Lecture 8: A Spatial Growth Framework

Economics 552

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Desmet and Rossi-Hansberg

- Economic growth and development vary widely across space and across sectors
- Need for a dynamic spatial theory with
  - Many locations and ordered space
  - Innovation being the outcome of profit-maximizing firms
  - Spatial micro-foundations of the macroeconomy
- Difficulty of developing such a theory
  - Dimensionality makes the problem intractable

Early Attempts

- Endogenous growth with two or more countries
  - Grossman and Helpman (1991)
  - Locations not ordered in space
  - Some "New Economic Geography" papers, but few locations
- Spatial dynamic problems
  - Quah (2002), Boucekkine et al. (2009), Brock and Xepapadeas (2008)
  - Includes either diffusion or capital mobility with immobile but fully forward-looking agents
  - Cannot be fully analyzed apart from special cases
- Spatial growth without endogenous innovation
  - Desmet and Rossi-Hansberg (2009)

This Paper I

- Propose a *dynamic spatial theory* that is
  - Simple enough to be tractable
  - Rich enough that it can be brought to the data
- Main elements of theory
  - Continuum of locations on a line
  - Two sectors: manufacturing and services
  - Firms invest in local innovation
  - Trade subject to transportation costs

This Paper II

- Apply the theory to the U.S. structural transformation, 1950-2005
- Account for macroeconomic stylized facts such as
  - Increasing share of employment in services
  - Drop in the relative price of manufactured goods
  - Increase in productivity growth of services relative to manufacturing
  - Aggregate growth did not slow down
- Account for spatial stylized facts:
  - Increase in land prices
  - Increase in dispersion of land prices
  - Increasing spatial concentration of services

The Model

- The economy consists of land and people located in the closed interval $[0, 1]$
- Density of land at each location ℓ equal to one
- Population size is $\overline{L}$
- Each agent is endowed with one unit of time each period
- Each agent owns a diversified portfolio of land and firms
- Agents are infinitely lived and have rational expectations
- Agents are freely mobile

Preferences and Consumer's Problem

- An agent at a particular location ℓ solves the following problem

$$\max_{\{c_i(\ell,t)\}_0^\infty} E \sum_{t=0}^{\infty} \beta U(c_M(\ell,t), c_S(\ell,t))$$

$$\text{s.t.} \quad w(\ell,t) + \frac{\bar{R}(t) + \Pi(t)}{\bar{L}} = p_M(\ell,t)\, c_M(\ell,t) + p_S(\ell,t)\, c_S(\ell,t)$$

- Free labor mobility and the absence of a savings technology imply that agents solve this problem period-by-period
- Numerical part of the paper will use CES preferences

Technology

- Firms specialize in one sector and use labor and one unit of land
- Production of a firm in location ℓ and time t is given by

$$
\begin{aligned}
M\left(L_M\left(\ell,t\right)\right) &= Z_M^{+}\left(\ell,t\right)^{\gamma} L_M\left(\ell,t\right)^{\mu_M} \\
S\left(L_S\left(\ell,t\right)\right) &= Z_S^{+}\left(\ell,t\right)^{\gamma} L_S\left(\ell,t\right)^{\mu_S}
\end{aligned}
$$

  where Z_M^{+} and Z_S^{+} are the technology levels after innovation

- We will later describe how Z_M^{+} and Z_S^{+} are determined

Diffusion

- Technology diffuses locally between time periods
- If $Z_i^+(r, t-1)$ was used at r in $t-1$, next period t location ℓ has access to

$$e^{-\delta|\ell - r|} Z_i^+(r, t-1)$$

- Hence, before the innovation decision in period t, location ℓ's technology is

$$Z_i^-(\ell, t) = \max_{r \in [0,1]} e^{-\delta|\ell - r|} Z_i^+(r, t-1)$$

where Z_i^- is the technology level before innovation

Idea Generation

- A firm can decide to buy a probability $\phi \leq 1$ of innovating at cost $\psi(\phi)$ in a particular industry i
- An innovation is a draw of a technology multiplier z_i from

$$\Pr[z < z_i] = \left(\frac{1}{z}\right)^a$$

- Conditional on innovation and technology Z_i, the expected technology is

$$E\left(Z_i^+(\ell,t)\,|Z_i^-, Innovation\right) = \frac{a}{a-1} Z_i^- \text{ for } a > 1.$$

- Expected technology for a given ϕ, not conditional on innovating, is

$$E\left(Z_i^+(\ell,t)\,|Z_i^-\right) = \left(\frac{\phi a}{a-1} + (1-\phi)\right) Z_i^- = \left(\frac{\phi + a - 1}{a-1}\right) Z_i^-.$$

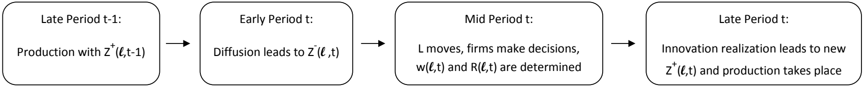
Spatial Correlation

- The innovation draws are i.i.d. across time, but not across space
- Conditional on an innovation, let $s(\ell, \ell')$ denotes the correlation in the realizations of $z_i(\ell)$ and $z_i(\ell')$
- We assume that $s(\ell, \ell')$ is non-negative, continuous, symmetric, and

$$\lim_{\ell \downarrow \ell'} s\left(\ell, \ell'\right) = 1 \text{ and/or } \lim_{\ell \uparrow \ell'} s\left(\ell, \ell'\right) = 1$$

- This, together with diffusion, will ensure that a firm's innovation decision today does not affect its innovation decision tomorrow (more later....)

Timing


Late Period t-1:
Production with $Z^{+}(\ell,t-1)$
Early Period t:
Diffusion leads to $Z^{-}(\ell,t)$
Mid Period t:
L moves, firms make decisions, $w(\ell,t)$ and $R(\ell,t)$ are determined
Late Period t:
Innovation realization leads to new $Z^{+}(\ell,t)$ and production takes place

Firm's Problem

- Firms maximize the expected present value of profits:

$$\max_{\{\phi_i(\ell,t), L_i(\ell,t)\}_{t_0}^{\infty}} E_{t_0}\left[\sum_{t=t_0}^{\infty}\beta^{t-t_0}\left(\begin{array}{c} p_i(\ell,t)\left(\left(\frac{\phi_i(\ell,t)}{a-1}+1\right)Z_i^-(\ell,t)\right)^{\gamma}L_i(\ell,t)^{\mu_i} \\ -w(\ell,t)\,L_i(\ell,t)-R(\ell,t)-\psi\left(\phi_i(\ell,t)\right)\end{array}\right)\right]$$

- Labor is freely mobile and firms compete for land and labor every period with potential entrants that, because of diffusion, have access to the same technology

- The problem of choosing L and R is therefore static

- Firms bid for land, with the bid rent

$$\begin{aligned} R_i(\ell,t) &= p_i(\ell,t)\left(\frac{\hat{\phi}_i(\ell,t)}{a-1}+1\right)^{\gamma} Z_i^-(\ell,t)^{\gamma}\,\hat{L}_i(\ell,t)^{\mu_i} \\ &\quad -w(\ell,t)\,\hat{L}_i(\ell,t)-\psi\left(\hat{\phi}_i(\ell,t)\right), \end{aligned}$$

  so ex-ante one-period profits are zero

Innovation I

Proposition 1

A firm's optimal dynamic innovation decisions maximize current period profits

- Keys to Proof
  - Today's innovations diffuse by tomorrow
  - A firm has access to the same technology as its neighbors
  - Continuity in the diffusion process and the spatial correlation in innovation realizations imply that a firm's own decisions today do not affect the expected technology it wakes up with tomorrow

- So firms solve

$$\max_{\phi_i} p_i(\ell, t)\left(\frac{\phi_i + a - 1}{a-1} Z_i^-(\ell, t)\right)^{\gamma} \hat{L}_i(\ell, t)^{\mu_i} - w(\ell, t)\hat{L}_i(\ell, t) - R(\ell, t) - \psi(\phi_i)$$

Innovation II

-
- Proposition 1 makes the dynamic spatial model solvable and computable
- Firms innovate in a competitive framework with zero profits
  - Innovation is local
  - In each location land is in fixed supply
  - Firms innovate to the extent that it enhances their bid for land
- Scale effect in innovation
- In the numerical exercise we set

$$\psi\left(\phi; w(\ell, t)\right) = w(\ell, t)\left(\psi_1 + \psi_2 \frac{1}{1-\phi}\right) \text{ for } \psi_2 > 0$$

Transport costs and Land Markets

- If one unit is transported from ℓ to r, only $e^{-\kappa|\ell-r|}$ units arrive in r
- The price of good i produced in ℓ and consumed in r has to satisfy

$$p_i(r,t) = e^{\kappa|\ell-r|} p_i(\ell,t)$$

- Land is assigned to its highest value, so land rents are such that

$$R(\ell,t) = \max\{R_M(\ell,t), R_S(\ell,t)\}$$

- Denote by $\theta_i(\ell) \in \{0,1\}$ the fraction of land at location ℓ used in the production of good i

Goods, Services and Labor Markets

- Let $H_i(\ell, t)$ denote the stock of excess supply of product i between locations 0 and ℓ
- Define $H_i(\ell, t)$ by $H_i(0, t) = 0$ and by the differential equation

$$\frac{\partial H_i(\ell, t)}{\partial \ell} = \theta_i(\ell, t)\, x_i(\ell, t) - c_i(\ell, t)\left(\sum_i \theta_i(\ell, t)\, \hat{L}_i(\ell, t)\right) - \kappa\, |H_i(\ell, t)|$$

  - $x_i(\ell, t)$ denotes production in industry i per unit of land *net* of real investment costs

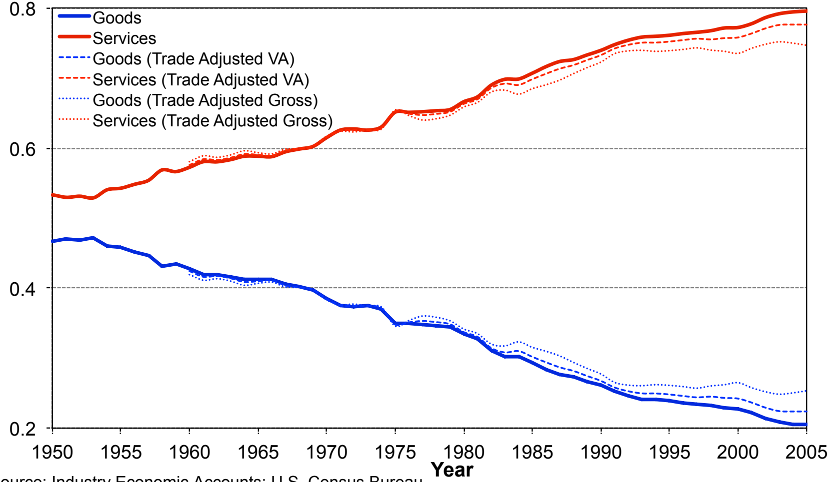
- Equilibrium in products markets is guaranteed by $H_i(1, t) = 0\ \forall i$
- Labor markets clear, so $\int_0^1 \sum_i \theta_i(\ell, t)\, \hat{L}_i(\ell, t)\, d\ell = \overline{L}\ \forall t$

Equilibrium: Definition and Uniqueness

- Definition of Equilibrium
  Given initial productivity functions $Z_i^-(\cdot, 1)$, for $i \in \{M, S\}$, an equilibrium is a set of real functions (c_i, $\hat{L}_i$, θ_i, H_i, p_i, R_i, w, Z_i^-, Z_i^+, ϕ_i) of locations $\ell \in [0, 1]$ and time $t = 1, \ldots$, for $i \in \{M, S\}$, such that all agents maximize utility, all firms maximize profits, land is assigned to the industry that values it the most, technologies diffuse as described before, and all markets clear

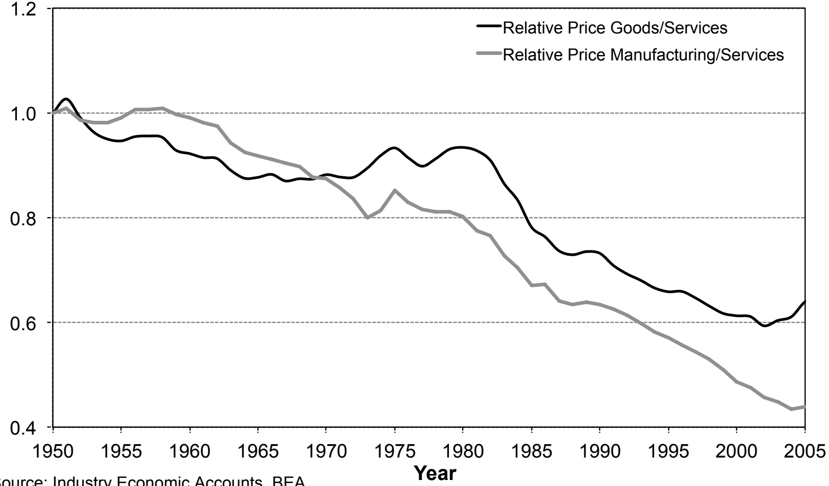
- Proposition 2
  The equilibrium of this economy exists and is unique if $\gamma + \mu_i \leq 1$ for all i

Employment Shares



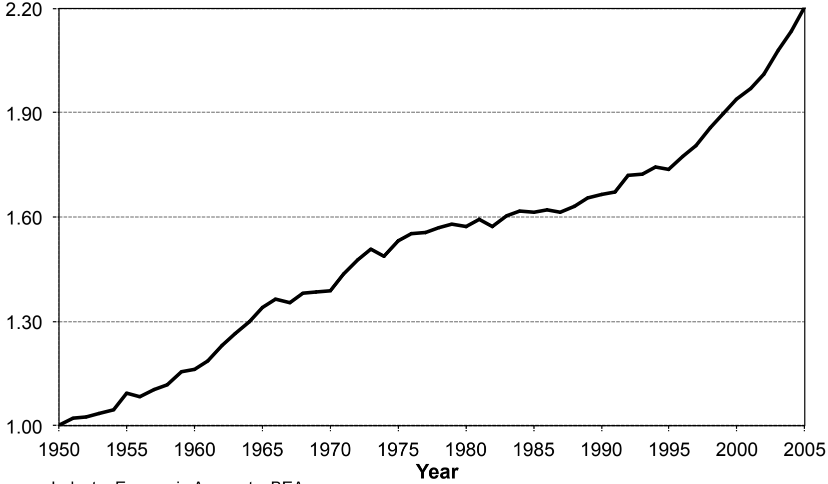
Source: Industry Economic Accounts; U.S. Census Bureau

Relative Prices



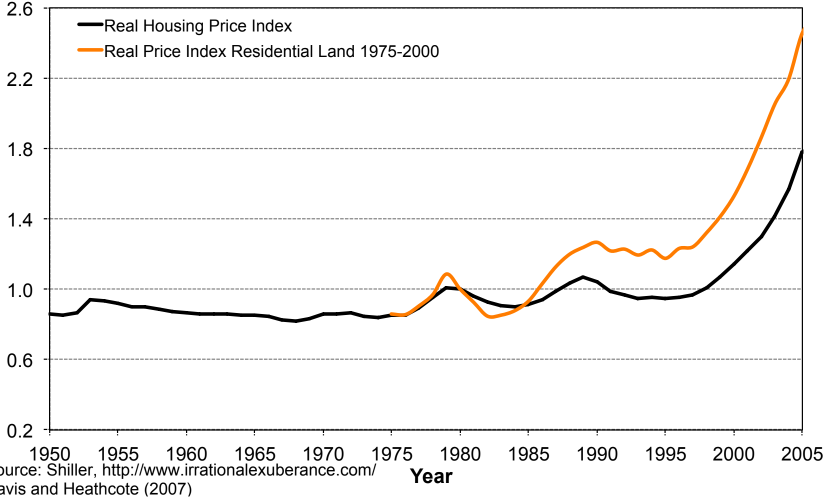
Source: Industry Economic Accounts, BEA

Value Added per Worker



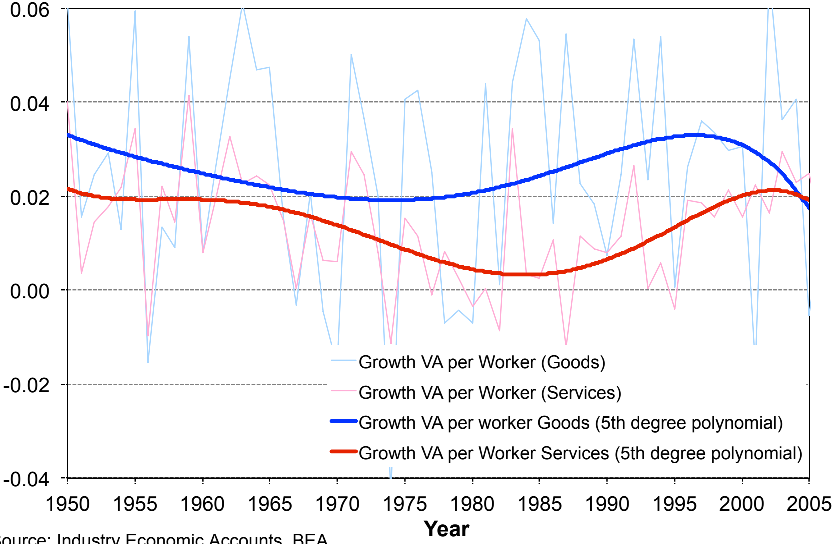
Source: Industry Economic Accounts, BEA

Real Housing and Land Price Index



Source: Shiller, http://www.irrationalexuberance.com/
Davis and Heathcote (2007)

Growth in Value Added per Worker



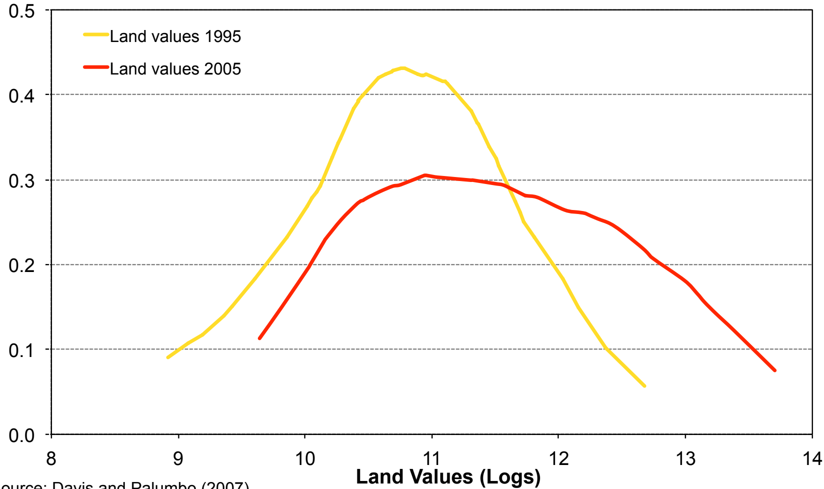
Source: Industry Economic Accounts, BEA

Spatial Concentration of Employment

	1950	1970	1980	1990	2000	2005
Log Employment Density						
Difference 70-30						
Goods	1.40	1.71	1.58	1.60	1.57	1.58
Services	1.14	1.24	1.34	1.39	1.44	1.46
Goods/Services	1.23	1.37	1.18	1.15	1.09	1.08
Standard deviation						
Goods	1.67	1.80	1.69	1.70	1.64	1.60
Services	1.42	1.51	1.52	1.58	1.59	1.59
Goods/Services	1.18	1.20	1.11	1.08	1.03	1.00

Source: REIS, Bureau of Economic Analysis; County and City Databooks

Land Values Distribution across MSAs



Source: Davis and Palumbo (2007)

Summary of Empirical Facts

- Between 1950s and early 1990s
  - Productivity in goods, relative to services, was growing fast
  - Employment in the goods-producing sectors was steadily falling
  - Real land rents did not exhibit significant changes
  - Manufacturing became increasingly dispersed relative to service
- Starting in mid-1990s
  - Land prices started to increase
  - Value added per worker growth accelerated
  - Service productivity growth took off
  - Changes in employment shares slowed down
  - The dispersion in land prices increased
  - The tendency towards greater dispersion of manufacturing relative to services slowed down

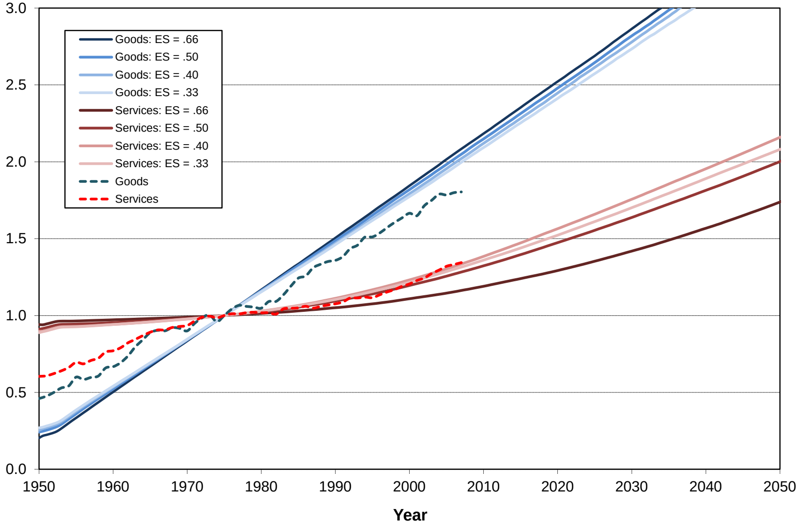
Calibration I

- Initial productivity functions
  - $Z_S(\ell, 0) = 1$ and $Z_M(\ell, 0) = 0.8 + 0.4\ell$
  - Innovation will happen earlier in manufacturing and in locations close to upper border
- Elasticity of substitution between manufacturing and service of 0.4
  - Key for endogenous take-off of innovation in service sector
- Follow Herrendorf and Valentiniyi (2008) and let $\mu = \sigma = 0.6$
- Preference parameters $h_S = 1.4 > h_M = 0.6$ set to match 1950 sectoral employment shares
- Transport cost parameter based on Ramondo & Rodríguez-Clare (2012)

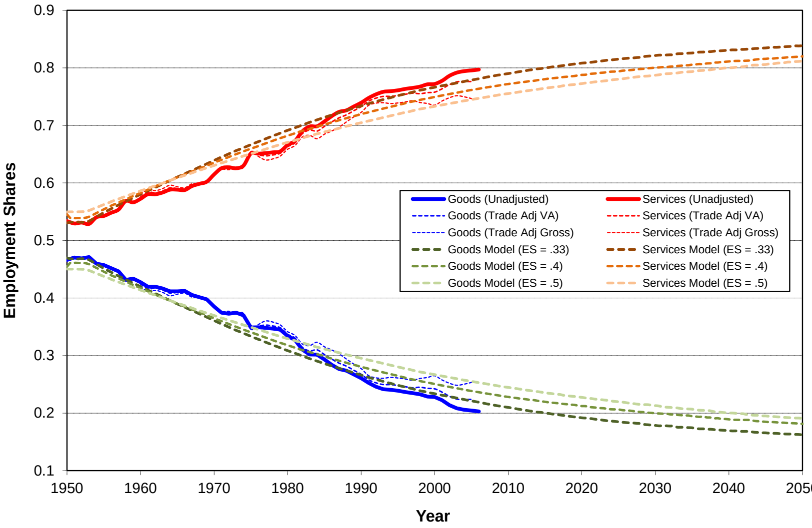
Calibration II

- Technology diffusion parameter based on Comin et al. (2012)
- Innovation parameters
  - Fixed cost parameter ψ_1, so no innovation in services in 1950
  - Variable cost parameter ψ_2 and the shape parameter of the Pareto distribution *a* chosen to match average productivity growth in manufacturing and services in the period 1980 to 2005
  - To ensure unique equilibrium, set $\gamma = 0.4$, so that $\mu_i + \gamma = 1$
- Divide the unit interval into 500 'counties'
  - Local shocks are still noticeable in the aggregate
  - Focus on the average outcome of 100 realizations

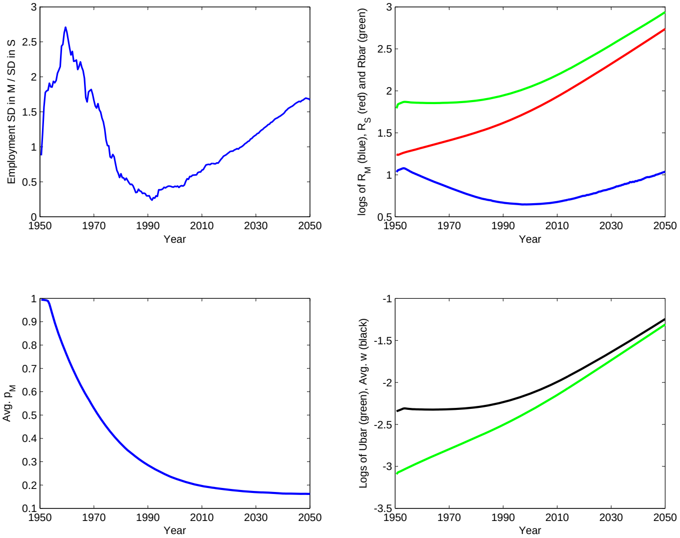
Basic Calibration: Aggregate Productivity



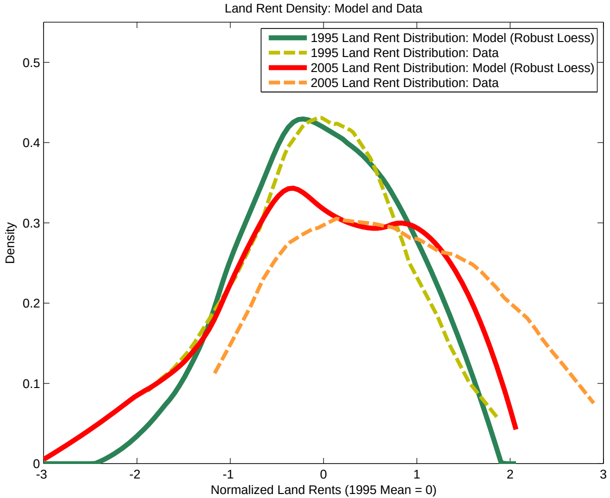
Basic Calibration: Employment Shares



Basic Calibration: Concentration, Rents, Prices and Utility



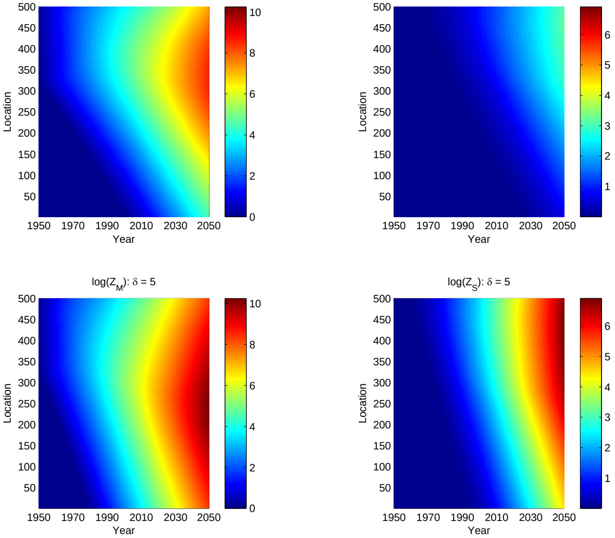
Basic Calibration: Distribution of Land Rents



Basic Calibration: Discussion

- Acceleration of innovation in services starting in the 1990s
  - Elasticity of substitution less than one
  - Share of employment in services increases
  - Endogenous takeoff of services
- The shift of workers moving into services slows starting in the 1990s
  - This happens once innovation in services takes off
- Because of trade costs, services and manufacturing collocate
  - Increasing concentration of services
- Once services start innovating
  - Wage growth accelerates
  - Acceleration in land rents
  - Greater dispersion in land rents

Diffusion of Technology I



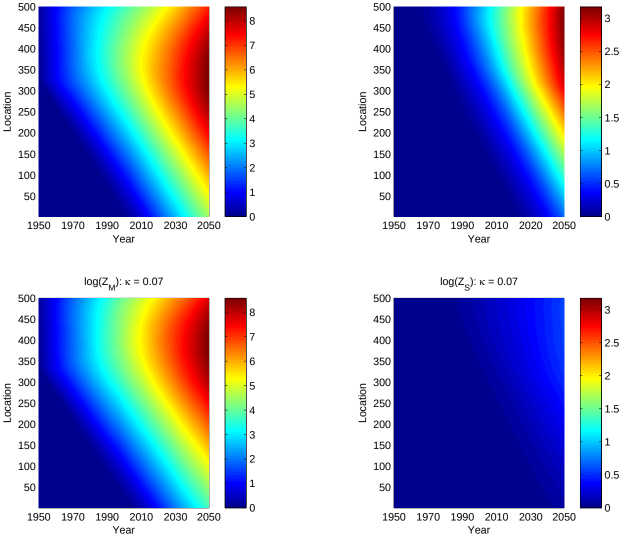
Diffusion of Technology II

- Benchmark case
  - Initially, high-productivity manufacturing located at upper border
  - Once services start innovating, they collocate
  - Service cluster emerges next to manufacturing cluster
  - Goods cluster moves down
- Stronger spatial diffusion
  - Service industry takes off earlier
  - Goods cluster moves down faster

Transport Costs I

- Transport costs have standard negative effect on static welfare
  - Goods are lost in transportation
- But higher transport costs also imply that it is more important to produce in areas close to locations where the other sector is producing
  - Leads to more agglomeration if one sector is already clustered
  - More incentives to innovate and earlier take-off of services
  - In contrast to standard economic geography models, where higher transport costs lead to more dispersion
- 'Second best' world in which frictions can be welfare enhancing
- Next graph: top is benchmark; bottom is lower transport costs

Transport Costs II



Conclusions I

- We have proposed a spatial dynamic growth model
- To make the model tractable and solvable
  - Labor is mobile
  - Innovation shocks are spatially correlated
  - Innovation diffuses across space
- Perfectly competitive environment in which firms' decisions are static
- We have illustrated potential of our theory by applying it to the evolution of the U.S. economy in the last half-century
  - Quantitatively match the main spatial and macro stylized facts

Conclusions II

- Desmet and Rossi-Hansberg (2015) apply this framework to quantitatively analyze the spatial economic impact of global warming
  - Integrated assessment model
  - Combines insights from climate change with spatial growth model
- Two-sector one-dimensional framework is appropriate
  - Global warming has differential impact across different latitudes
  - Global warming has differential impact across sectors
- Importance of mobility as a way of adapting to global warming
  - Some locations gain and others lose
  - Changing trade patterns
  - Migration

The Geography of Development

- Where a person lives determines his productivity, income and well-being
- But a person's location is neither a permanent characteristic nor a free choice
  - How do migratory restrictions shape the economy of the future?
  - How do they interact and affect the spatial distribution of productivity and amenities?
- We propose a theory of development that explicitly takes into account
  - The geography of economic activity
  - The mobility restrictions and transport costs associated with it

A Theory of the Geography of Development

- Each location is unique in terms of its
  - Amenities
  - Productivity
  - Geography
- Each location has firms that
  - Produce and trade subject to transport costs
  - Innovate
- Static part of model
  - Allen and Arkolakis (2013) and Eaton and Kortum (2002)
  - Allow for migration restrictions
- Dynamic part of model
  - Desmet and Rossi-Hansberg (2014)
  - Land competition and technological diffusion

Population Density and Income

- Model predicts that the correlation between population density and income per capita should increase with development
  - Dynamic agglomeration economies greater in attractive places
    - Attractive due to amenities, productivity, or geography
  - Mobility to those locations increase market size and, therefore, innovation
- Appears consistent with
  - Cross-section of $1^\circ \times 1^\circ$ cells for the whole world
  - Evidence from U.S. zip codes

Population Density and Income

Correlation between population density and real income per capita

- Across all cells of the world: -0.38
- Weighted average across cells within countries: 0.10
- Across richest and poorest cells of the world
  - 50% poorest cells: -0.02
  - 50% richest cells: 0.10
- Weighted average across richest and poorest cells within countries
  - 50% poorest cells: 0.14
  - 50% richest cells: 0.23
- Across cells of different regions
  - Africa: -0.04
  - Asia: 0.06
  - Latin America and Caribbean: 0.14
  - Europe: 0.15 (Western Europe: 0.20)
  - North America: 0.28
  - Australia and New Zealand: 0.48 (Oceania: -0.08)

Population Density and Income

- Evidence from U.S. zip codes

Correlation between Population Density and Per Capita Income (logs)*

Year	$<$ 25th	25-50th	50th-75th	$>$75th	$<$ Median	$\geq$ Median
2000	-0.1001***	0.0495***	0.1499***	0.2248***	-0.0609***	0.3589***
2007-2011	-0.0930***	0.0175	0.0733***	0.2420***	-0.0781***	0.3234***

*Percentiles based on per capita income

- Also holds across zip codes within CBSAs

Endowments and Preferences

- Economy occupies a two-dimensional surface S
  - Location is point $r \in S$
  - S is partitioned into C countries
- $\bar{L}$ agents each supplying one unit of labor
- An agent's period utility

$$u_t^i\left(\bar{r}_-, r\right) = a_t\left(r\right)\left[\int_0^1 c_t^\omega\left(r\right)^\rho d\omega\right]^{\frac{1}{\rho}} \varepsilon_t^i\left(r\right)\prod_{s=1}^{t} m\left(r_{s-1}, r_s\right)^{-1}$$

  - $\varepsilon_t^i\left(r\right)$ is a location preference shock that is iid Fréchet (Ω)
  - $m\left(r_{s-1}, r_s\right)$ is the cost of moving from r_{s-1} to r_s
  - amenities take the form

$$a_t\left(r\right) = \bar{a}\left(r\right)\bar{L}_t\left(r\right)^{-\lambda}$$

- Agents earn income from work and from local ownership of land

Equilibrium: Definition, Existence and Uniqueness

- Assumption 1: $m(r,s) = m_1(r)\, m_2(s)$ and $m(r,r) = 1$ for all $r, s \in S$
- Then an agent's value function can be written as

$$V\left(r_0, \bar{\varepsilon}_1^i\right) = \frac{1}{m_1(r_0)}\left[\max_{r_1} \frac{u_1(r_1)}{m_2(r_1)}\varepsilon_1^i(r_1) + \beta \mathbf{E}\left(\max_{r_2}\left[\frac{u_2(r_2)}{m_2(r_2)}\varepsilon_2^i(r_2) + V\left(r_2, \bar{\varepsilon}_3^i\right)\right]\right)\right]$$

where

$$u_t(r) = a_t(r)\left[\int_0^1 c_t^\omega(r)^\rho\, d\omega\right]^{\frac{1}{\rho}}$$

- Hence, current location only influences current utility and not future decision

Equilibrium: Definition, Existence and Uniqueness

- An agent's expected period-t utility including taste shocks is then given by

$$\mathbf{E}\left[u_t\left(r\right)\varepsilon_t^i\left(r\right)\right] = \Gamma\left(1-\Omega\right) m_2\left(r\right)\left[\int_S u_t\left(s\right)^{1/\Omega} m_2\left(s\right)^{-1/\Omega} ds\right]^{\Omega}$$

- The fraction of agents choosing to be at r in period t is

$$\frac{H\left(r\right)\overline{L}_t\left(r\right)}{\overline{L}} = \frac{u_t(r)^{1/\Omega} m_2\left(r\right)^{-1/\Omega}}{\int_S u_t(s)^{1/\Omega} m_2\left(s\right)^{-1/\Omega} ds}$$

Technology

- Production per unit of land of a firm producing good $\omega \in [0,1]$ is

$$q_t^{\omega}(r) = \phi_t^{\omega}(r)^{\gamma_1} z_t^{\omega}(r) L_t^{\omega}(r)^{\mu}$$

where $\phi_t^{\omega}(r)$ is an innovation requiring $\nu \phi_t^{\omega}(r)^{\xi}$ units of labor

  - ★ If $\gamma_1 < 1$, there are decreasing returns to local innovation

- $z_t^{\omega}(r)$ is the realization of a r.v. drawn from a Fréchet distribution

$$F(z,r) = e^{-T_t(r) z^{-\theta}}$$

where $T_t(r) = \tau_t(r) \overline{L}_t(r)^{\alpha}$ and

$$\tau_t(r) = \phi_{t-1}(r)^{\theta\gamma_1} \left[\int_S \eta_{t-1}(r,s) \tau_{t-1}(s) \, ds \right]^{1-\gamma_2} \tau_{t-1}(r)^{\gamma_2}$$

  - If $\gamma_2 < 1$, we get global diffusion of technology

Productivity Draws and Competition

- Firms face perfect local competition and innovate
  - Productivity draws are i.i.d. across time and goods, but correlated across space (with perfect correlation as distance goes to zero)
  - Firm profits are linear in land, so for any small interval there is a continuum of firms that compete in prices
  - Firms bid for land up to point of making zero profits after covering investment in technology
- Dynamic profit maximization simplifies to sequence of static problems
  - Next period all potential entrants have access to same technology (Desmet and Rossi-Hansberg, 2014)

$$\max_{L_t^\omega(r),\phi_t^\omega(r)} p_t^\omega(r,r)\,\phi_t^\omega(r)^{\gamma_1} z_t^\omega(r)\,L_t^\omega(r)^\mu - w_t(r)\,L_t^\omega(r) - w_t(r)\,\nu\phi_t^\omega(r)^\xi - R_t(r)$$

- **Lemma 1: In any $r \in S$, $L_t^\omega(r)$ and $\phi_t^\omega(r)$ are identical across goods ω**

Prices, Export Shares and Trade Balance

- Price of good produced at r and sold at r

$$p_t^{\omega}(r,r) = mc_t(r) / z_t^{\omega}(r)$$

  - From the point of view of the individual firm the input cost is given
  - Productivity draws affect prices without changing the input cost

- Probability that good produced in r is bought in s

$$\pi_t(s,r) = \frac{T_t(r)\left[mc_t(r)\varsigma(r,s)\right]^{-\theta}}{\int_S T_t(u)\left[mc_t(u)\varsigma(u,s)\right]^{-\theta} du} \text{ all } r,s \in S$$

- Trade balance location by location

$$w_t(r) H(r) \bar{L}_t(r) = \int_S \pi_t(s,r) w_t(s) H(s) \bar{L}_t(s) ds \text{ all } r \in S$$

Equilibrium: Definition, Existence and Uniqueness

- Standard definition of dynamic competitive equilibrium
- Equilibrium implies

$$\left[\frac{\bar{a}(r)}{u_t(r)}\right]^{-\frac{\theta(1+\theta)}{1+2\theta}} \tau_t(r)^{-\frac{\theta}{1+2\theta}} H(r)^{\frac{\theta}{1+2\theta}} \bar{L}_t(r)^{\lambda\theta-\frac{\theta}{1+2\theta}\chi}$$

$$= \kappa_1 \int_S \left[\frac{\bar{a}(s)}{u_t(s)}\right]^{\frac{\theta^2}{1+2\theta}} \tau_t(s)^{\frac{1+\theta}{1+2\theta}} H(s)^{\frac{\theta}{1+2\theta}} \varsigma(r,s)^{-\theta} \bar{L}_t(s)^{1-\lambda\theta+\frac{1+\theta}{1+2\theta}\chi} ds$$

where $\chi = \left[\alpha - 1 + \left[\lambda + \frac{\gamma_1}{\xi} - [1-\mu]\right]\theta\right]$

- **Lemma 3: An equilibrium exists and is unique if**

$$\frac{\alpha}{\theta} + \frac{\gamma_1}{\xi} < \lambda + 1 - \mu + \Omega$$

  - Iterative procedure converges to unique equilibrium
  - Weaker condition guarantees that model can be solved backward

Balanced Growth Path

- In a balanced growth path (BGP) the spatial distribution of employment is constant and all locations grow at the same rate
- **Lemma 4: There exists a unique BGP if**

$$\frac{\alpha}{\theta} + \frac{\gamma_1}{\xi} + \frac{\gamma_1}{[1-\gamma_2]\,\xi} \leq \lambda + 1 - \mu + \Omega$$

  - This condition is stronger than the condition for uniqueness and existence of the equilibrium

- In a BGP aggregate welfare and real consumption grow according to

$$\frac{u_{t+1}(r)}{u_t(r)} = \left[\frac{\int_0^1 c_{t+1}^{\omega}(r)^{\rho}\, d\omega}{\int_0^1 c_t^{\omega}(r)^{\rho}\, d\omega}\right]^{\frac{1}{\rho}} \propto \left[\int_S \overline{L}(s)^{\frac{\theta\gamma_1}{[1-\gamma_2]\xi}}\, ds\right]^{\frac{1-\gamma_2}{\theta}}$$

  - Growth depends on population size and its distribution in space

Calibration: Summary

1. Preferences	
$\rho = 0.75$	Elasticity of substitution of 4 (Bernard et al., 2003)
$\lambda = 0.32$	Relation between amenities and population
$\Omega = 0.5$	Elasticity of migration flows with respect to income (Monte et al., 2015)
2. Technology	
$\alpha = 0.06$	Elasticity of productivity to density (Carlino et al., 2007)
$\theta = 6.5$	Trade elasticity (Simonovska and Waugh, 2014)
$\mu = 0.8$	Labor or non-land share in production (Greenwood et al., 1997; Desmet and Rappaport, 2014)
$\gamma_1 = 0.319$	Relation between population distribution and growth
3. Evolution of productivity	
$\gamma_2 = 0.993$	Relation between population distribution and growth
$\xi = 125$	Desmet and Rossi-Hansberg (2014a)
$\nu = 0.15$	Initial world growth rate of real GDP of 2%
4. Trade Costs	
	Allen and Arkolakis (2014) and Fast Marching algorithm
$\Upsilon = 0.393$	Elasticity of trade to distance of -0.93 (Head and Mayer, 2014)

Calibration: Amenity and Technology Parameters

- Amenity parameter λ:

$$\log(a(r)) = E(\log(\bar{a}(r))) - \lambda \log \bar{L}(r) + \varepsilon(r)$$

  - Estimate using data on amenities and population for 192 U.S. MSAs
  - Instrument for $\bar{L}$ using productivity

- Technology parameters γ_1 and γ_2
  - Use cell level population data from G-Econ to estimate BGP relation

$$\log y_{t+1}(c) - \log y_t(c) = \alpha_1 + \alpha_2 \log \sum_{S_c} L_c(s)^{\alpha_3}$$

    where α_1, α_2 and α_3 are functions of γ_1 and γ_2
  - BGP relation is used as simplification
  - Technology parameters are consistent with 2% average growth rate in real GDP per capita today

Simulation: Amenities and Productivity

- Use data on land, population and wages from G-Econ 4.0 to derive spatial distribution of $\overline{a}(r)/u_0(r)$ and $\tau_0(r)$ by inverting the model
- Lemma 6: inversion yields a unique set of $\overline{a}(r)/u_0(r)$ and $\tau_0(r)$
- The inversion does not separately identify $\overline{a}(r)$ and $u_0(r)$
  - Not a problem in models with free mobility (Roback, 1982)
  - Not reasonable here
    - Congo would have very attractive amenities
- We need additional data on utility: subjective wellbeing
  - Correlates well with log of income (Kahneman and Deaton, 2010)
  - Once we have $u_0(r)$, amenities identified as $\overline{a}(r) = \frac{\overline{a}(r)}{u_0(r)} u_0(r)$

Subjective Well-Being

- Data on subjective well-being from the Gallup World Poll
  - Cantril ladder from 0 to 10 Map subjective well-being
    - 0 is worst possible life and 10 is best possible life
  - Linear relation between subjective well-being and the log of real income
    - Within and across countries (Deaton and Stone, 2013)
- In the model: $u^i(r) = a(r)\, y(r)\, \varepsilon^i(r)$ absent moving costs
- Deaton and Stone (2013): $\tilde{u}^i(r) = \frac{1}{\psi} \ln y^i(r) + v(r) + \varepsilon^i_{DS}(r)$
- Hence, relation between utility in model and subjective well-being is

$$u^i(r) = e^{\psi \tilde{u}^i(r)}$$

  - Deaton and Stone (2013) find $\psi = 1.8$

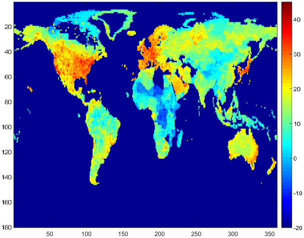
Moving Costs and Counterfactuals

- Use data on population distribution in two consecutive years to identify moving costs
  - Lemma 7: given $\overline{L}_0(r)$ and $\overline{L}_1(r)$, moving costs can be uniquely identified up to a constant
  - Set constant so that $\min m_2(r) = 1$
- Once we have values for $m_2(r)$, simulate model forward using moving costs

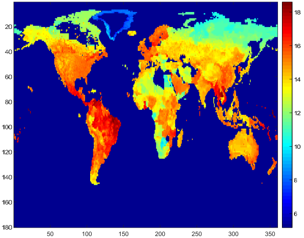
$$\widetilde{m}_2(r) = m_2(r)^{\vartheta}$$

- Counterfactual migration scenarios
  - Keep moving costs unchanged ($\vartheta = 1$)
  - Eliminate moving costs ($\vartheta = 0$)
  - Partial mobility (ϑ between 0 and 1)
    - ⋆ Keeps ranking of moving costs unchanged

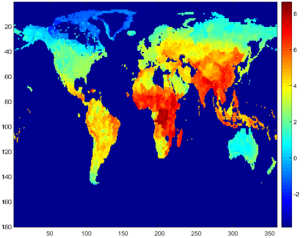
Results from Inversion and Moving Costs



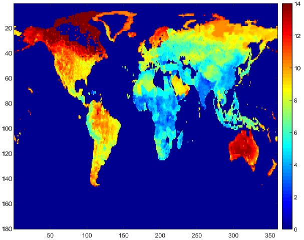
a. Fundamental Productivities: $\tau_0(r)$



b. Fundamental Amenities: $\overline{a}(r)$



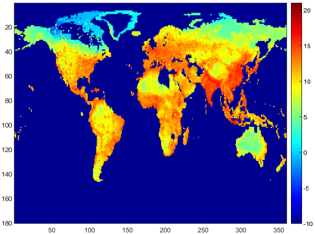
c. Amenities Over Utility: $\overline{a}(r)/u_0(r)$



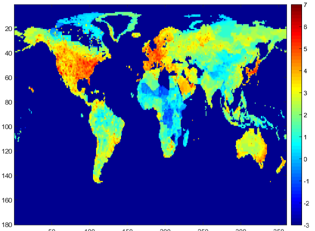
d. Moving Costs: $m_2(r)$

Correlation amenities

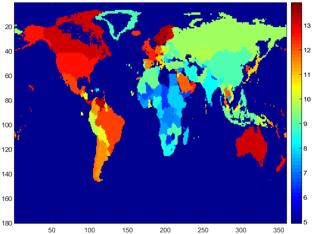
Benchmark Calibration: Period 1



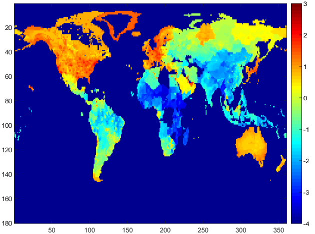
a. Population Density



b. Productivity: $\left[\tau_t\left(r\right)\overline{L}_t\left(r\right)^{\alpha}\right]^{\frac{1}{\theta}}$

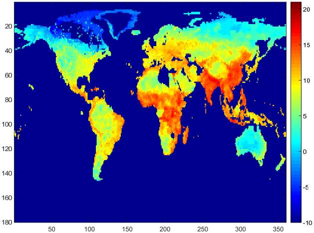


c. Utility

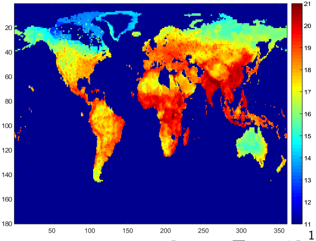


d. Real Income per Capita

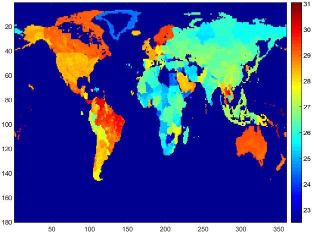
Keeping Migratory Restrictions Unchanged: Period 600



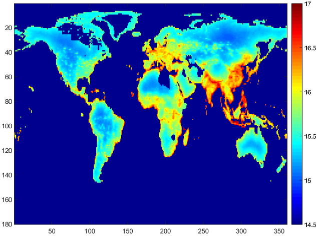
a. Population Density



b. Productivity: $\left[\tau_t\left(r\right)\overline{L}_t\left(r\right)^{\alpha}\right]^{\frac{1}{\theta}}$

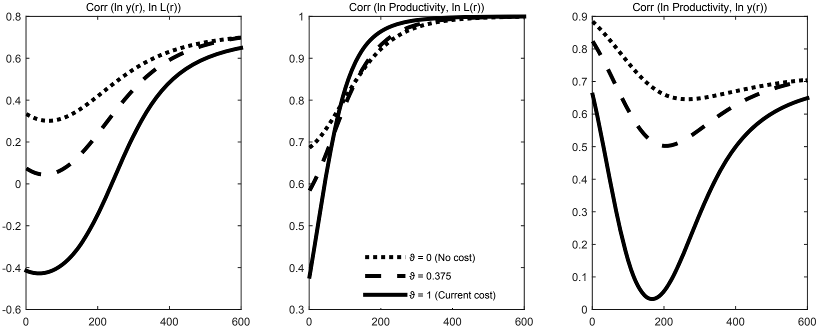


c. Utility



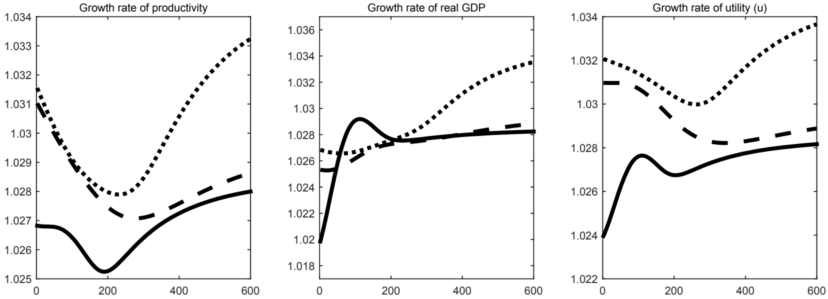
d. Real Income per Capita

Correlations under Different Scenarios

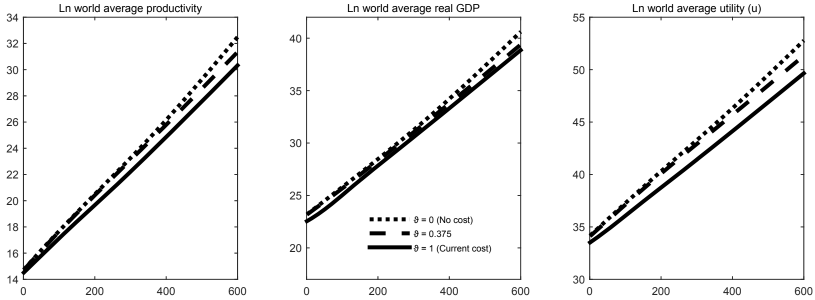


▸ Empirical correlation density and income

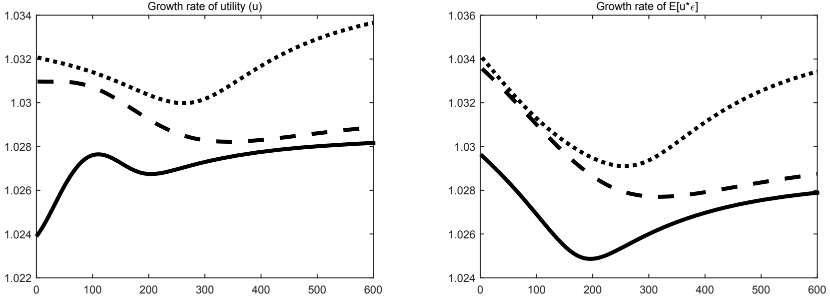
Growth Rates under Different Scenarios



Levels under Different Scenarios



Growth Rates and Levels: Different Measures of Utility

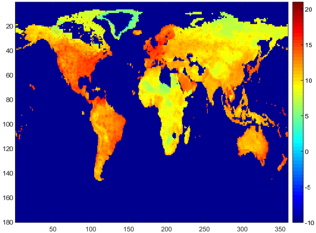


Backcasting

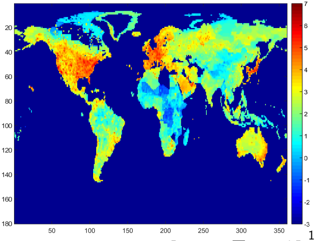
- Using the calibrated model for the year 2000 we can use the dynamics of the model to backcast the past
- We solve the model backwards
  - We show that there is a unique sequence of past allocations consistent with today's allocation
  - Simple iterative algorithm is guaranteed to converge under the assumptions of Lemma 3 or 4
- Compare model's implications for past population across countries with the data
  - Overidentification test since no past data is used

Correlations Model vs Data	Penn World Tables 8.1					Maddison	
Year t	1990	1980	1970	1960	1950	1913	1870
Corr. log population t	0.993	0.991	0.982	0.974	0.965	0.842	0.681
Corr. pop. $\%\Delta$ from t to 2000	0.414	0.535	0.504	0.671	0.742	0.462	0.344
Number of countries	152	131	131	102	53	76	76

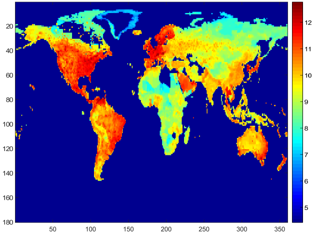
Free Mobility: Period 1



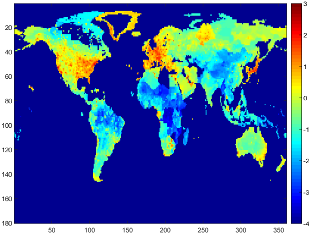
a. Population Density



b. Productivity: $\left[\tau_t\left(r\right)\overline{L}_t\left(r\right)^{\alpha}\right]^{\frac{1}{\theta}}$

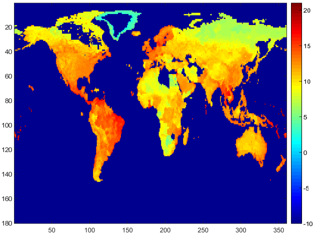


c. Utility

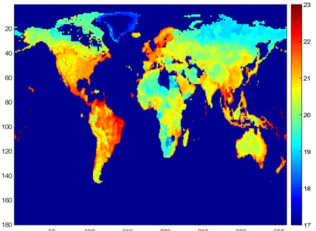


d. Real Income per Capita

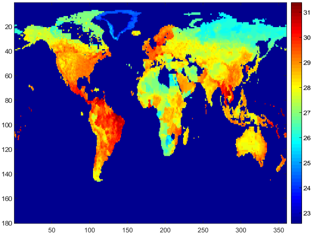
Free Mobility: Period 600



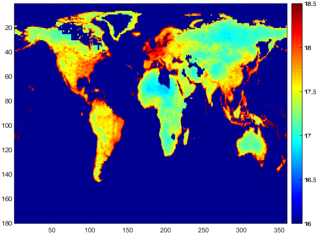
a. Population Density



b. Productivity: $\left[\tau_t\left(r\right)\overline{L}_t\left(r\right)^{\alpha}\right]^{\frac{1}{\theta}}$



c. Utility



d. Real Income per Capita

Welfare and Migratory Restrictions

Mobility	Discounted Real Income*	Discounted Utility**	Migration Flows***
ϑ	$\%\Delta$ w.r.t. $\vartheta = 0$	$\%\Delta$ w.r.t. $\vartheta = 0$	
1^a	0%	0%	0.30%
0.75	30.6%	59.8%	21.2%
0.5	69.2%	144.3%	43.2%
0.25	101.6%	228.8%	60.2%
0^b	125.8%	305.9%	70.3%

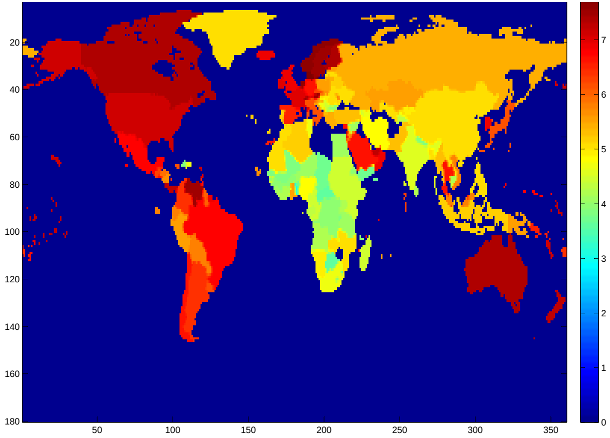
We use $\beta = 0.965$. a: Current Moving Costs. b: No Costs. *: Population-weighted average of cells' real GDP. **: Population-weighted average of cells' utility levels. ***: Share of world population moving to countries that grow between period 0 and 1 (immediately after the change in ϑ).

Conclusion

- Interaction between geography and economic development through trade, technology diffusion and migration
- Connect to real geography of the world at fine detail
- Relaxing migration restrictions can lead to very large welfare gains
- Level of migration restrictions will have important effect on which regions of the world will be the productivity leaders of the future
  - Correlation between density and productivity increases over time

Map Subjective Well-Being

Subjective Well-being from the Gallup World Poll (Max = 10, Min = 0)



▸ Return

Correlation Amenities

	Correlations with Estimated Amenities (logs)				
	(1) All cells	(2) U.S.	(3) One cell per country	(4) Placebo of (1)	(5) Placebo of (3)
A. Water					
Water $<$ 50 km	0.2198***	0.1286***	0.1232**	0.1064***	-0.1363**
B. Elevation					
Level	-0.4152***	-0.1493***	-0.2816***	-0.2793***	0.1283**
Standard deviation	-0.4599***	-0.2573***	-0.3099***	-0.3285***	0.1121*
C. Precipation					
Average	0.4176***	0.08643***	0.3851***	0.3185***	0.1830***
Maximum	0.4408***	0.1068***	0.3128***	0.4286***	0.3200***
Minimum	0.2035***	0.2136***	0.2108***	-0.0096	-0.1965**
Standard deviation	0.4160***	0.0212	0.2746***	0.4715***	0.4535***
D. Temperature					
Average	0.6241***	0.6928***	0.3087***	0.6914***	0.5692***
Maximum	0.5447***	0.7388***	0.1276***	0.6589***	0.4635***
Minimum	0.6128***	0.6060***	0.2931***	0.6565***	0.5389***
Standard deviation	-0.5587***	-0.3112***	-0.3313***	-0.5539***	-0.3679***
E. Vegetation					
Desert, ice or tundra	-0.3201***	-0.3993***	-0.1827***	-0.2440***	-0.1291*

- Correlations using all cells, U.S. cells, or one cell per country are similar (see 1, 2 and 3)
  - Also consistent with Albouy et al. (2014) and Morris & Ortalo-Magné (2007)
- Placebo correlations under free mobility are not (see 5)

Return