

# Lecture 5: Matching and Trade

Economics 552

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# Offshoring in a Knowledge Economy, Antras, Garicano and Rossi-Hansberg (QJE, 2006)

- Globalization leads to the formation of international teams
- Study the impact of cross-country team formation on
  - ▶ the organization of work
  - ▶ the structure of earnings
- Key Idea: Agents can match with different agents than otherwise. This leads to new teams, new matching and affects
  - ▶ occupational choices
  - ▶ size distribution of teams (firms)
  - ▶ wages and rents

# The Framework

- Production requires physical inputs and knowledge
- Agents of heterogeneous ability sort into teams competitively
- Agents specialize in either production work (workers) or knowledge work (managers)
- Complementarity between worker and manager skills
  - ▶ Skilled managers increase the productivity of their team
  - ▶ Skilled workers save manager's time allowing them to manage larger teams and leverage their skills
  - ▶ Results in positive sorting or assortative matching
- Natural interpretation of communication technology:
  - ▶ Represented by the time managers spend communicating knowledge to workers.
- We can then study the wage schedule in equilibrium and characterize the universe of firms in the economy
  - ▶ Wages serve an allocative role: sustain the universe of firms
  - ▶ Units of skill are priced differently

# Related Papers

- Pattern of trade and matching:
  - ▶ Grossman and Maggi (2000)
  - ▶ Kremer and Maskin (2003)
- Offshoring and matching
  - ▶ Nocke and Yeaple (2006)
- Equilibrium analysis in a knowledge economy:
  - ▶ Garicano and Rossi-Hansberg (2003 and 2004)

# Globalization

- Study the effect of globalization by comparing equilibrium in the closed economy and the integrated economy
- Analyze two countries that differ only in their ability distributions
- Globalization leads to:
  - ▶ Production job creation, Knowledge job destruction, and smaller firms in the South
  - ▶ Production job destruction and Knowledge job creation in the North, and smaller dispersion of firm sizes
  - ▶ Trade deficit in manufacturing in North; trade surplus in services
- Leads to higher wage inequality in South (as in empirical literature)
- Leads to higher wage inequality in North if offshoring partner relatively unskilled or communication costs are low (Feenstra and Hanson (1996b and 1999))
- Globalization always hurts some workers
  - ▶ If communication costs low, it hurts low-skilled workers in North
  - ▶ If communication costs high, it hurts low skilled workers in South
- We show southern workers in multinationals earn relatively more than other southern workers (Aitken et. al. (1996))

# The Model

- Production requires time and knowledge
- Agents are endowed with 1 unit of time and knowledge level  $z$
- The distribution of skills in the population is given by a cumulative distribution function  $G(z)$ , with support in  $[0, \bar{z}]$  for  $\bar{z} \leq 1$
- Agents order consumption according to a linear utility

$$\max \{R(z), w(z)\}$$

- Workers face problems while producing. If they know the solution, production occurs and one unit of a numeraire good is produced
- There is a continuum of possible problems indexed in  $[0, 1]$
- An agent with skill  $z$  can (has the require knowledge to) solve all problems between 0 and  $z$ 
  - ▶  $z$  denotes the proportion of problems a particular worker can solve

# The Model

- Knowledge can be communicated
  - ▶ Agents **can ask** about problems when they do not know the solution
- This allows an agent to combine his/her own knowledge with the time of other agents
- There is a cost  $h$  of communicating knowledge:
  - ▶ Cost incurred whether answer is known or not
  - ▶ Average cost of helping a worker with knowledge  $z_p$  is thus

$$(1 - z_p)h$$

- Agents form hierarchical teams:
  - ▶ Some workers specialize in problem solving (managers) and others in production (workers)
- Output of a two-layer team with  $n$  production workers with knowledge  $z_p$  and one problem solver with knowledge  $z_m$  is

$$y = z_m n$$

- Subject to the helping time constraint of managers

$$h(1 - z_p)n = 1$$

# The Model: Production Function

- The latter constraint pins down team size

$$n(z_p) = \frac{1}{h(1 - z_p)}$$

- Output is thus

$$y = \frac{z_m}{h(1 - z_p)}$$

- Complementarity  $\rightarrow$  Positive assortative matching

# The Model: Production Function

- 1 Assignment of agents to positions – production workers, managers
  - 2 Matching of production workers to managers (matching function)
  - 3 Earnings function such that no agent has an incentive to deviate
- An equilibrium is such that agents with skills in  $[0, z^*]$  become workers and agents with skills in  $[z^*, \bar{z}]$  become managers

# The Model: Wage Function

- The problem of a manager with ability  $z_m$  is to choose the ability of her workers to maximize

$$R(z_m) = \max_{z_p} (z_m - w(z_p)) n(z_p) = \max_{z_p} \frac{z_m - w(z_p)}{h(1 - z_p)}$$

- The first order condition of this problem implies

$$w'(z_p) = \frac{z_m - w(z_p)}{1 - z_p} \quad (1)$$

- If this condition was not satisfied, managers would choose to form different teams

# The Model: Assignment

- Let  $m(z)$  denote the ability of the manager that hires worker  $z$ ; positive sorting implies  $m'(z) > 0$
- Equilibrium in the labor market then implies that  $m(z^*) = \bar{z}$  and

$$\underbrace{\int_0^{z_p} g(z) dz}_{\text{Supply of Workers}} = \underbrace{\int_{m(0)}^{m(z_p)} n(m^{-1}(z)) g(z) dz}_{\text{Demand of Workers}} \quad \text{all } z_p \leq z^*$$

- Hence, in equilibrium  $m(z^*) = \bar{z}$ ,  $m(0) = z^*$  and

$$m'(z) = h(1-z) \frac{g(z)}{g(m(z))} \quad (2)$$

- One differential equation, two boundary conditions  $\rightarrow$  obtain  $m(z)$  and  $z^*$

# The Model: Occupational Choice

- Agents order consumption according to a linear utility,

$$\max \{R(z), w(z)\}$$

- This condition implies

$$R(z^*) = w(z^*)$$

which together with (1) pins down  $w(z)$

- In order to guarantee that managers maximize rents we also need to guarantee that  $w'(z^*) < R'(z^*)$

# Existence and Uniqueness

## Theorem

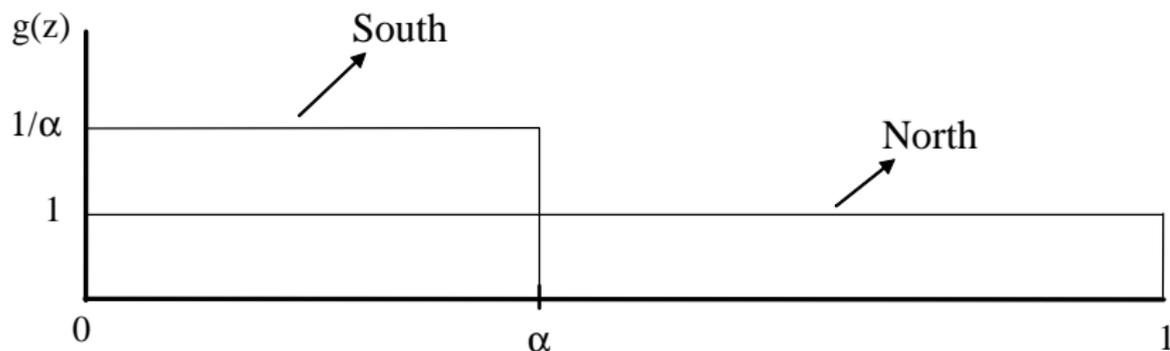
*There exists a threshold  $h^* > 0$  such that if  $h \in [0, h^*]$  there exists a unique competitive equilibrium of this economy. In equilibrium the set of managers and the set of workers are connected and the equilibrium exhibits positive sorting. Furthermore, the equilibrium allocation is efficient.*

- In what follows let  $h < 0.85$  so we can guarantee existence

# Equilibrium in the Closed Economy

Two countries, North and South, identical except for

- $G_N(z) = z$  for  $z \in [0, 1]$ , with density  $g_N(z) = 1$
- $G_S(z) = z/\alpha$  for  $z \in [0, \alpha]$ , and density  $g_S(z) = 1/\alpha$



# Equilibrium in the Closed Economy

Two countries, North and South, identical except for  $G$

- Matching:

$$m_S(z) = z_S^* + hz \left(1 - \frac{1}{2}z\right)$$
$$z_S^* = \frac{1 + h - (1 + h^2 + 2h(1 - \alpha))^{\frac{1}{2}}}{h}$$

- ▶  $z_S^*$  is decreasing in  $h$  and increasing in  $\alpha$

- Wages:

$$w_S(z) = z_S^* - \sigma_S(1 - z) + \frac{1}{2}hz^2$$
$$\sigma_S = \frac{hz_S^* \left(1 + \frac{1}{2}hz_S^*\right)}{1 + h - hz_S^*}$$

- ▶  $w'_S(z) = \sigma_S + hz : \begin{cases} \text{Competition effect in } \sigma \\ \text{Complementarity effect in } hz \end{cases}$

# Equilibrium in the Closed Economy

- Rents:

$$R_S(z) = \frac{z - w_S \left( m_S^{-1}(z) \right)}{h \left( 1 - m_S^{-1}(z) \right)}, \quad R'_S(z) = \frac{1}{h \left( 1 - m_S^{-1}(z) \right)}$$

- Production:

$$Y_S = \int_0^{z_S^*} m_S(z) g_S(z) dz = \frac{1}{6\alpha} z_S^{*2} (6 + 3h - h z_S^*)$$

- ▶ Decreases with  $h$  and increases with  $\alpha$

# Equilibrium in the World Economy

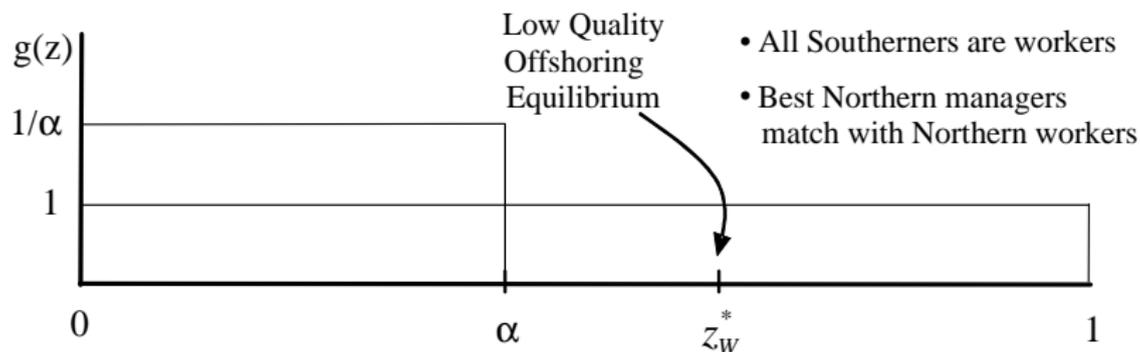
- The distribution of skills in the world population is given by the sum of the distribution of skills in the South and in the North, namely,

$$G(z) = \begin{cases} \frac{1+\alpha}{\alpha} z & \text{if } 0 < z < \alpha \\ z & \text{if } \alpha < z < 1 \end{cases}$$

- Depending on the value of  $h$  and  $\alpha$  there are two types of equilibria in the world

# Equilibrium in the World Economy

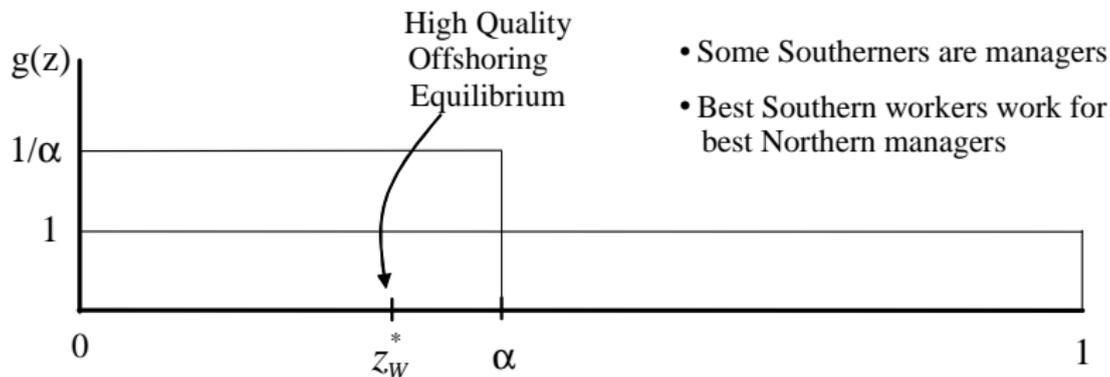
- *Low Quality Offshoring Equilibrium (LQE):*  $z_W^* > \alpha$



- International teams are associated with the worst managers in the world and therefore with the smallest and least productive firms

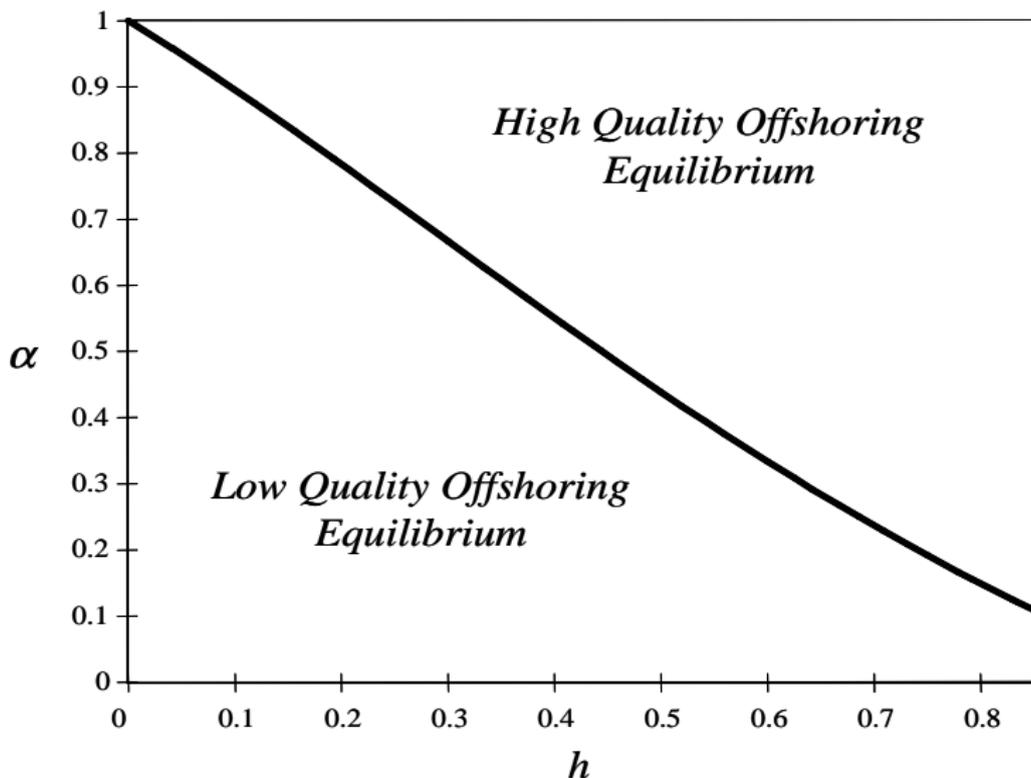
# Equilibrium in the World Economy

- *High Quality Offshoring Equilibrium (HQE):*  $z_W^* < \alpha$

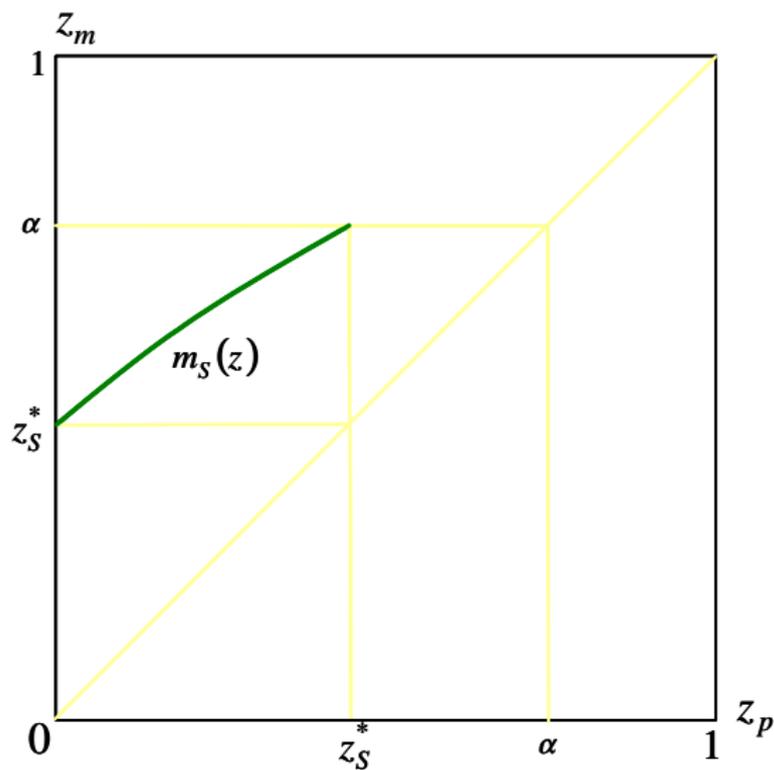


- Hence, international teams are associated with the best managers in the world and therefore with the largest and most productive firms

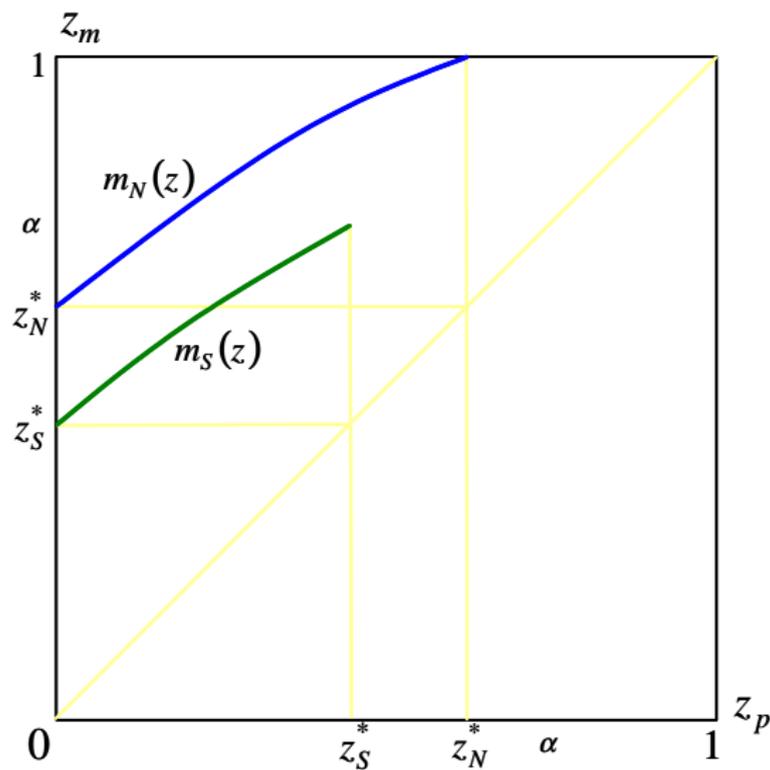
# Equilibrium in the World Economy



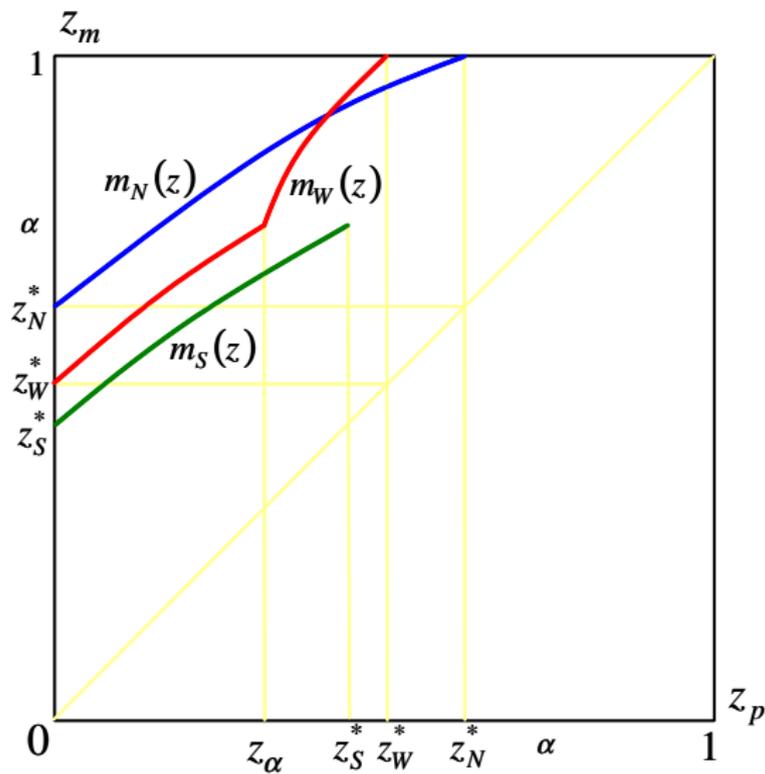
# Effects of Globalization: Matching



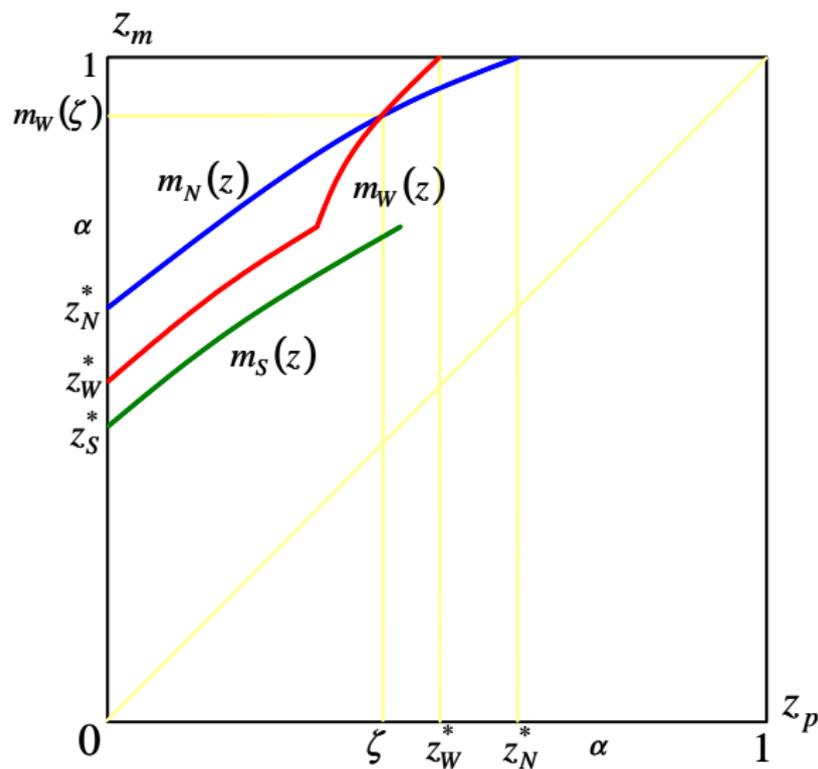
# Effects of Globalization: Matching



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# Effects of Globalization: Matching

## Proposition 1 (Effects on team formation):

- (i) *The mass of southern workers and the mass of northern managers both increase, i.e.,  $z_S^* < z_W^* < z_N^*$*
- (ii)
  - ① *Southern workers improve their match*
  - ② *Southern managers worsen their match*
- (iii)
  - ① *Northern workers with  $z < \zeta$  worsen their match; those with  $z > \zeta$  improve it*
  - ② *Northern managers with  $z < m_W(\zeta)$  improve their match; those with  $z > m_W(\zeta)$  worsen it*

# Effects of Globalization: Organization

- If firms are identified by the managers than run them, a corollary is that:

**Corollary 1:** *Globalization leads to production job creation and firm destruction in the South, and to production job destruction and firm creation in the North. Furthermore, it compresses the size distribution of firms in both countries and reduces the size of all surviving southern firms*

- Positive sorting implies that relatively more skilled workers in the South are matched with relatively better managers, who are in the North

**Corollary 2:** *The best workers in the South work for northern managers and receive higher wages than southern workers that are employed by southern managers*

# Effects of Globalization: Wages

- Our measure of within-worker inequality is  $w(z^*) - w(0) = \int_0^{z^*} w'(z) dz$
- We decompose this in three effects:
  - 1 *Occupational Choice Effect*: Changes in the skill set of agents that become workers
  - 2 *Competition Effect*: Changes in the baseline price of a unit of skill, i.e.,  $w'(0)$
  - 3 *Complementarity Effect*: Changes in the reward for agents that possess a certain number of units of skills
    - ▶ Because of the complementarity effect not all units of skill command the same compensation

# Effects of Globalization: South Wage Inequality

- Globalization increases wage inequality in the South:
  - ▶ ( $\uparrow$ ) Workers skills are relatively more scarce in the world economy than in the South: Competition effect ( $\sigma_{WL} > \sigma_S$  and  $\sigma_{WH} > \sigma_S$ )
  - ▶ ( $\uparrow$ ) More workers competing for the same manager so possessing more skills improves the assignment more: Complementarity effect
  - ▶ ( $\uparrow$ ) More agents become workers: Occupational choice effect

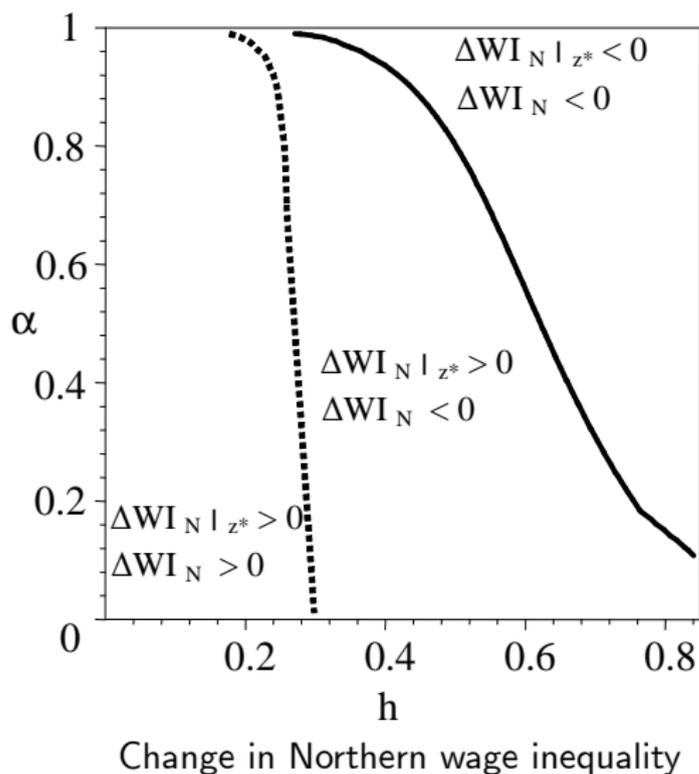
**Proposition 2:** *Globalization increases within-worker wage inequality in the South. Furthermore, it increases the marginal return to skill for southern workers at all skill levels*

# Effects of Globalization: North Wage Inequality

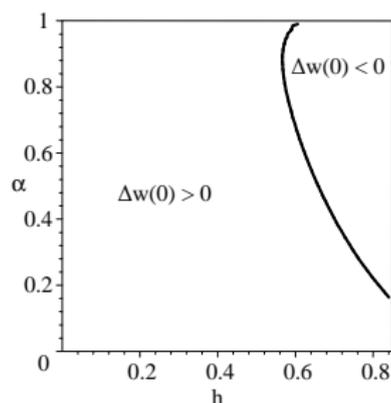
- Globalization increases wage inequality if  $h$  and  $\alpha$  low
  - ▶ ( $\downarrow$ ) Workers skills are relatively less scarce in the world than in the North: Competition effect
  - ▶ ( $\uparrow$ ) More workers competing for the same manager: Complementarity effect
  - ▶ ( $\downarrow$ ) Less agents become workers: Occupational choice effect

**Summary 1:** *Globalization increases within-worker wage inequality in the North if  $h$  and  $\alpha$  are sufficiently low, but it decreases it if  $h$  and  $\alpha$  are sufficiently high.*

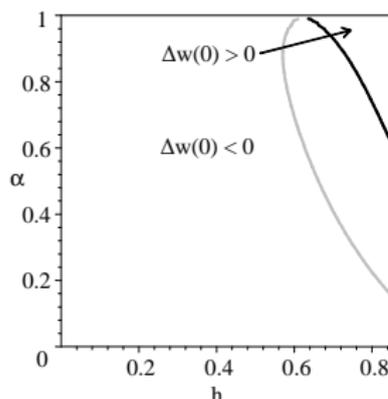
# Effects of Globalization: North Wage Inequality



# Effects of Globalization: Winners and Losers



$w(0)$  in the South



$w(0)$  in the North

**Summary 2:** Globalization has the following effects on wages:

- (i) Increases the wages of low skilled southern workers for low  $h$  and  $\alpha$ , but decreases them for high  $h$  and  $\alpha$
- (ii) Decreases the wages of low skilled northern workers for low  $h$  and  $\alpha$ , but increases them for high  $h$  and  $\alpha$
- (iii) It decreases the wage of at least some low skilled agents

# Effects of Globalization: Aggregates

**Corollary 4:** *Globalization increases total production in the world economy. That is, there are gains from trade*

**Summary 3:** *Globalization has the following effects on manufacturing production, consumption, and the trade balance:*

- (i) *It increases manufacturing production in the South and decreases manufacturing production in the North*
- (ii) *It increases consumption (and thus welfare) in both countries*
- (iii) *If knowledge transactions are not reported, globalization generates a trade surplus in the South and a trade deficit in the North*

# A Fall in Communication Costs

- As communication costs decrease, managers can deal with larger teams
- The measure of knowledge jobs decreases; the measure of production jobs increases
- The least productive firms disappear. All surviving firms become larger in size
- *Quantity of offshoring*, measured as the proportion of southern workers that work for international team, **increases**
- *Quality of offshoring*, measured as the average skill level of the workers that form international teams relative to the skill level of all southern workers, **decreases**

# An Increase in the Skill Overlap

- As the skill overlap increases the mass of agents with high skill increases
- This again relaxes the time constraint of managers
  - ▶ In the world, there are more production jobs and less knowledge jobs
  - ▶ But in the South, production jobs actually (weakly) decrease (and knowledge jobs weakly increase)
- The quantity of international offshoring weakly decreases. But its quality weakly increases

# Diversity and Trade, Grossman and Maggi (AER, 2000)

- Large amount of trade between countries with similar aggregate factor endowments and similar technologies
- The existing theoretical explanations for this type of trade are based on IRS or imperfect competition.
  - ▶ These models predict IIT and share the implication that the pattern of trade is essentially arbitrary
  - ▶ They don't predict which country exports which products
- But it seems that some of this trade is “inter-industry” and that there are regularities in trade patterns:
  - ▶ Germany, Sweden, Japan have comparative advantage in large scale manufacturing
  - ▶ US, Italy have comparative advantage in activities requiring creative ideas and problem solving
- They explore the possibility that these regularities might be explained by:
  - ▶ countries differ in the distribution of talent (“diversity”)
  - ▶ diversity of talents may be “good” for problem-solving activities, while consistency of talents may be “good” for large scale manufacturing

# Diversity and Trade, Grossman and Maggi (AER, 2000)

TABLE 1—DIVERSITY IN LITERACY SKILLS  
PERSONS AGED 16–65, 1994–1995

	Prose			Document			Quantitative		
	$\frac{\sigma}{\mu}$	$\frac{\Phi_{0.05}}{\mu}$	$\frac{\Phi_{0.95}}{\mu}$	$\frac{\sigma}{\mu}$	$\frac{\Phi_{0.05}}{\mu}$	$\frac{\Phi_{0.95}}{\mu}$	$\frac{\sigma}{\mu}$	$\frac{\Phi_{0.05}}{\mu}$	$\frac{\Phi_{0.95}}{\mu}$
United States	0.25	0.50	1.34	0.26	0.47	1.37	0.25	0.50	1.37
United Kingdom	0.24	0.57	1.32	0.26	0.48	1.35	0.25	0.53	1.35
Canada	0.23	0.52	1.30	0.26	0.54	1.35	0.23	0.55	1.34
Sweden	0.17	0.71	1.27	0.18	0.72	1.27	0.18	0.71	1.28
Netherlands	0.16	0.72	1.23	0.17	0.71	1.24	0.17	0.70	1.25
Germany	0.16	0.72	1.27	0.16	0.73	1.27	0.15	0.74	1.25

*Notes:* Based on scales with a range of 0–500 points.  $\sigma/\mu$  is the standard deviation of scale score divided by the mean.  $\Phi_{0.05}/\mu$  is the fifth percentile score divided by the mean.  $\Phi_{0.95}/\mu$  is the ninety-fifth percentile score divided by the mean.

*Source:* OECD and Human Resources Development Canada (1997 Tables 1.1 and 1.11).

# Production Technology

- Two tasks that are indivisible and both required
- $Y = F(t_A, t_B)$  where  $t_j$  is talent of worker performing task  $j$ ; output measured in quality-adjusted units
- $F$  symmetric:  $F(t, t') = F(t', t)$ .
- $F$  homogeneous of degree one (constant returns to talent) - increasing returns to talent could itself generate trade.

# Supermodularity

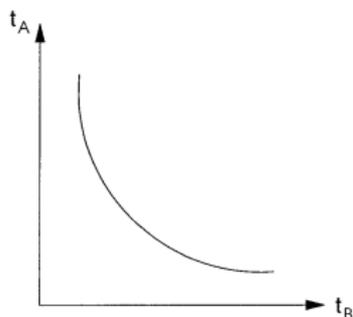
- $F$  is *supermodular* if

$$F(t_A, t_B) + F(t'_A, t'_B) \leq F[\min(t_A, t'_A), \min(t_B, t'_B)] \\ + F[\max(t_A, t'_A), \max(t_B, t'_B)]$$

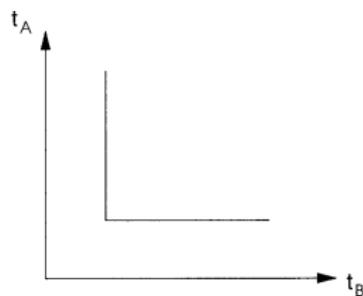
- Given two teams, you can (weakly) increase output by rearranging the teams and pairing the worse type in task A with the worse type in task B
- Essentially equivalent to  $F_{12} \geq 0$ .
- Tasks are *complementary*

# Supermodularity

- Note,  $F$  h.d.1 and  $F_{12} \geq 0$  imply  $F$  quasi-concave, so isoquants are convex. Example,  $F(t_A, t_B) = [t_A^\theta + t_B^\theta]^{1/\theta}$ , with  $\theta < 1$



a: moderate complementarity



b: extreme complementarity

- $F$  supermodular  $\Rightarrow$  self-matching efficient.
  - Consider two firms, each employing types  $(t_1, t_2)$ ; you are better off pairing the  $t_1$  types together, and the  $t_2$  types together

# Submodularity

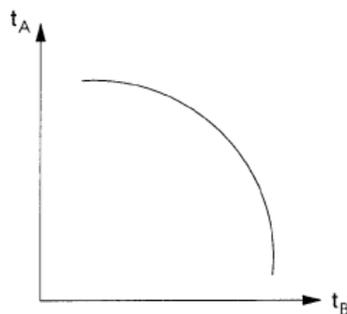
This is the opposite of supermodularity.  $F$  is *submodular* if

$$F(t_A, t_B) + F(t'_A, t'_B) \geq F[\min(t_A, t'_A), \min(t_B, t'_B)] \\ + F[\max(t_A, t'_A), \max(t_B, t'_B)]$$

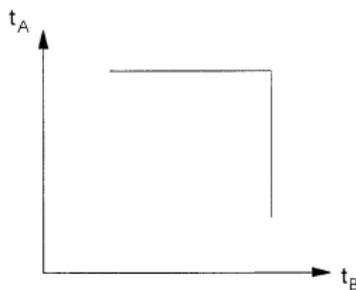
- Essentially equivalent to  $F_{12} \leq 0$ .
- Talents are *substitutable*.
- Note,  $F$  h.d.1 and  $F_{12} \leq 0$  imply  $F$  quasi-concave, so isoquants are concave.

# Submodularity

- Example,  $F(t_A, t_B) = [t_A^\theta + t_B^\theta]^{1/\theta}$ , with  $\theta > 1$ .



a: moderate substitutability



b: extreme substitutability

- The firm will use only the best of the two draws
- $F$  submodular  $\Rightarrow$  cross-matching efficient: For any four workers, output is max when the two with the greatest and least talent work together and those with intermediate talents work together.
  - ▶ Cross-matched pairs will be observed in a competitive equilibrium, because firms employing superstars will be less willing to pay for a supporting cast than firms stocked with workers of more modest talents.

# Trade with Two Supermodular Sectors

- Economy has two sectors,  $C(\text{omputers})$  and  $S(\text{oftware})$
- Output by pair of workers in sector  $i$ :  $F^i(t_A, t_B)$
- $F^i$  symmetric, constant returns to talent, supermodular for  $i = C, S$  (degree of supermodularity, e.g.  $\theta$  in case of CES, can differ across industries)
- Two countries
  - ▶ Continuum of workers:  $L, L^*$
  - ▶ Talent distributed according to continuous cdf's:  $\Phi(t), \Phi^*(t)$ . Densities are symmetric about their means. These are exogenous, and reflect e.g. education policies.
  - ▶ Talent observable by workers and firms (will relax this)
  - ▶ Identical, homothetic preferences
- Note CRS at industry level (if an industry doubles its employment at every talent level, output doubles). There exists a competitive, free-trade equilibrium

# Properties of Competitive Equilibrium

- Take a social-planner shortcut to characterizing the competitive equilibrium: this maximizes value of national output, *given prices*  $\Rightarrow$  Each sector technically efficient

**Lemma 1:** If  $F^i$  is supermodular, then for any allocation of resources to sector  $i$ , output is maximized when  $t_A = t_B$  in all firms in  $i$

Let's derive the PPF:

- $F^i(t, t) = F^i(1, 1)t \equiv \lambda_i t \Rightarrow Y_i = \lambda_i T_i / 2$  where  $T_i$  is the total allocation of talent to sector  $i \Rightarrow$  constant marginal rate of transformation ( $= -\frac{\lambda_C}{\lambda_S}$ )
- Note, same MRT in both countries.

**Proposition 1:** If technology is supermodular in each sector, the free-trade equilibrium has no trade.

# Trade with One Supermodular and One Submodular Sector

- $F^C$  is supermodular,  $F^S$  is submodular
- First look for the efficient organization of production within a sector, conditional on the resources devoted to that sector:

**Lemma 2:** For any allocation of resources  $\Phi^S(t)L$  to sector  $S$ , output is maximized when each worker of talent  $t$  is matched with a worker of talent  $m(t) = 2\bar{t} - t$

- Then look for the efficient allocation of resources between sectors, for given  $p$ :

**Lemma 3:** For a given output of good  $S$ , the output of good  $C$  is maximized if this sector is allocated a symmetric group of workers with talents closest to the median

# Trade with One Supermodular and One Submodular Sector

- Intuition:

- ▶ First note that marginal products are homogeneous of degree zero, so  $t_A F_{11} + t_B F_{12} = 0$ , so  $F_{11}$  has the opposite sign from  $F_{12}$
- ▶ So  $F_{12} > 0$  implies  $F_{ii} < 0$  and  $F_{12} < 0$  implies  $F_{ii} > 0$
- ▶ Lemma 3 says that the  $S$  sector attracts the most (and least) talented workers
- ▶ This allocation reflects the fact that there are increasing returns to a single worker's talent in the sector with substitutable tasks;  $F_{11}^S, F_{22}^S > 0$ , whereas  $F_{11}^C, F_{22}^C < 0$ .
- ▶ And once a highly talented worker has been assigned to the  $S$  sector, there are diminishing marginal costs of reducing the talent of his partner.

# Comparative Advantage

- Note that PPFs are concave: producing more of the S good is more and more costly in terms of good C.

**Proposition 2:** If  $\Phi = \Phi^*$  (but possibly  $L \neq L^*$ ) the free-trade equilibrium has no trade.

- If the two countries are scalar replicas of one another, the potential matches in each country are the same.
- The larger country simply can have more of each type of match.
- Since there are constant returns to scale at the industry level, more matches means proportionally more output.
- In short, countries with the same technologies and same distributions of talent face the same trade-off in producing one good relative to the other.
- PPFs have the same shape.

# Comparative Advantage

- Next consider the case in which every worker in one country has a similar fraction (or multiple) of the skills of his counterpart in the talent distribution of the other:

**Proposition 3:** If  $\Phi(t) = \Phi^*(\beta t)$ , the free-trade equilibrium has no trade.

- This is because of constant returns to talent.
- The potential matches in the foreign country are similar to those in the home country.
- Except that every member of a pair in the foreign country has  $\beta$  times as much talent as his counterpart in a pairing in the home country.
- This means that each potential team in the foreign country can produce  $\beta$  times as much output as the analogous team in the home country.
- Once again, PPFs have the same shape.

# Comparative Advantage

- Consider differences in diversity:

## Definition

The distribution of talent  $\Phi$  is more *diverse* than  $\Phi^*$  if  $\Phi(t) > \Phi^*(t)$  for  $t < t'$  and  $\Phi(t) < \Phi^*(t)$  for  $t > t'$ , for some  $t' > t_{\min}$ .

- If  $\Phi$  is more diverse than  $\Phi^*$  and  $\bar{t} = \bar{t}^*$ , then  $\Phi$  must be a mean preserving spread of  $\Phi^*$ .
- However, the combination of equal mean and greater diversity is a bit more restrictive than a mean-preserving spread, because we allow the two c.d.f.'s to cross only once.
- This notion of diversity corresponds to what in decision theory is known as a *simple increase in risk*.

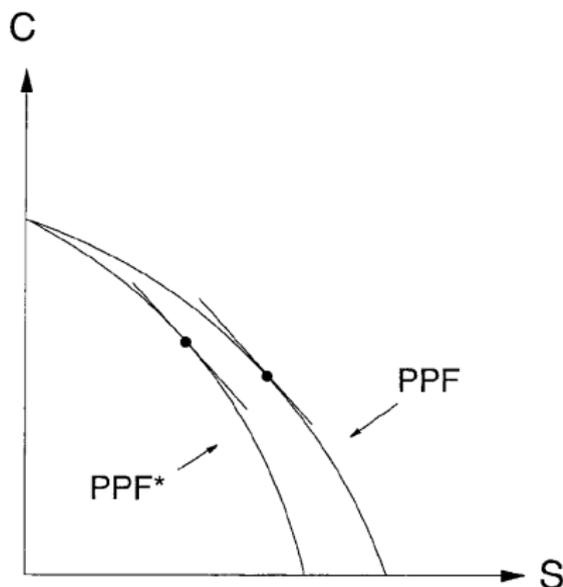
# Comparative Advantage

**Proposition 4:** If  $\bar{t} = \bar{t}^*$  and  $\Phi$  is more *diverse* than  $\Phi^*$ , home exports  $S$  and imports  $C$  in a free trade equilibrium.

- The more diverse country has the greater possibilities for pairing very talented individuals with ones at the opposite end of the talent distribution.
- Hence the maximum potential output of good  $S$  is higher in this country.
- Productivity in  $C$  depends only on the average quality of the pairs, which is the same in two countries.
- Hence the maximum potential output of good  $C$  is the same in both countries.
- It follows that comparative advantage in the  $S$  industry resides in the more diverse country.

# Comparative Advantage

- The key of the proof is to show a comparative-statics result; as you increase diversity, you increase the output ratio  $S/C$  at constant prices (or MRT).
- The trade pattern follows immediately from this



## Extension to Countries with Different Mean Talent Levels

- Consider a hypothetical home economy that has the same mean talent level as that abroad.
- In particular, define  $\Phi^{ma}(\cdot)$  such that  $\Phi^{ma}(rt) = \Phi(t)$  for all  $t \in [t_{\min}, t_{\max}]$ , where  $r \equiv \bar{t}^*/\bar{t}$ .
- The mean of  $\Phi^{ma}$  is  $r$  times the mean of  $\Phi$ , and thus is equal to  $\bar{t}^*$  by construction.
- By Proposition 3, the PPF for the hypothetical economy has the same shape as that for the home economy; the one is a radial expansion of the other.
- If the hypothetical economy has a more diverse distribution of talent than the foreign country, the proof of Proposition 4 tells us that it must produce relatively more of good  $S$  when the two face the same price.
- It follows that the home country also produces relatively more of good  $S$  when  $MRT = MRT^*$ .

**Proposition 4'**: Define  $\Phi^{ma}$  so that  $\Phi^{ma}(rt) = \Phi(t)$  for all  $t \in [t_{\min}, t_{\max}]$ , where  $r = \bar{t}^*/\bar{t}$ . If  $\Phi^{ma}$  is more diverse than  $\Phi^*$ , home exports  $S$  and imports  $C$  in a free trade equilibrium.

# Imperfect Matching

- The model has an unrealistic prediction: the more diverse country has absolute advantage in S, but not a disadvantage in C.
- But the country with a more homogenous workforce may have an absolute advantage in a sector where consistency of talents is valuable.

Need for an environment with imperfect matching:

- Talent  $t_i = q_i e_i$ , where  $q_i$  is observable to all and  $e_i$  is not observable (even to the worker)
- $q_i$  may represent years of schooling; distributed  $\Phi(q)$ ,  $\Phi^*(q)$  (symmetric distributions with equal means and  $\Phi$  equally or more diverse than  $\Phi^*$ )
- $e_i$  may represent “schooling effectiveness”; distributed  $\Psi(e)$ ,  $\Psi^*(e)$  (symmetric distributions with  $\Psi$  m.p.s. of  $\Psi^*$ ). Normalize  $Ee_i = 1$ . Differences between  $\Psi(e)$  and  $\Psi^*(e)$  can arise from differences in educational policy.
- $e$  independent of  $q$
- $EF^c(q_A, q_B)$  is supermodular in  $(q_A, q_B)$ , and  $EF^s(q_A, q_B)$  submodular  $\implies$  firms match according to observable  $q$ .

# Trade with Imperfect Matching

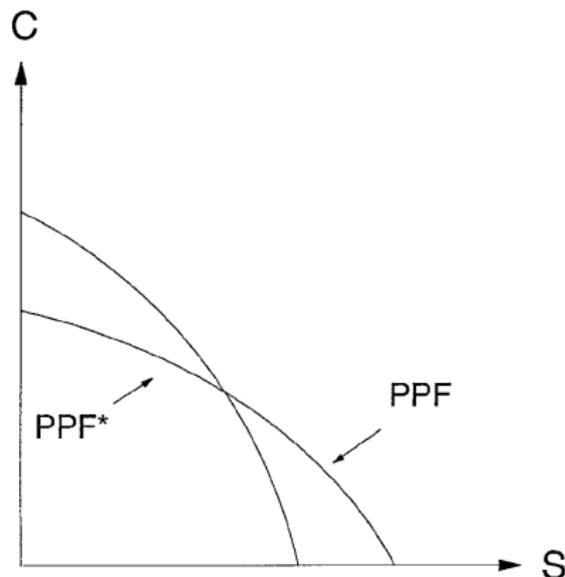
- With one supermodular and one submodular sector, imperfect observability strengthens the forces of comparative advantage:

**Proposition 6:** Let  $\Phi(q)$  and  $\Phi^*(q)$  be symmetric distributions of observable talent with  $\Phi$  more diverse. Let  $\Psi(e)$  and  $\Psi^*(e)$  be symmetric distributions of unobservable talent with  $\Psi$  noisier. Then home exports good  $S$  and imports good  $C$  in a free-trade equilibrium.

- $\bar{C}$  is higher in the less diverse country.
- The reason is mismatches in the pairing of workers producing good  $C$
- The average mismatch in the home country will be larger than that in the foreign country, because the unobserved component of talent has a noisier distribution there
- It follows that the country with a more homogeneous workforce has a competitive advantage in producing good  $C$ .

## Trade with Imperfect Matching

- And the higher noise adds to the home country's advantage in  $S$ .
- Not only do the matches in this country involve workers of more disparate observable talent, on average, but in each match the noisiness of  $e_A$  and  $e_B$  raises expected output



- Intuitive that the home country imports good  $C$  and exports good  $S$ .

# Matching and Inequality in the World Economy, Arnaud Costinot and Jonathan Vogel (2009)

- **Question:** How do changes in factor supply or factor demand affect factor prices and factor allocation in **high-dimensional** environments?
- **Why do we care about high-dimensional environments?**
  - ① Large changes in inequality and in factor allocation occur at high levels of disaggregation
    - ① *Top income inequality*, e.g. Piketty and Saez (2003)
    - ② *Income polarization*, e.g. Autor, Katz, and Kearney (2008)
    - ③ *Job polarization*, e.g. Goos and Manning (2003)
    - ④ *Within and between- inequality*, e.g. Juhn, Murphy, and Pierce (1993)
  - ② Large changes occurring at low levels of disaggregation (e.g. skill premium) reflect average changes over a large number of factors

# How to Answer this Question?

Weak assumptions, weak results

- **Approach #1:** Start from a standard neoclassical model with low dimensionality (e.g. Heckscher-Ohlin) and increase it
- **Problems with Approach #1:**
  - ① Predictions are **unintuitive**: Is the number of goods greater than the number of factors in the economy?
  - ② Predictions are **weak**, e.g. Jones and Scheinkman's (1977) "Friends and Enemies" result states that a rise in the price of *some* good causes an even larger proportional increase in the price of *some* factor

# How to Answer this Question?

Strong assumptions, strong results

- **Approach #2:** Directly start from an assignment model with high dimensionality (e.g. Roy)
- **Problems with Approach #2:**
  - ① General results focus on **cross-sectional predictions**: PAM (Becker 1973, Shimer and Smith 2000, Legros and Newman 2002)
  - ② Comparative statics use **strong functional form assumptions** on:
    - ★ *Production function*, e.g. Teulings (1995), Garicano and Rossi-Hansberg (2006)
    - ★ *Distribution of factors*, e.g. Kremer and Maskin (2003), Antras, Garicano and Rossi Hansberg (2006), Gabaix and Landier (2008)
    - ★ *Utility function*, e.g. Teulings (2005), Blanchard and Willman (2008), Tervyo (2008)

# This Paper

- **Contribution:**

- ① Develop concepts and techniques to do **robust monotone comparative statics** in a Roy-like assignment model
  - ★ Deepen our understanding of an important class of models in the labor and trade literature
- ② Use results to **revisit consequences of globalization** on factor prices and factor allocation in high-dimensional environments
  - ★ Go from weak to strong predictions even in such environments
  - ★ Offer a unifying perspective on North-South trade, North-North trade, offshoring, and skill-biased technological change
  - ★ Broaden the scope of standard trade theory to discuss phenomena such as pervasive changes in inequality and wage and job polarization

# Roadmap of the Talk

- ① The Closed Economy
- ② Comparative Statics in the Closed Economy
- ③ The World Economy
- ④ Technological Change in the World Economy

# The Closed Economy: The Basic Environment

- A set of intermediate goods/tasks with skill-intensity  $\sigma \in \Sigma \equiv [\underline{\sigma}, \bar{\sigma}]$
- A set of workers with skill  $s \in S \equiv [\underline{s}, \bar{s}]$
- $V(s) > 0$  is the inelastic supply of workers with skill  $s$
- Good and labor markets are perfectly competitive

## The Basic Environment (Cont.)

- Workers are perfect substitutes in the production of each task:

$$Y(\sigma) = \int_{s \in S} A(s, \sigma) L(s, \sigma) ds$$

- $A(s, \sigma) > 0$  is strictly log-supermodular:

$$\frac{A(s, \sigma)}{A(s, \sigma')} > \frac{A(s', \sigma)}{A(s', \sigma')}, \text{ for all } s > s' \text{ and } \sigma > \sigma'$$

- Output of the final good is given by the following CES aggregator:

$$Y = \left\{ \int_{\sigma \in \Sigma} B(\sigma) [Y(\sigma)]^{\frac{\varepsilon-1}{\varepsilon}} d\sigma \right\}^{\frac{\varepsilon}{\varepsilon-1}}$$

- $B(\sigma) > 0$  is an exogenous technological parameter

# Definition of a Competitive Equilibrium

A competitive equilibrium is a set of functions  $(Y, L, p, w)$  such that:

- 1 Final good producers maximize profit

$$Y(\sigma) = I \times [p(\sigma) / B(\sigma)]^{-\varepsilon}$$

- 2 Intermediate good producers maximize profit

$$p(\sigma) A(s, \sigma) - w(s) \leq 0, \text{ for all } s \in S$$

$$p(\sigma) A(s, \sigma) - w(s) = 0, \text{ for all } s \in S \text{ such that } L(s, \sigma) > 0$$

- 3 The intermediate market clears

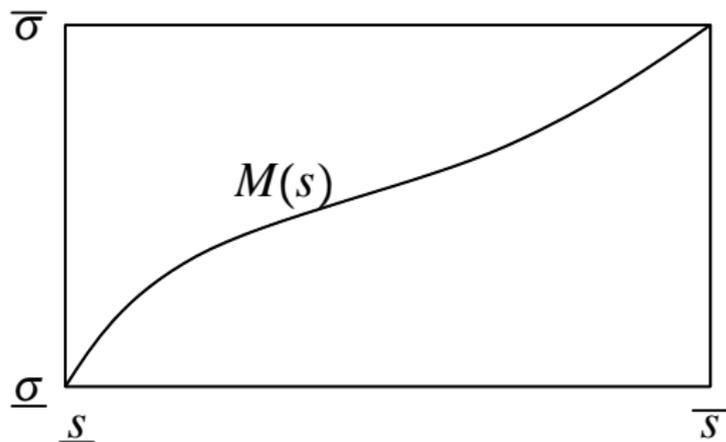
$$Y(\sigma) = \int_{s \in S} A(s, \sigma) L(s, \sigma) ds, \text{ for all } \sigma \in \Sigma$$

- 4 The labor market clears

$$V(s) = \int_{\sigma \in \Sigma} L(s, \sigma) d\sigma, \text{ for all } s \in S$$

# Properties of a Competitive Equilibrium

**Lemma 1** In a competitive equilibrium, there exists an increasing bijection  $M : S \rightarrow \Sigma$  such that  $L(s, \sigma) > 0$  if and only if  $M(s) = \sigma$



# Properties of a Competitive Equilibrium (Cont.)

- **Lemma 2** In a competitive equilibrium,  $M$  and  $w$  satisfy

$$\frac{dM}{ds} = \frac{A[s, M(s)] V(s)}{I \times \{p[M(s)] / B[M(s)]\}^{-\varepsilon}} \quad (3)$$

$$\frac{d \ln w(s)}{ds} = \frac{\partial \ln A[s, M(s)]}{\partial s} \quad (4)$$

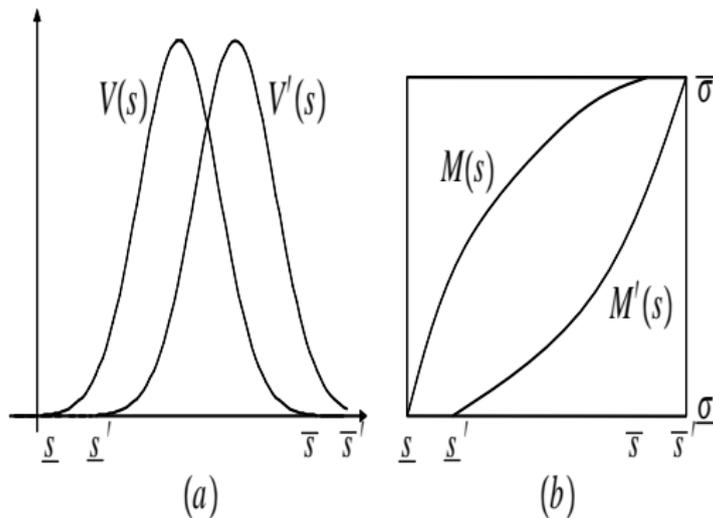
with  $M(\underline{s}) = \underline{\sigma}$ ,  $M(\bar{s}) = \bar{\sigma}$ , and  $p[M(s)] = w(s) / A[s, M(s)]$ .

# Comparative Statics in the Closed Economy: Change in Factor Supply (I): Skill Abundance

Definition

**Definition**  $V'$  is *skill-abundant* relative to  $V$ , denoted  $V' \succeq_a V$ , if

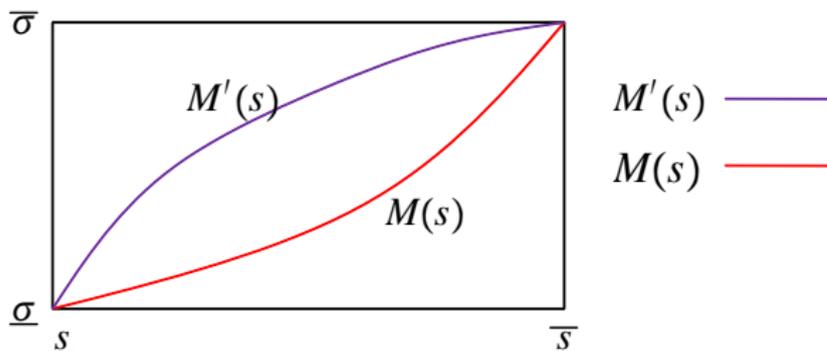
$$\frac{V(s)}{V(s')} \geq \frac{V'(s)}{V'(s')}, \text{ for all } s \leq s'$$



# Change in Factor Supply (I): Skill Abundance

## Matching

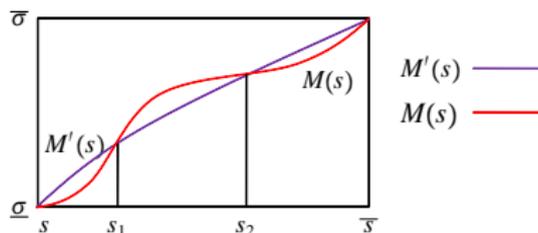
- **Lemma 3** Suppose  $V \succeq_a V'$ . Then  $M'(s) \geq M(s)$  for all  $s \in S$



- From a task standpoint: *worker/skill downgrading*
- From a worker standpoint: *task upgrading*

# Change in Factor Supply (I): Skill Abundance

Sketch of Proof



- 1  $M'(s_1) = M(s_1) = \sigma_1$ ,  $M'(s_2) = M(s_2) = \sigma_2$ , and  $\frac{M'_s(s_1)}{M'_s(s_2)} < \frac{M_s(s_1)}{M_s(s_2)}$
- 2 Equation (3)  $\implies \frac{V'(s_2) Y'(\sigma_1)}{V'(s_1) Y'(\sigma_2)} > \frac{V(s_2) Y(\sigma_1)}{V(s_1) Y(\sigma_2)}$
- 3  $V \succeq_a V' \implies \frac{V(s_2)}{V(s_1)} \geq \frac{V'(s_2)}{V'(s_1)}$
- 4 Equation (4) + zero profits  $\implies \frac{d \ln p}{d \sigma} = - \frac{\partial \ln A[M^{-1}(\sigma), \sigma]}{\partial \sigma}$
- 5  $M^{-1}(\sigma) < M'^{-1}(\sigma)$  for  $\sigma \in (\sigma_1, \sigma_2)$  + A log-spm  $\implies \frac{p(\sigma_1)}{p(\sigma_2)} \leq \frac{p'(\sigma_1)}{p'(\sigma_2)}$
- 6  $\frac{p(\sigma_1)}{p(\sigma_2)} \leq \frac{p'(\sigma_1)}{p'(\sigma_2)}$  + CES  $\implies \frac{Y(\sigma_1)}{Y(\sigma_2)} \geq \frac{Y'(\sigma_1)}{Y'(\sigma_2)}$

# Change in Factor Supply (I): Skill Abundance

## Inequality

- Moving from  $V$  to  $V' \preceq_a V$  implies *pervasive rise in inequality*:

$$\frac{w'(s)}{w'(s')} \geq \frac{w(s)}{w(s')}, \text{ for all } s > s'$$

- The mechanism is simple:

- 1 Profit-maximization implies

$$\frac{d \ln w}{ds} = \frac{\partial \ln A[s, M(s)]}{\partial s} \text{ and } \frac{d \ln w'}{ds} = \frac{\partial \ln A[s, M'(s)]}{\partial s}$$

- 2 Since  $A$  is log-supermodular, task upgrading implies

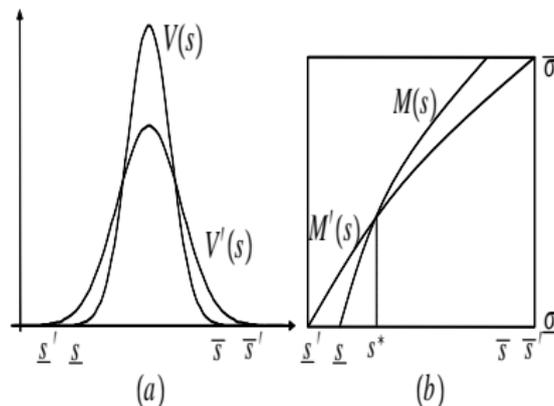
$$\frac{d \ln w'}{ds} \geq \frac{d \ln w}{ds}$$

# Change in Factor Supply (II): Skill Diversity

## Definition

**Definition**  $V'$  is more diverse than  $V$ , denoted  $V' \succeq_d V$ , if there exists an  $\hat{s} \in (\underline{s}, \bar{s})$  such that

$$V' \succ_a V, \text{ for all } s \geq \hat{s}$$
$$V \succ_a V', \text{ for all } s < \hat{s}$$



# Change in Factor Supply (II): Skill Diversity

Matching

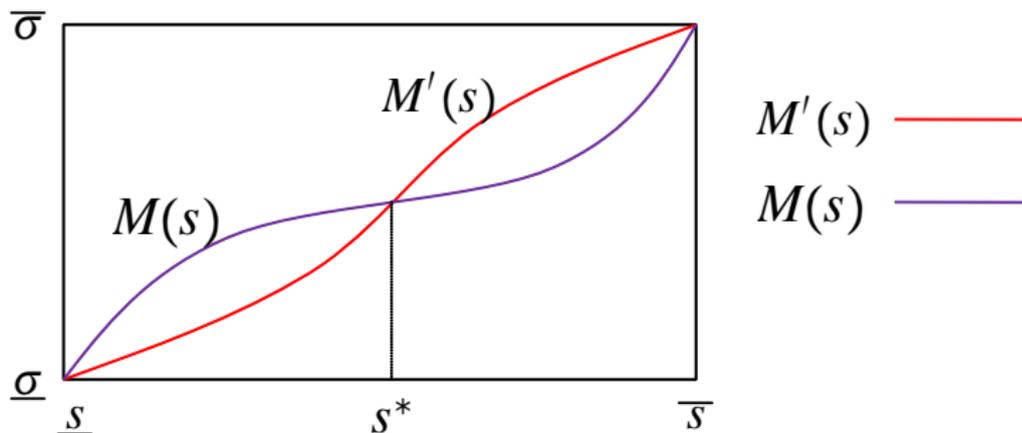
Moving from  $V$  to  $V' \preceq_d V$  implies:

- 1 Skill upgrading for low- $\sigma$  tasks (task downgrading for low  $s$ ):

$$M'(s) \leq M(s), \text{ for all } s < s^*$$

- 2 Skill downgrading for high- $\sigma$  tasks (task upgrading for high  $s$ ):

$$M'(s) \geq M(s), \text{ for all } s^* < s$$



# Change in Factor Supply (II): Skill Diversity

## Inequality

Moving from  $V$  to  $V' \preceq_d V$  implies:

- ① *Pervasive fall in inequality* among low-skilled workers:

$$\frac{w'(s)}{w'(s')} \leq \frac{w(s)}{w(s')}, \text{ for all } s' < s \leq s^*$$

- ② *Pervasive rise in inequality* among high-skilled workers:

$$\frac{w'(s)}{w'(s')} \geq \frac{w(s)}{w(s')}, \text{ for all } s^* \leq s' < s$$

# Change in Factor Demand (I): SBTC

## Definition

- **Definition**  $B'$  is skill-biased relative to  $B$ , denoted  $B' \succeq_s B$ , if

$$\frac{B'(\sigma)}{B'(\sigma')} \geq \frac{B(\sigma)}{B(\sigma')}, \text{ for all } \sigma > \sigma'$$

# Change in Factor Demand (I): SBTC

## Matching and Inequality

Moving from  $B$  to  $B' \succeq_s B$  implies:

① *Skill downgrading:*

$$M'(s) \geq M(s), \text{ for all } s$$

② *Pervasive rise in inequality:*

$$\frac{w'(s)}{w'(s')} \geq \frac{w(s)}{w(s')}, \text{ for any } s > s'.$$

# Change in Factor Demand (II): EBTC

## Definition

- **Definition**  $B'$  is extreme-biased relative to  $B$ , denoted  $B' \succeq_e B$ , if there exists an  $\hat{\sigma} \in (\underline{\sigma}, \bar{\sigma})$  such that

$$B \succeq_s B' \text{ for all } \sigma < \hat{\sigma}$$

$$B' \succeq_s B \text{ for all } \sigma \geq \hat{\sigma}$$

# Change in Factor Demand (II): EBTC

## Matching and Inequality

Moving from  $B$  to  $B' \succeq_e B$  implies:

① *Job Polarization:*

$$M'(s) \leq M(s), \text{ for all } s < s^*$$

and

$$M'(s) \geq M(s), \text{ for all } s^* < s$$

② *Wage Polarization:*

$$\frac{w'(s)}{w'(s')} \leq \frac{w(s)}{w(s')}, \text{ for all } s' < s \leq s^*$$

and

$$\frac{w'(s)}{w'(s')} \geq \frac{w(s)}{w(s')}, \text{ for all } s^* \leq s' < s$$

# The World Economy

## Setup

- Two countries, Home ( $H$ ) and Foreign ( $F$ )
- Workers are internationally immobile, final good is not traded, and all intermediate goods are freely traded
- Factor productivity differences across countries are Hicks-neutral:

$$A_i(s, \sigma) \equiv \gamma_i A(s, \sigma) \text{ for } i = H, F$$

# The World Economy

## Free Trade Equilibrium

- A competitive equilibrium in the world economy under free trade is s.t.

$$\frac{dM_T}{ds} = \frac{A[s, M_T(s)] V_W(s)}{I_W \times \{p_T[M_T(s)] / B_W[M_T(s)]\}^{-\varepsilon}},$$

$$\frac{d \ln w_T(s)}{ds} = \frac{\partial \ln A[s, M_T(s)]}{\partial s},$$

where:

$$M_T(\underline{s}) = \underline{\sigma} \text{ and } M_T(\bar{s}) = \bar{\sigma}$$

$$p_T[M_T(s)] = w_T(s) / \gamma_H A[s, M_T(s)]$$

$$B_W[M_T(s)] \equiv \left\{ (I_H / I_W) B_H[M_T(s)]^\varepsilon + (I_F / I_W) B_F[M_T(s)]^\varepsilon \right\}^{1/\varepsilon}$$

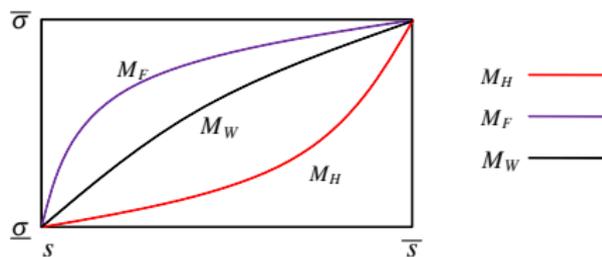
$$V_W \equiv V_H + V_F$$

# Consequences of North-South Trade

## The Role of Cross-Country Differences in Factor Endowments

- Assumption:  $V_H \succeq_a V_F$  and  $B_H = B_F$
- If  $V_H \succeq_a V_F$ , then  $V_H \succeq_a V_W \succeq_a V_F$
- Continuum-by-continuum extensions of two-by-two HO results
  - ① *Changes in skill-intensities:*

$$M_H(s) \leq M_T(s) \leq M_F(s), \text{ for all } s$$



- ② *Strong Stolper-Samuelson effect:*

$$\frac{w_H(s)}{w_H(s')} \leq \frac{w_T(s)}{w_T(s')} \leq \frac{w_F(s)}{w_F(s')}, \text{ for all } s > s'$$

# Consequences of North-South Trade (Cont.)

## The Role of Cross-Country Differences in Skill Biases

- Assumption:  $V_H = V_F$  and  $B_H \succeq_s B_F$
- If  $B_H \succeq_s B_F$ , then  $B_W$  satisfies  $B_H \succeq_s B_W \succeq_s B_F$
- Exact same logic leads to the exact opposite conclusion

① *Matching:*

$$M_H(s) \geq M_T(s) \geq M_F(s), \text{ for all } s$$

② *Inequality:*

$$\frac{w_H(s)}{w_H(s')} \leq \frac{w_T(s)}{w_T(s')} \leq \frac{w_F(s)}{w_F(s')}, \text{ for all } s > s'$$

# Consequences of North-South Trade (Cont.)

## Summary

- **Observation #1:**  
Predictions regarding the impact of trade integration crucially depend on the correlation between supply and demand considerations
- **Observation #2:**  
Acemoglu (1998, 2002, 2003a, 2003b, 2007) argues that skill-abundant countries tend to use skill-biased technologies
- **Conclusion #1:**  
Similar countries may have different globalization experiences depending on which of these two forces, supply or demand, dominates
- **Conclusion #2:**  
Overall effect of trade liberalization on factor allocation and factor prices may be small in practice

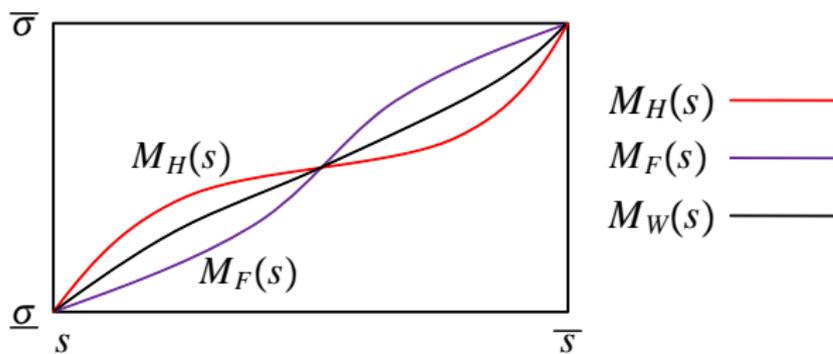
# Consequences of North-North Trade

## Matching

- Assumption:  $V_H \succeq_d V_F$  and  $B_H = B_F$
- If  $V_H \succeq_d V_F$ , then  $V_W$  satisfies  $V_H \succeq_d V_W \succeq_d V_F$
- Changes in matching: Job polarization at Home

$$M_T(s) \leq M_H(s), \text{ for all } s < s_H;$$
$$M_T(s) \geq M_H(s), \text{ for all } s_H < s.$$

and the converse in Foreign



# Consequences of North-North Trade (Cont.)

## Inequality

- Changes in Inequality:

- 1 *Wage polarization* in the more diverse country

$$\frac{w_T(s)}{w_T(s')} \leq \frac{w_H(s)}{w_H(s')}, \text{ for all } s' < s \leq s_H$$
$$\frac{w_T(s)}{w_T(s')} \geq \frac{w_H(s)}{w_H(s')}, \text{ for all } s_H \leq s' < s$$

- 2 *Wage convergence* in the less diverse country

$$\frac{w_T(s)}{w_T(s')} \geq \frac{w_F(s)}{w_F(s')}, \text{ for all } s' < s \leq s_F$$
$$\frac{w_T(s)}{w_T(s')} \leq \frac{w_F(s)}{w_F(s')}, \text{ for all } s_F \leq s' < s$$

# Consequences of North-North Trade (Cont.)

## Summary

- **Conclusion #1:**

North-North trade has no clear implications for overall inequality: Relative wage *between* high- and low-skill workers—as well as relative price of goods they produce—may either increase or decrease

- **Conclusion #2:**

Consequences of North-North trade are to be found at a higher level of disaggregation: changes in inequality occur *within* low- and high-skill workers, respectively

# Technological Change in the World Economy

Global SBTC

- Assumption:  $V_H \succeq_a V_F$  and  $\gamma_H \geq \gamma_F$
- Moving from  $B_W$  to  $B'_W \succeq_s B_W$  implies:
  - ① *Skill downgrading/task upgrading in both countries:*

$$M_T(s) \leq M'_T(s), \text{ for all } s$$

- ② *Pervasive rise in inequality in both countries:*

$$\frac{w'_T(s)}{w'_T(s')} \geq \frac{w_T(s)}{w_T(s')}, \text{ for all } s > s'.$$

- ③ *An increase in inequality between countries:*

$$I'_H / I'_F \geq I_H / I_F$$

# Technological Change in the World Economy

## Offshoring Tasks

- Assumption:  $V_H \succeq_a V_F$  and  $\gamma_H \geq \gamma_F$
- Moving from  $\gamma_F$  to  $\gamma'_F \geq \gamma_F$  implies

① *Skill downgrading/task upgrading in both countries:*

$$M_T(s) \leq M'_T(s), \text{ for all } s$$

② *Pervasive rise in inequality in both countries:*

$$\frac{w'_T(s)}{w'_T(s')} \geq \frac{w_T(s)}{w_T(s')}, \text{ for all } s > s'.$$

- **Intuition:** Offshoring makes the world relatively less skill-abundant, which leads to sector upgrading around the world, thereby increasing the marginal return to skill in all countries

# Conclusions

- **Contribution (I):** Derive sufficient conditions for robust monotone comparative statics predictions—without functional form restrictions on the distribution of skills or worker productivity—in a Roy-like assignment model where goods neither have to be perfect substitutes nor perfect complements
- **Contribution (II):** Show how these general results can be used to derive sharp predictions about the consequences of globalization in economies with an arbitrarily large number of both goods and factors, thereby broadening the scope of standard trade theory