuals then solve

$$\max_{c_t, z_t} \log(c_t) + Az_t + \varepsilon_{z_t} \quad \text{s.t.} \quad c_t + q(z_t) = y_t \quad , \quad t = 0, 1.$$

The preference shocks ε_z are assumed to be independently drawn across locations z and follow a Gumbel distribution with shape parameter θ.

We show numerically below that, while this model may deliver that low-y individuals downgrade relatively more than high-y individuals in response to an adverse income shock if preference heterogeneity is large enough, we also highlight that such a configuration comes with additional stark implications that are at odds with what we find empirically in Section 4.

To numerically investigate the location responses of individuals to income shocks, we must also specify whether idiosyncratic preferences for locations ε_z are a permanent attribute of the individual, or whether they are re-drawn every period. Since our empirical analysis follows individuals over no more than four years, we find the assumption of permanent shocks more plausible. Nevertheless, we also present results when shocks are drawn anew every period.

We choose standard parameter values. θ is set to 3, a common value in the literature. To obtain a housing-to-income ratio close to a third, we set $q(z) = z^{1.1}$. Finally, we split disposable income into assets and wages. We consider three types of individuals: poor ($a_0 = 0$), medium ($a_0 = 1$) and rich ($a_0 = 2$). All individuals receive a wage of $w_0 = 1$. We then consider individuals in the second period who earn a lower wage $w_1 = 0.5$. We choose $A = 2$. The results of our simulations are presented in Figure 10 below.

While the rightmost panels of Figure 10(a) and (b) reveal that our calibration indeed predicts that poor individuals may downgrade their location relative more than rich individuals on average, it also predicts the following counterfactual observations. First, medium and rich individuals downgrade their location after receiving the negative income shock (middle panels). Second, medium individuals downgrade their location relative to rich individuals on average (right panels).

D Calibration and Additional Exercises

D.1 Calibration

We calibrate our infinite horizon economy to an annual level with two income states $N = 2$ for CRRA utility $u(c) = \frac{c^{1-\frac{1}{\sigma}}-1}{1-\frac{1}{\sigma}}$. We choose the parameter values in Table VIII.

Most of those values are standard. For instance, if we interpret the low income state y_1 as unemployment and the high income state y_2 as employment, we can compute the stationary unemployment rate in this economy through the invariant distribution of the Markov chain transition matrix Λ'. At our current values, we obtain a stationary non-employment rate of 14%, consistent with the prime-age male non-employment rate in France.

Our value of the Intertemporal Elasticity of Substitution σ (IES) is within the accepted range. The median skill we use is $s_0 = 1$. Given our house rent schedule and the equilibrium city choice, this implies that the idiosyncratic component of income y_t represents between 5% and 15% of total labor income $y_t + s_0 z_t$ depending on where individuals are in the state space. Persistent income $s_0 z_t$ thus represents between 85% and 95%. This reflects the large observed differences in wages across cities. The differences in location between the best city 1 and the lowest city 0.5 individuals locate in, imply an income change of 0.5, which is of the order of magnitude of the high idiosyncratic income state.

Table VIII: Calibration Parameters

Parameter	Notation	Value
Preferences		
Discount Factor	β	0.95
Intertemporal Elasticity of Substitution	σ	0.2
Idiosyncratic Income		
Reference skill	s	1
Skill distribution		Uniform on $[0.8, 1.2]$
Low Income State	y_1	0.05
High Income State	y_2	0.2
Transition Probability From Low to High	Λ_{12}	0.6
Transition Probability From High to Low	Λ_{21}	0.1
Financial Markets		
Risk-Free Rate	R	1.04
Credit Constraint	$\underline{a}$	0.00
Cities		
Best City	$\overline{z}$	1.00
Worst City	$\underline{z}$	0.3
House Rents Slope	$q'(z)$	$0.29 + 0.87 \cdot ((z - 0.3)/0.7)^{1.5}$
House Rents	$q(z)$	$\int_{\underline{z}}^{z} q'(x)dx$

Finally, our house rents schedule is constructed in such a way that unconstrained individuals of skill $s = 1.2$ locate at the best available city, and are free to downgrade as much as they like. It also implies housing expenses of about one third of total labor income, consistent with its empirical counterpart reported in Davis and Ortalo-Magne (2011).

To solve the model numerically, we adapt the method of endogenous grid points of Carroll (2006).

D.2 Additional Exercises and Results

In Figure 11 we present the percentage gain in consumption, and in consumption equivalent welfare from using the location asset. The values are calculated starting from the ideal city for unconstrained individuals, $\mathcal{Z}^U(s)$, and we keep the skill of the individual fixed, as in Figure 3. Figure 11 then plots the relative consumption and welfare from using the location asset as a function of the starting asset level, as well as the invariant distribution of assets in the right panel.[43] It presents the gains for agents with a current high or low income realization. Clearly, because we are not estimating the parameters of the model for a particular circumstance, the level of the gains provides only a rough indication of what is at stake from using the location asset. In contrast, the qualitative patterns are more interesting. Most consumption gains happen close to the constraint for low-income individuals who are dis-saving. These consumption gains quickly fade away as we consider individuals with higher levels of assets. However, because those consumption gains occur precisely in the high marginal utility states, they translate into welfare gains of 0.5% close to the constraint.[44] The figure also shows that agents in the low income state benefit more than agents in the high income state, as they are the most likely to use the 'location asset'.

Figure 11: Consumption and welfare gains from the use of the Location Asset

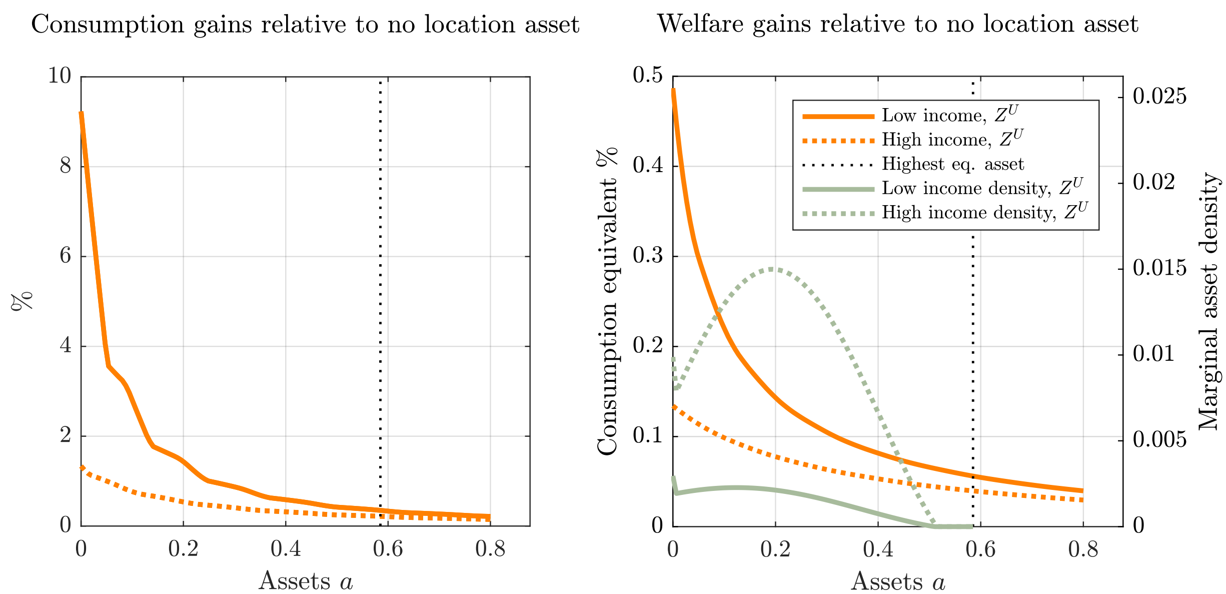


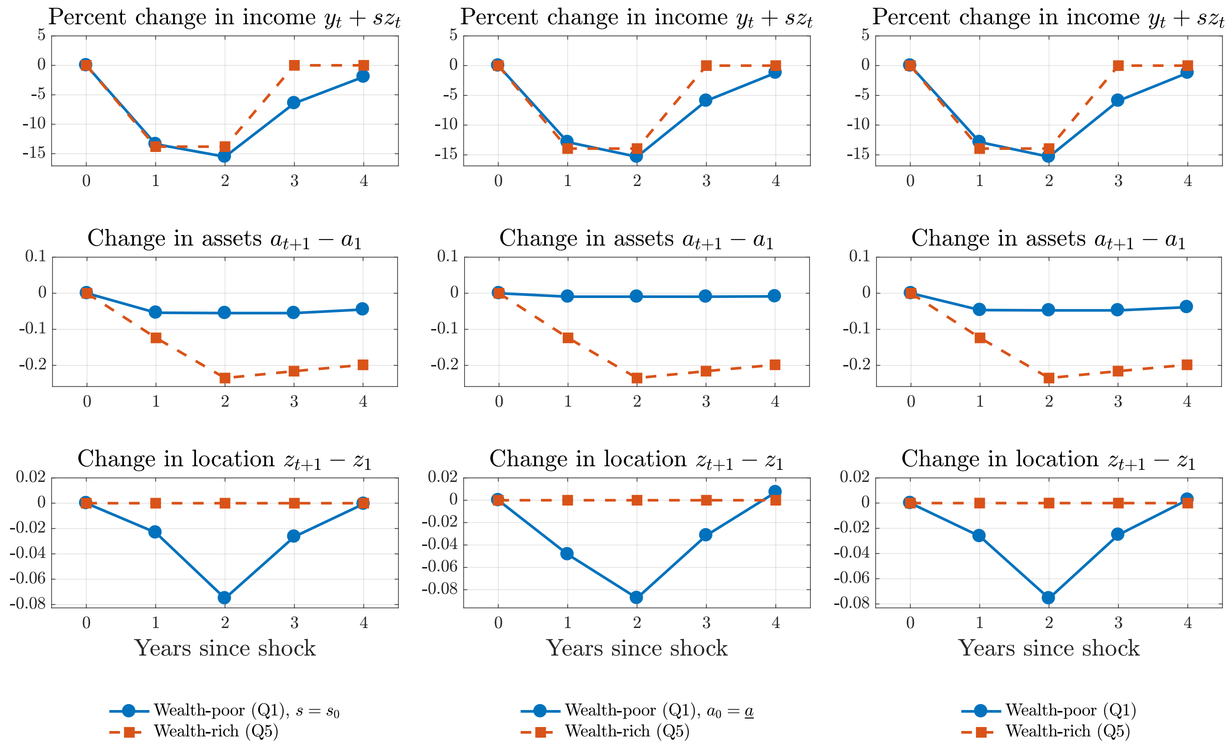
Figure 12 shows these individuals' asset and location over time. In column (a), we select only individuals of the same skill $s = s_0 = 1$, where $s_0 = \mathcal{S}^U(z_0)$ is the skill of the unconstrained individuals who reside in location z_0. Selecting individuals of the same skill

[43]Because financially constrained individuals borrow with the location asset, our two-asset model predicts more individuals at the financial constraint than the standard one-asset formulation. This has interesting implications for macroeconomic policy. For instance, tax rebate shocks would be partly saved by financially constrained individuals by upgrading location.

[44]Gains in flow consumption can be as high as 10% for low-income individuals close to the constraint that live in their unconstrained preferred location. These gains are larger than the ones depicted in Figure 3. This is the case because individuals usually start downgrading location one period before they hit the constraint. Note also that the small kinks in the consumption gains are due to kinks in the consumption policy functions, when individuals hit the constraint next period.

Figure 12: Disposable income, asset and location response to a front-loaded idiosyncratic income shock, by wealth quintile.

(a) Same z_0 and s (b) Same z_0 and poor constrained (c) Same z_0 only



s_0 in both groups implies that everyone must be unconstrained, and so the low-wealth individuals must hold just enough financial assets to be unconstrained but sufficiently little to be in the bottom quintile. The second row of column (a) shows that, as a response to the negative income shock, wealthy individuals dis-save their financial assets to smooth consumption. By contrast, low-wealth individuals do not dis-save much because they hit the credit constraint rapidly. As shown in the last row, these low-wealth, credit-constrained, individuals smooth consumption using the 'location asset' and downgrading their location. By contrast, wealthy individuals stay in their unconstrained location z_0. After the idiosyncratic component of income reverts to the high state, the initially wealthy individuals start saving again in financial assets. The credit-constrained individuals save in the 'location asset' by upgrading their location. Because we select individuals of the same skill s_0 among both low-wealth and wealthy individuals in column 4, z_0 is the unconstrained location for individuals of both groups. Thus, the low-wealth individuals also revert to z_0 in period 4 when they all accumulate assets and become unconstrained.

In column (b) of Figure 12, we select only wealth-poor individuals who are exactly constrained $a_0 = \underline{a}$ when entering the high income state in period 0. Constrained and unconstrained individuals choose to live in the same location only if the unconstrained wealthy individuals have a lower s and are, therefore, less location-elastic. Hence, we cannot condition on s. The second row of column (b) shows that the asset downgrading of initially low-wealth individuals is minimal, since individuals now hold only the assets they managed to accumulate in period zero, when they had high income. Since low-wealth constrained individual have a higher s than wealthy ones, they are already 'borrowing' with the location asset. As in column (a), however, they downgrade location even more to weather the low income shock. Once the idiosyncratic component of income reverts to the high state, however, they upgrade their location towards an even better location than where they started (this is evident in the last row of column (b) in period 4). In fact, we know that they will start accumulating assets and will stop upgrading their location only when they reach a better location than their wealthy counterparts, since we know they have a higher s.

In column (c), which is identical to Figure 4 in the main text, we consider all individuals who satisfy our initial criteria. As a result, the impulse response of assets and location are a weighted average of those in columns (a) and (b). Overall, all three columns reveal similar patterns.

E Data Description and additional results

E.1 Data Description and Sample Selection

Our main data sources are the 'Déclaration de Données Sociales' (DADS) Panel, as well as the 'Données Sociales et Fiscales' from the 'Echantillon Demographique Permanent' (EDP). Both are administrative tax data from the French statistical institute (INSEE).

DADS. The DADS is a matched employer-employee dataset based on tax returns filed by employers. It has rich information on a representative sample of workers who receive taxable labor income in France. It is a panel of all workers in France born in October of even years (approximately 8%). In this dataset we can track the same individual throughout her employment spells for the period 2002-2015. We start in 2002 to observe workers for a long enough period and estimate long-term returns to mobility.

We extract the following variables from the dataset: (i) anonymized individual identifier, common to DADS and EDP; (ii) total net wage earnings; (iii) age and gender; (iv) municipality of residence and workplace; (v) 2-digit occupation. We extract the highest paying employment spell for each individual and each quarter. We then aggregate wage at the annual level, and select municipality and occupation based on the highest paying spell in the year.

EDP. The fiscal data in the EDP dataset starts in 2008 and contains income tax return information for French households that are sampled in the DADS or in the baseline EDP sample. The EDP sample contains individuals born in January 2-5, April 1-4, July 1-4, and October 1-4. We link it to the DADS Panel through a common individual identifier.

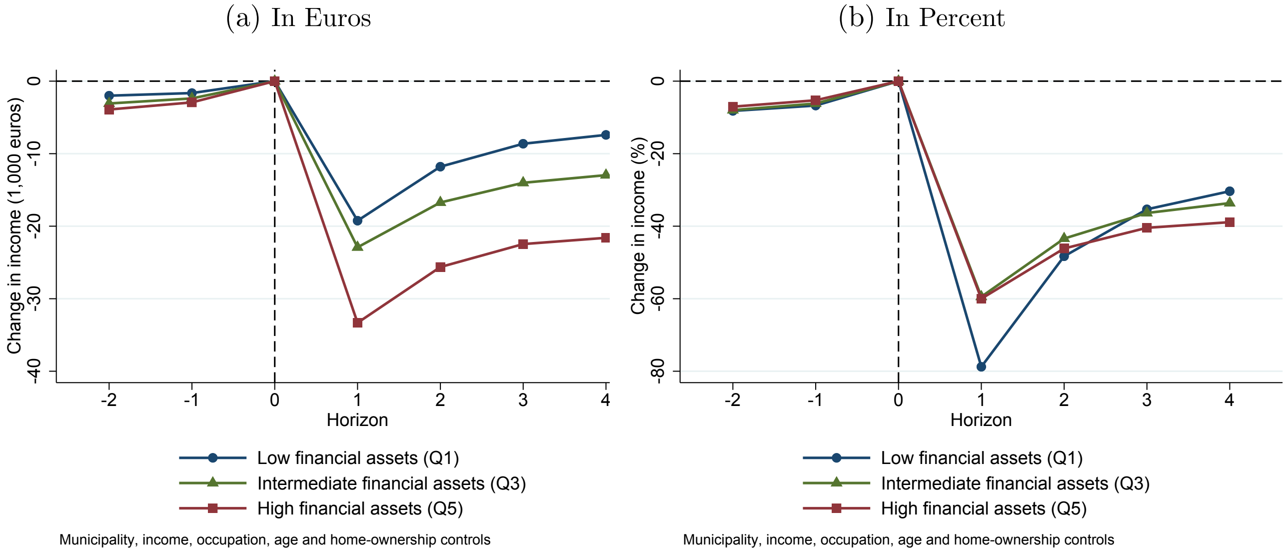
We extract the following variables from the dataset: (i) anonymized individual identifier, common to DADS and EDP; (ii) income from financial assets: annuities, housing rents, net of expenses (mortgage payments, repairs, etc.), stocks, mutual funds, bonds, taxable bank accounts, excluding capital gains, imputed non-taxable income (life insurance, certain types of bank accounts, etc.). We excluded private equity from the analysis because in many cases it corresponds to ownership of a practice (lawyers, medical doctors, etc.) that it highly illiquid and hard to separate from the worker and sell. We use the residence information from the DADS rather than the fiscal residence information from the EDP due to well-known concern that the fiscal residence is oftentimes different from the actual residence. We indeed find that using the fiscal residence implies a annual migration rate that is an order of magnitude lower than what we find in the DADS, and implausibly low. We must restrict attention to years 2010 to 2015 so that all variables are non-missing.

We compute asset quintiles in every year based on the within-municipality distribution of assets to be fully consistent with our theory. The correlation with unconditional asset quintiles is 0.9. As shown in Figure 5, local wealth quintiles appear to capture substantial variation in monetary terms.

Additional data. We complement our main sample with additional data from two sources. *Amenities.* We use two measures of amenities. First, we use the 'Base Permamente des Equipements' in 2007 to construct a measure of amenities. 2007 is the year prior to which the closest year available before our sample with financial income starts. It reports data the number of 136 types of establishments in health services (e.g. hospitals), education services (e.g. pre-schools), public services (e.g. police stations), and commercial services (e.g. perfumeries). We first compute the number of these establishments per capita in each municipality. Then, we extract the first principal components of the corresponding covariance matrix. For each municipality, we obtain the loading on this principal component. We choose the sign of the principal component such that the loadings correlate positively with our measure of z. Finally, we rank these loadings between 0 and 1. This rank is our first measure of amenities. Our second measure of local amenities are the amenities recovered through the structural model in Bilal (2020). We refer to that paper for details.

Commuting distance. We obtain data on the centroids of each municipality in France from a database publicly available from the French government at `https://www.data.gouv.fr/en/datasets/listes -des-communes-geolocalisees-par-regions-departements-`

Figure 13: Wage income effect of a negative income shock by financial assets quintile.



Note: Difference between wage income of individuals with low financial assets (Q1) and individuals with high financial assets (Q5) $\alpha_{1,1,t} - \alpha_{5,1,t}$ following a negative income shock relative to individuals who do not receive the shock. $t = 0$ is the year before the income shock. Confidence intervals omitted for readability. The set of controls includes: fixed effects for the time-0 municipality, log wage income at period 0, fixed effects for the time-0 2-digit occupation, 5-year age bin fixed effects, and a home-ownership (HO) fixed effect.

`circonscriptions-nd/`. We then compute the geodesic distance between each residence-workplace municipality pair, and use this distance as our measure of commuting distance.

Background on the French geography. The French mainland territory is partitioned in about 96 districts ('Départements') and 36,552 municipalities ('Communes'). Départements are fairly large areas (median area is 8,763 km2 and median population is 531,380 inhabitants), while municipalities are much smaller (median area is slightly above 10 km2, and median population is 432 inhabitants).

Construction of the z variable. To determine how desirable a municipality is, we compute average annual wage earnings in each municipality in the DADS. We then rank municipalities and compute the corresponding percentile for each municipality.

E.2 Income Shock

Figure 13 presents the average wage income shock by wealth quintile that we use for the exercises in Section 4.