Lecture 7: Growth and Trade

Economics 552

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Growth and Interdependence, Ventura (1997)

- Two of the most interesting facts of the post-war international growth experience are :
  - Conditional convergence
  - The East Asian Miracle
- Standard explanations for conditional convergence rely on :
  - Establishing diminishing returns to investment
  - Assuming low level of economic integration
- Problems with standard explanations :
  - If diminishing returns are important, why have the growth rates of the East Asian economies not been declining quickly enough as these economies accumulate capital?
  - Inability of investors to exploit arbitrage opportunities

This Paper

- Model of trade and growth that combines conditional version of FPE with the neoclassical growth model
- Technology exhibits diminishing returns, yet countries' ability to trade and eliminate price differentials implies that **these diminishing returns are global but not local**
- Holding constant differences in labor productivity, poor countries grow faster than rich ones if and only if factor prices do not change too fast as the world economy grows
- East Asian economies were able to beat the curse of diminishing returns through their ability to trade

The Model : Countries and Goods

- World economy with J countries
- One final good used for consumption and investment
- 2 factors of production: Capital and Labor
- π_j is the share of world's population located in country j
  - These shares are constant and population grows at the rate n
- Final good is non-traded and is the numeraire
- Costless trade in intermediate goods $\Rightarrow$ Countries share the same intermediate good price $p_1(t)$ and $p_2(t)$, where $t \in [0, \infty)$
- No international factor movements
- $w_j(t)$ is the wage rate while $r_j(t)$ is the rental rate in country j

The Model : Representative agent

- Representative agent in each country consumes $c_j(t)$ and owns a capital stock of $k_j(t)$
- Capital does not depreciate
- The consumer maximizes

$$\int_0^\infty \ln c_j(t) e^{-(\rho-n)t} dt$$

- The budget constraint of the consumer is

$$c_j + \dot{k}_j + n \cdot k_j = r_j \cdot k_j + w_j$$

The Model : Production

- Final goods sector
  - Many competitive firms
  - $(x_{1j}^b + x_{2j}^b)^{1/(1-b)}$, where $x_{ij}(t)$ denotes the purchases of good i by the representative firm of country j at date t
  - $\sigma = (1-b)^{-1}$ is the elasticity of substitution, $b < 1$
- Intermediate goods sector
  - A_j is a measure of labor productivity
  - One worker produces A_j units of good 1
  - One unit of capital produces one unit of good 2

Competitive Equilibrium

- The competitive equilibrium of the world economy consists of a sequence of prices and quantities such that consumers and firms optimize and markets clear
- The first order conditions are :

$$
\begin{aligned}
r_j &= \rho + \frac{\dot{c}_j}{c_j} \\
\dot{k}_j &= (r_j - n) \cdot k_j + w_j - c_j \\
\lim_{t \to \infty} \frac{k_j}{c_j} e^{-(\rho - n)t} &= 0
\end{aligned}
$$

- The demands of intermediates as a function of prices and the demand of final goods are given by

$$
\begin{aligned}
x_{ij} &= p_i^{1/(1-b)} \cdot (r_j \cdot k_j + w_j) \text{ where} \\
1 &= p_1^{b/(1-b)} + p_2^{b/(1-b)}
\end{aligned}
$$

- Factor prices satisfy the following relationships :

$$
\begin{aligned}
w_j &= A_j \cdot p_1 \\
r_j &= p_2
\end{aligned}
$$

- Rental rates are equalized across countries despite the absence of international factor movements
- Since $x_{1j}/x_{2j} = A/k$ so market clearing in world commodity markets leads to

$$p_1/p_2 = (k/A)^{1-b}$$

- The dynamics of c and k (world averages) are

$$
\begin{aligned}
\dot{c}_j/c_j &= (A^b + k^b)^{(1-b)/b} \cdot k^{b-1} - \rho \\
\dot{k}_j &= (A^b + k^b)^{1/b} - n \cdot k - c
\end{aligned}
$$

- Long run growth if aggregate technology is capable of sustaining the MP_k above ρ
  - $\sigma > 1 \Rightarrow \text{MP}_k$ is bounded below at $1 \Rightarrow$endogenous growth model
  - $\sigma < 1 \Rightarrow \text{MP}_k$ eventually equals $\rho \Rightarrow$exogenous growth model

Autarky

- Diminishing returns ensure that additional investment is less productive $\Rightarrow$ poor countries grow faster than rich ones
- Assuming that the world is a collection of autarkic economies allows us to extend the properties of time-series graph to cross-section graphs
- However one needs to be aware that the convergence test is a joint test of diminishing returns and the view that international linkages do not matter for the growth process

Free trade

- In models of trading economies, the law of diminishing returns applies only to world averages
- Assume that countries are so interdependent that differences in rates of return are arbitraged away
- Define $k_j^R = k_j / k$ and $A_j^R = A_j / A$
- The law of motion of k_j^R is given by

$$\dot{k_j^R} = \phi \cdot (k_j^R - A_j^R),$$

where ϕ is defined as follows

$$\phi = \frac{A}{k} \cdot \left[(\rho - n) \cdot \int_t^\infty p_1 \cdot e^{-\int_t^\tau (p_2 - n) dv} . d\tau - p_1\right]$$

- The above equation provides a full characterization of the cross section of growth rates

Free trade

- Ceteris paribus, if the growth of wages is low, ϕ is negative, and countries that have a low stock of capital relative to their labour productivity parameter, accumulate capital at a higher rate
- The world is populated by countries that act as permanent income consumers
  - Countries exhibit identical rates of wealth accumulation, which in turn, is a weighted average of the growth rates of the stock of capital and the net present value of wages
  - The growth rate of the latter is the same for all countries
  - If this growth rate is low, countries must be accumulating capital at a rate that exceeds that of total wealth, depending on how large the share of capital is in consumer's wealth
  - The lower this share the higher is the growth rate
- Symmetric argument for the case in which the net present value of wages grows at a high rate

Dynamics

- Define $z = c/k$ and note that $\phi = \dot{z}/z$
- The distribution of capital stocks approaches (moves away) from the distribution of labour productivities if and only if the average consumption-capital ratio decreases (increases), $\phi < 0$

$$
\begin{aligned}
\dot{z}/z &= z - \rho + n - (A^b + k^b)^{(1-b)/b} \cdot A^b \cdot k^{-1} \\
\dot{k} &= (A^b + k^b)^{1/b} - (n + z) \cdot k
\end{aligned}
$$

- In the steady state z is constant and the distribution of capital stocks does not change
- During the transition, the behavior of z depends crucially on the assumed elasticity of substitution between capital and labor

Case 1 :

$\sigma > 1$

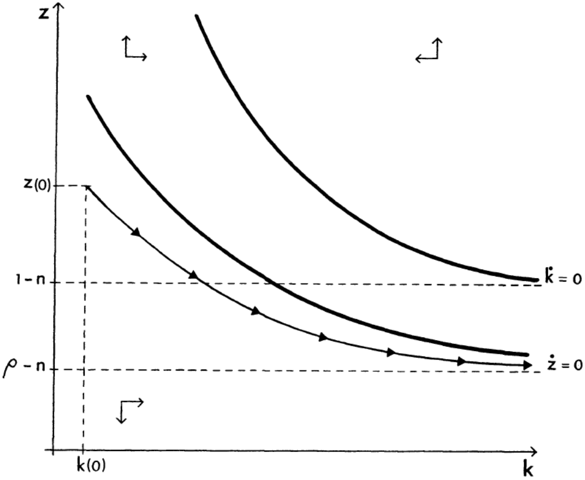


FIGURE I

but still high enough

$\sigma < 1$

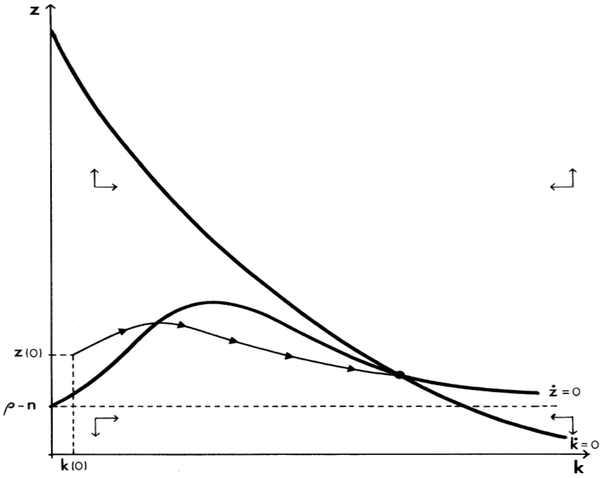


FIGURE II

and low enough

$\sigma < 1$

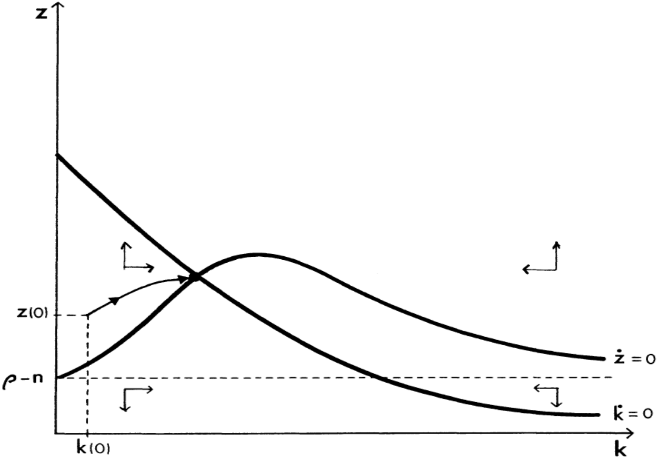


FIGURE III

Cross Sections and Time Series

- If the aggregate technology exhibits a high σ
  - We would observe a downward-sloping cross section of growth rates
  - We would also observe a (downward-sloping, but) almost flat time-series graph for the growth rate
- If the aggregate technology exhibits a low σ
  - We would observe an upward-sloping cross section of growth rates
  - We would also observe a (downward-sloping, but) steep time-series graph for the growth rate
- Since the existence of diminishing returns does not rule out any configuration for a cross section of growth rates, one cannot use the conditional convergence finding as evidence of diminishing returns
- However the conditional convergence finding can be used as evidence that σ is high and that the world economy might be closer to a endogenous growth model

Making Miracles

- Growth in Miracle Economies
  - Rental rate in a miracle economy is $p_2 \cdot (1 + \theta_j)$, where θ_j is the discrepancy between domestic and foreign rates, reflecting different taxation systems, investment subsidies, etc
  - A small economy has the following dynamics for relative capital stock :

$$\dot{k}_j^R = (\phi + p_2 \cdot \theta_j + \rho - \rho_j) \cdot k_j^R - \phi_j \cdot A_j^R$$

  where ϕ_j is defined as follows

$$\phi_j = \frac{A}{k} \cdot [(\rho_j - n) \cdot \int_t^{\infty} p_1 \cdot e^{-\int_t^{\tau}(p_2(1+\theta_j)-n_j)dv} \cdot d\tau - p_1]$$

  - Miracle economies are those in which $p_2 \cdot \theta_j + \rho - \rho_j > 0$
  - For the small economy to remain small asymptotically, population growth must not be too large
  - Even when $\sigma \leq 1$, a small economy that combines a high savings rate with a low rate of population growth can beat diminishing returns by adopting an open trade regime
  - This economy behaves as if it had a linear technology

$$GDP(autarky) = (k_j^b + A_j^b)^{1/b} \qquad GDP(free - trade) = p_1 \cdot A_j + p_2 \cdot k_j$$

Making Miracles

- Structural Transformation versus Capital Deepening
  - In autarkic models of economic growth, growth is basically equivalent to capital deepening
  - In this model, as the capital-labor ratio increases, instead of using more capital-intensive techniques in each sector, the miracle economies absorb the extra capital by expanding capital-intensive sectors and contracting labor-intensive ones - economic growth leads to structural transformation
  - The share of manufacturing exports in GDP is

$$(1-\mu) \cdot \frac{\mu \cdot (k_j^R - A_j^R)}{\mu \cdot (k_j^R - A_j^R) + A_j^R}$$

The World Income Distribution, Acemoglu & Ventura (2002)

- The picture of the world income distribution raises two questions
  - Why are there such large income differences across countries?
  - Why has the world income distribution been relatively stable since 1960?
- Existing explanations are built on two assumptions :
  - Diminishing returns to capital
  - Technological spillovers

This paper

- This paper offers an alternative framework for analyzing the world income distribution, based on international trade and specialization
- Countries that accumulate capital faster than average experience declining export prices, reducing the value of the marginal product of capital
- These terms-of-trade effects introduce de facto diminishing returns at the country level and ensure the stability of the world income distribution
- Cross-country differences in economic policies, saving rates, and technology lead to differences in relative incomes, not in long-run growth rates
- This result depends essentially on some degree of specialization

This paper

- In the steady state, countries with lower rates of time preference and lower price of investment goods will have lower rental rates, hence higher relative capital and income
- Countries with better technologies will be richer because they have higher rental rates for a given level of relative capital and income
- The speed of conditional convergence depends on the strength of the terms-of-trade effects, not on the capital share in output
- Holding technology and other determinants of steady-state income constant, a 1 % faster growth is associated with a 0.6 % deterioration in the terms of trade.
- This explains a significant fraction of cross-country income differences

The Model

- Continuum of countries with mass 1
- Capital is the only factor of production
- Continuum of intermediate goods $z \in [0, M]$
- 2 final goods - consumption good and investment good
- Free trade in intermediate goods, no trade in final goods
- Each country is defined by a triplet (μ, ρ, ϕ)
  - μ - technology
  - ρ - rate of time preference
  - ϕ - effect of policies and institutions on the incentives to invest
- The joint distribution of these characteristics is given by $G(\mu, \rho, \phi)$

The Model

- Each country has a representative consumer with utility function :

$$u = \int_0^\infty \ln c(t) \cdot e^{-\rho \cdot t} \cdot dt$$

- The budget constraint is given by

$$p_I \cdot \dot{k} + p_c \cdot c = y \equiv r \cdot k$$

- Products are differentiated by origin
- μ is the measure of intermediates produced by the (μ, ρ, ϕ) country with $\int \mu \cdot dG = M$
- Intermediates are produced by competitive firms. One unit of capital is used to produce one intermediate good

The Model continued

- Each country also contains many competitive firms in the consumption and investment goods sectors with unit cost functions :

$$
\begin{aligned}
B_c(r, p(z)) &= r^{1-\tau} \cdot \left(\int_0^M p(z)^{1-\epsilon} \cdot dz \right)^{\tau/(1-\epsilon)} \\
B_I(r, p(z)) &= \phi^{-1} \cdot r^{1-\tau} \cdot \left(\int_0^M p(z)^{1-\epsilon} \cdot dz \right)^{\tau/(1-\epsilon)}
\end{aligned}
$$

  where $p(z)$ is the price of intermediate with index z
- τ is the share of intermediates in production and will turn out to be the ratio of exports to income. It is usually interpreted as a measure of openness
- $\epsilon > 1$ is the elasticity of substitution and as well as the price elasticity of foreign demand for the country's products. The inverse of this elasticity is often interpreted as a measure of the degree of specialization

World Equilibrium

- The competitive equilibrium of the world economy consists of a sequence of prices and quantities such that consumers and firms optimize and markets clear
- Consumer optimization leads to the following first-order conditions:

$$\frac{r + \dot{p_I}}{p_I} - \frac{\dot{p_c}}{p_c} = \rho + \frac{\dot{c}}{c}$$

$$\lim_{t \to \infty} \frac{p_I \cdot k}{p_c \cdot c} \cdot e^{-\rho \cdot t} = 0$$

- The optimal rule is given by

$$p_c \cdot c = \rho \cdot p_I \cdot k$$

- Firm maximization in the intermediate sector leads to

$$p = r$$

- Price index for intermediates is the numeraire

$$\int_0^M p(z)^{1-\epsilon} \cdot dz = \int \mu \cdot p(z)^{1-\epsilon} \cdot dG = 1$$

World Equilibrium

- p is also the terms of trade
- Firms in the consumption and investment sectors take prices as given and choose factor inputs to maximize profits

$$\begin{aligned} p_c &= r^{1-\tau} \\ p_I &= \phi^{-1} \cdot r^{1-\tau} \end{aligned}$$

- Market clearing for capital, by Walras' Law, is equivalent to imposing trade balance

$$y = \mu \cdot p^{1-\epsilon} \cdot Y$$

where $Y = \int y \cdot dG$ is world income

World Dynamics

- The state of the world economy is fully described by a distribution of capital stocks. The law of motion for the world economy is given by

$$\begin{aligned} \dot{k}/k &= \phi \cdot r^{\tau} - \rho \\ r \cdot k &= \mu \cdot r^{1-\epsilon} \cdot \int r \cdot k \cdot dG \end{aligned}$$

- Define $x \equiv \dot{Y}/Y$ and $y_R \equiv y/Y$
- Setting the same growth rate for all countries, we obtain the steady-state cross section of rental rates as

$$r^* = (\frac{\rho + x^*}{\phi})^{1/\tau}$$

- The world distribution is stable because countries that accumulate more face lower terms of trade, reducing the interest rate and the incentives for further accumulation

World distribution of income

$$y_R^* = \mu \cdot (\frac{\phi}{\rho + x^*})^{(\epsilon-1)/\tau}$$

$$\int \mu \cdot (\frac{\phi}{\rho + x^*})^{(\epsilon-1)/\tau} \cdot dG = 1$$

$$\text{terms of trade} = p = (\frac{\mu}{y_R})^{1/(\epsilon-1)}$$

$$\text{rate of return} = \frac{r + \dot{p}_I}{p_I} - \frac{\dot{p}_c}{p_c} = \phi \cdot p^\tau$$

- For countries with greater patience and better economic policies, lower terms of trade are sufficient to ensure accumulation
- Lower terms of trade and better technology is translated into a greater relative income level y_R
- Countries with low values for ρ and high values for ϕ and μ will be richer
- The strength of diminishing returns depends on the volume of trade τ and the extent of specialization ϵ

Empirical Implications

- Relation between steady-state income and savings rate $(\frac{x^*}{\rho+x^*})$ is given

$$y_R^* = \mu \cdot \phi^{(\epsilon-1)/\tau} \cdot (\frac{s}{x^*})^{(\epsilon-1)/\tau}$$

- $\tau = 0.3$ (share of exports) and $\epsilon = 2.6$ (changes in terms of trade)
- The model's predictions for cross-country income differences are identical to those of the Solow model with $\alpha = 0.85$
- Yields a large elasticity of output to savings and generates cross-country income differences even larger than those observed in the data
- Furthermore, the speed of convergence is 1.1 percent a year, which is slower than observed in the data

Empirical Evidence

- Equation to be tested

$$\pi_t = (g_t - x_t)/(\epsilon - 1) + \Delta \ln \mu_t$$

Empirical Evidence

- Since changes in technology, as captured by $\Delta \ln \mu_t$ are directly correlated with changes in income, the above relationship will be biased
- A possible source of variation in growth would come from countries growing at different rates because they have started in different positions relative to their steady state income level and are therefore accumulating at different rates to approach their steady state. This should be orthogonal to technological change
- The estimating equation is

$$\pi_t = \delta \cdot g_t + Z_t' \cdot \omega + v_t$$

where g_t is instrumented for using

$$g_t = -\beta \cdot \ln y_{t-1} + Z_t' \cdot \theta + u_t$$

TABLE I

IV REGRESSIONS OF GROWTH RATE OF TERMS OF TRADE

	Main regression	Detailing schooling	Adding political indicat	Adding change in Sch	Adding change in Sch	Nonoil sample
	(1)	(2)	(3)	(4)	(5)	(6)
Panel A: Two-stage least squares						
GDP Growth 1965–1985	−0.595 (0.265)	−0.578 (0.261)	−0.458 (0.221)	−0.561 (0.248)	−0.455 (0.187)	−0.620 (0.354)
Years of schooling 1965	−0.001 (0.002)		−0.002 (0.002)	−0.000 (0.002)		−0.001 (0.002)
Years of primary schooling 1965		−0.002 (0.003)				
Years of secondary schooling 1965		−0.002 (0.006)				
Years of higher schooling 1965		0.019 (0.034)				
Log of life expectancy 1965	0.043 (0.024)	0.045 (0.024)	0.034 (0.021)	0.020 (0.027)		0.046 (0.030)
OPEC dummy	0.091 (0.009)	0.090 (0.009)	0.092 (0.009)	0.086 (0.010)	0.087 (0.009)	
War dummy			−0.013 (0.005)			
Political instability			0.007 (0.023)			
Log black market premium			−0.005 (0.012)			
Change in years of schooling 1965–1985				0.008 (0.004)	0.009 (0.003)	
Change in log of life expectancy 1965–1985				−0.000 (0.078)	−0.042 (0.045)	
Panel B: First-stage for GDP growth						
Log of GDP 1965	−0.019 (0.004)	−0.020 (0.004)	−0.024 (0.004)	−0.020 (0.004)	−0.020 (0.004)	−0.016 (0.004)
R^2	0.35	0.36	0.54	0.47	0.47	0.34
Panel C: Ordinary least squares						
GDP Growth 1965–1985	0.037 (0.106)	0.037 (0.107)	0.038 (0.107)	0.041 (0.112)	−0.005 (0.103)	0.116 (0.114)
N. of obs	79	79	70	79	79	74

"Growth Rate of Terms of Trade" is measured as the annual growth rate of export prices minus the growth rate of import prices. The OPEC dummy takes value 1 for five countries in our sample (Algeria, Indonesia, Iran, Iraq, and Venezuela). The political instability variable is the average of the number of assassinations per million inhabitants per year and the number of revolutions per year, the war variable is a dummy for countries that fought at least one war over the period 1965–1985, and the log black market premium is the average of the logarithm of the black market premium over the period 1965–1985. All the data are from the Barro-Lee data set.

Excluded instrument is log of output in 1965 in columns (1), (2), (3), and (4) and (6), while in column (5) excluded instruments are log of output in 1965, years of schooling in 1965, and the log of life expectancy in 1965.

Extensions

- **Add labour as a factor of production**
  - Production of consumption good requires labor
  - The determinants of the steady state distribution of income and world growth rate are still the same
  - Wages tend to be higher in rich countries (those with high μ and high ϕ). Follows from a greater demand for consumption by richer countries, increasing the demand for labor and wages
  - The cost of living tend to be higher in rich countries since consumption goods are more expensive there
- **Endogenous determination of the number of varieties**
  - Firms in all countries know how to produce an infinite mass of intermediates

$$\mu^* = \tau \cdot \frac{\rho + x^* \cdot [1 - (1-\gamma) \cdot (1-\tau)]}{\rho \cdot [\tau + \epsilon \cdot (1-\gamma) \cdot (1-\tau)] + \tau \cdot x^*}$$

  - Countries with low ρ endogenously specialize in the production of more goods

Product Development and International Trade, Grossman and Helpman (1989)

- This paper develops a multicountry, dynamic, general equilibrium model of product innovation and international trade to study the creation of comparative advantage through research and development (R&D) and the evolution of world trade over time
- World economy with 3 activities :
  - A "traditional" commodity produced under competitive conditions
  - A continuum of varieties of "modern" industrial product (horizontally differentiated
  - R&D that leads to the acquisition of the know-how needed to produce new brands of the industrial good
- Forward-looking potential producers conduct R&D and enter the product market whenever profit opportunities exist
- New products substitute imperfectly for the old, and prices, interest rates and the pattern of trade evolve over time as more commodities become available
- Trade has both intra-industry and inter-industry components
- International capital flows take place to finance R&D and in some circumstances, multinational corporations may emerge

Main Results

- If both R&D and the production of differentiated goods are more human capital-intensive activities than the production of a traditional good and if all activities bear Leontief production technologies, then the human capital-rich country will be a net exporter of differentiated products and an importer of the traditional good at every moment in time
- The model predicts a rising share of trade in world GNP, at least when R&D is the most human capital intensive of the three activities
- The human capital-rich country has both a greater incentive to invest and a greater incentive to save (per capita)
- For certain compositions of factor endowments, the extent of multinationality, as measured by output, employment of subsidiaries, or the number of multinational firms, expands over time, at least initially and as the world economy approaches the steady state

A Dynamic Model of R&D

Consumers

$$U = \int_0^\infty e^{-\rho t} \log u(.) dt$$

$$u = [\int_0^n c_x(i)^\alpha di]^{s_x/\alpha} c_y^{1-s_x}, \qquad \alpha, s_x \in (0,1)$$

- The consumer's maximization problem can be solved in two stages
- First stage : Consumer chooses $\{c_x(i)\}$ and c_y to maximize u(.)

$$\begin{aligned} c_x(i) &= s_x E \frac{p(i)^{-\sigma}}{\int_0^{n(t)} p(j)^{-\sigma} dj} \\ c_y &= (1-s_x)E/p_y \end{aligned}$$

- The budget constraint is given by

$$\int_t^\infty e^{-[R(\tau)-R(t)]} E(\tau) d\tau = \int_t^\infty e^{-[R(\tau)-R(t)]} I(\tau) d\tau + A(t)$$

- Second stage : The F.O.C. for maximizing U s.t. the B.C. is $\frac{\dot{E}}{E} = \dot{R} - \rho$

A Dynamic Model of R&D

Producers

- $\phi_x(w_f) \rightarrow$ unit cost of production
- $\phi_n(w_f) \rightarrow$ cost of developing a brand
- The number of potential products is infinite
- Entrepreneur takes as given the aggregate level $E(t)$ and the pricing policy of competitors (of measure n(t))and sets the price of his brand so as to maximize profits
- In a symmetric equilibrium, output per variety $x(i) = x$and prices $p(i) = p$ $\forall i \in [0, n(t)]$ satisfy

$$x = s_x \frac{E}{pn}$$

and

$$\alpha p = \phi_x(w_f)$$

- The resulting operating profits per variety are

$$\pi = (1 - \alpha) s_x \frac{E}{n}$$

A Dynamic Model of R&D

Producers continued

- An entrepreneur has perfect foresight regarding the evolution of spending E and the number of firms n
- In equilibrium, we must have

$$\int_t^{\infty} e^{-[R(\tau)-R(t)]}\pi(\tau)d\tau = \phi_n[w_f(t)]$$

- Prices are normalized so that

$$1 = \phi_n[w_f(t)]$$

- This choice of numeraire implies that

$$\pi(t) = \dot{R}(t)$$

- The traditional good is produced subject to CRS. Therefore,

$$p_y = \phi_y(w_f)$$

A Dynamic Model of R&D

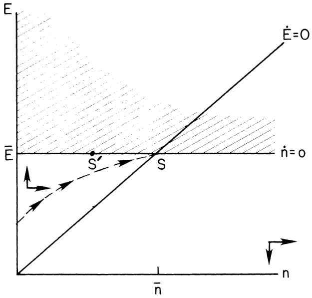
Integrated Equilibrium

- Differential equation for the rate of change of spending

$$\frac{\dot{E}}{E} = (1-\alpha)s_x\frac{E}{n} - \rho$$

- Differential equation for the change in the number of brands

$$\dot{n} = v(E)$$



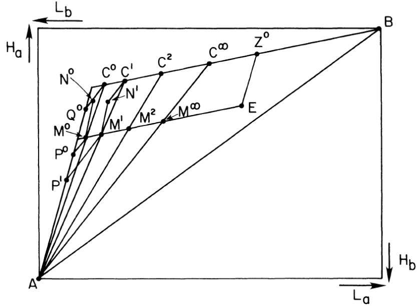
A Dynamic Model of R&D

Special Case

- Two factors of production - skilled and unskilled labor
- The traditional sector is the least human capital intensive and the overall human capital to labor ratio satisfies $a_{Hy}/a_{Ly} < H/L < a_{Hj}/a_{Lj}$ for $j = x, n$
- Proposition
  - (a) $\dot{w}/w > \dot{p_y}/p_y > \dot{p}/p > \dot{r}/r$
  - (b) $\dot{p}_y > 0$
  - (c) $\dot{r} < 0$
  - (d) $\dot{p} > 0$ if and only if $a_{Hn}/a_{Ln} > a_{Hx}/a_{Lx}$
  - (e) $\partial\dot{n}/\partial t < 0$
  - (f) $\dot{X} > 0$
  - (g) $\dot{Y} < 0$ if and only if $a_{Hn}/a_{Ln} > a_{Hx}/a_{Lx}$
  - (h) $\dot{E} > 0$

Pattern of trade in a two-country world

- Two countries a (human capital rich) and b with common tastes and technologies
- Factor services and blueprints aren't tradeable



- A sufficient condition for factor price equalization to obtain along the path of the trade equilibrium is that the human capital to labor ratios of the two countries be bounded by the less human capital-intensive among R&D and production of differentiated products

- At an arbitrary point in time, full employment conditions are given by

$$\begin{aligned} L_i &= a_{Ly} L_i + a_{Lx} \frac{s_x E}{np} n_i + a_{Ln} \dot{n}_i \\ H_i &= a_{Hy} L_i + a_{Hx} \frac{s_x E}{np} n_i + a_{Hn} \dot{n}_i \end{aligned}$$

  for i=a,b
- Combining the above two equations :

$$\dot{n}_i + b(t) n_i = k_i$$

- Solving the above equation :

$$n_i(t) = k_i \int_0^t e^{-\int_z^t b(\tau) d\tau} dz$$

- $\dot{n}_a(t)/\dot{n}_b(t)$ is a constant and equal to k_a/k_b, as is the ratio of the total outputs of modern goods X_a/X_b

- Ratio of outputs of traditional goods

$$\frac{Y_a(t)}{Y_b(t)} = \frac{L_a}{L_b}\frac{[h_c(t) - (H_a/L_a)]}{[h_c(t) - (H_b/L_b)]}$$

where $h_c(t)$ is the human capital to labor ratio in the composite activity

- Can be shown that $\frac{Y_a(t)}{Y_b(t)} < \frac{L_a}{L_b}$
    - $h_c(t)$ is bounded by $h_n \equiv a_{Hn}/a_{Ln}$ and $h_x \equiv a_{Hx}/a_{Lx}$
    - Under conditions for FPE, each of these exceed H_i/L_i
- Ratio of demands for the traditional goods

$$\frac{E_a(t)}{E_b(t)} = \frac{E_a(0)}{E_b(0)} = \frac{L_a}{L_b}\frac{\int_0^\infty e^{-R(t)}[w(t) + r(t)h_a]dt}{\int_0^\infty e^{-R(t)}[w(t) + r(t)h_b]dt}$$

- Can be shown that $c_{ya}(t) > Y_a(t)$
    - $h_a/h_b > 1$ implies that $\frac{E_a(0)}{E_b(0)} > \frac{L_a}{L_b}$
    - $\frac{c_{ya}(t)}{c_{yb}(t)} > \frac{L_a}{L_b}$
    - Follows immediately that $\frac{c_{ya}(t)}{c_{yb}(t)} > \frac{Y_a(t)}{Y_b(t)}$
    - Use market clearing condition $c_{ya}(t) + c_{yb}(t) = Y_a(t) + Y_b(t)$

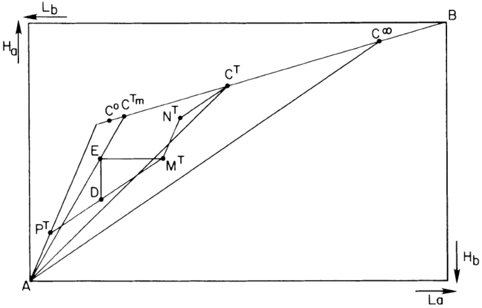
- Each differentiated product is manufactured in only one country but consumed worldwide$\Rightarrow$ The direction of trade in individual products is clear-cut
- Aggregate pattern of trade $\longrightarrow \frac{c_{xa}}{c_{xb}} < \frac{X_a}{X_b}$
  - Have already shown that $\frac{X_a}{X_b}$ is constant over time
  - So is $\frac{c_{xa}}{c_{xb}}$
  - Can't have $\frac{c_{xa}}{c_{xb}} > \frac{X_a}{X_b} \Rightarrow$ Country A
- The volume of trade as a fraction of world spending

$$\frac{VT}{E} = (1 - s_x)(\frac{Y_b}{Y} - s_b) + s_x(s_a\frac{X_b}{X} + s_b\frac{X_a}{X})$$

- If product development is human capital intensive relative to the production of differentiated products, the volume of world trade grows faster than world spending and GDP

Multinationals

- Headquarter services are produced with human capital only and the production plants use these services and unskilled labor only
- Headquarter services must be provided in the country in which the brand was developed



Learning by Doing and the Dynamic Effects of International Trade, Young (1991)

- Develops a model in which endogenous growth is generated by learning-by-doing
- Learning by doing incorporates two important characteristics :
  - Substantial spillovers in the development of knowledge across industries
  - Existence of strong diminishing returns in the learning-by-doing process
- Learning by doing is bounded in each good
- The knowledge generated by learning-by-doing is in the public domain
- The unit output labor requirement of each good evolves over time
- At any given time, learning-by-doing will have been exhausted in a subset of goods, but will continue in the remainder

Learning by Doing and the Dynamic Effects of International Trade, Young (1991)

- Over time, growth will involve the production of a changing basket of goods, with both the variety and quantity of goods consumed increasing
- Although production takes place under conditions of perfect competition, there are gains from increasing variety
- In the absence of international diffusion of knowledge, the effect of trade on technical progress and growth will depend on which goods the economy specializes in based on its static comparative advantage
- Trade improves the intertemporal welfare of consumers in economies in which it accelerates technical progress and growth
- However even in a country in which technical progress slows down, consumers might experience an improvement in welfare

The Model

- Large number of consumers
- Perfectly competitive firms
- Goods indexed by s along $[B, \infty)$ ordered according to the degree of sophistication
- Labor is the sole factor of production
- Unit labor requirements are $a(s, t)$ where $a(s, t)$ is continuous and $\lim_{t \to \infty} a(s, t) = \infty$
- Lower bound on potential input requirements, $\overline{a}(s)$ which is non-increasing in s
- For each $t \exists\ s_t^*$ s.t. $a(s, t) \geq a(s_t^*, t)$

Learning-by-doing

- Bounded learning-by-doing :
  - If $a(s,t) = \overline{a}(s)$, then
$$\partial a(s,t)/\partial t = 0$$
  - If $a(s,t) > \overline{a}(s)$, then
$$\frac{\partial a(s,t)/\partial t}{a(s,t)} = -\int_B^\infty B(s,v,a(v,t)/\overline{a}(v))L(v,t)dv$$

  where $L(v,t)$ is the amount of labor devoted to the production of good v at time t and $B(s,v,a(v,t)/\overline{a}(v))$ are the learning-by-doing coefficients
- $B(s,v,a(v,t)/\overline{a}(v)) \geq 0$ for all v
- $B(s,v,1) = 0$
- $B(s,v,a(v,t)/\overline{a}(v)) > 0$ for all $v \in (s - \alpha_s, s + \alpha_s)$ such that $a(v,t) > \overline{a}(v)$
- $\sup_s \sup_v \; B(s,v,a(v,t)/\overline{a}(v)) < \infty$
- $B(s,v,a(v,t)/\overline{a}(v))$ is continuous in s

Consumer optimization

- Instantaneous utility function :

$$V(t) = \int_B^\infty U(C(s,t))ds \qquad \text{with } U\prime(0) < \infty, U(0) = 0$$

- $U(\cdot)$ is strictly concave and continuously differentiable $\Rightarrow$ A strong, but not unbounded, preference for variety
- No storage technology
- Consumers inelastically supply one unit of labor for wage W
- Budget constraint is given by

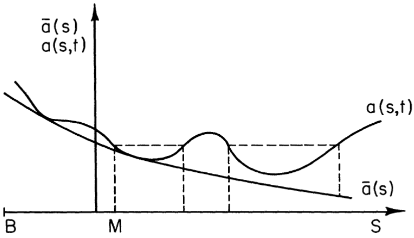
$$1 \geq \int_B^\infty a(s,t)C(s,t)ds$$

  where we use the relation $P(s,t) = Wa(s,t)$
- The first order condition is given by

$$\frac{U'(C(s,t))}{a(s,t)} = \frac{U'(C(v,t))}{a(v,t)} \quad \forall\ s, v \text{ such that } C(s,t) > 0,\ C(v,t) > 0$$

- Since all goods enter symmetrically into the utility function, the consumer should consume the cheapest of all goods, i.e. good s_t^*
- For finite quantities of good s_t^* consumed, there exists some good M such that as the price of goods approached $a(M,t)$ from below, consumption goes to zero
- For all goods x such that $a(x,t) \geq a(M,t)$, consumption is zero
- For all goods s whose price is less than $a(M,t)$, consumption quantities are determined by

$$U'(C(s,t)) = a(s,t)U'(0)/a(M,t)$$



Growth Rate

- Growth rate is measured by

$$g(t) = \frac{\int_B^\infty a(s,t)\partial X(s,t)/\partial t ds}{\int_B^\infty a(s,t)X(s,t)ds} - \frac{dL(t)/dt}{L(t)}$$

  where $L(t)$ equals the population at time t and $X(s,t)$ equals aggregate output of good s at time t
- Using $\int_B^\infty a(s,t)X(s,t)ds = L(t)$, it follows that

$$g(t) = \frac{\int_B^\infty -\partial a(s,t)/\partial t X(s,t)ds}{L(t)}$$

- Allowing S_L to denote the set of goods s such that $C(s,t) > 0$ and $a(s,t) > \overline{a}(s)$ and using $X(s,t) = C(s,t)L(t)$ under autarky

$$g(t) = \int_{s \in S_L} a(s,t)C(s,t)(\int_B^\infty B(s,v,a(v,t)/\overline{a}(v))L(v,t)dv)ds$$

- It can be shown that if $\lim_{s\to\infty} \overline{a}(s) = 0$, then $\lim_{s\to\infty} V(t) = \infty$

Dynamic Effects of International Trade

- Intertemporal utility function :

$$P = \int_t^{\infty} V(x) e^{-\rho(x-t)} dx$$

where $V(x)$ denotes instantaneous utility at time x, which is given by

$$V(x) = \int_B^{\infty} \log\left(C(s,x) + 1\right) ds$$

where $C(s,x)$ denotes consumption of good s at time x and B is a large negative number

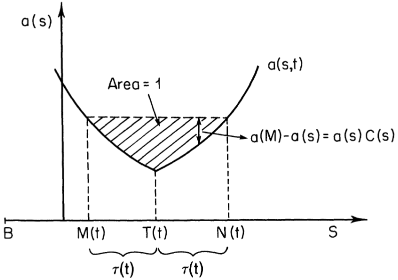
- The lower bound on potential unit labor requirements is given by $\overline{a}(s) = \overline{a}e^{-s}$
- $B(s, v, a(v,t)/\overline{a}(v)) = 2$ for all s and v such that $a(v,t)/\overline{a}(v) > 1$
- Unit labor requirements are given by

$$a(s,t) = \overline{a}e^{-s} \ \forall \ s \leq T(t); \ \ a(s,t) = \overline{a}e^{-T(t)} e^{s-T(t)} \ \forall \ s \geq T(t)$$

with $T(t)$ evolving according to the learning-by-doing equation

$$\frac{dT(t)}{dt} = \int_{T(t)}^{\infty} L(s,t) ds$$

Dynamic Effects of International Trade



- For all goods s which are consumed in positive quantities :

$$a(s)C(s) = a(M) - a(s)$$

- Since $a(s)$ is symmetric around T, to each M there corresponds a N such that $a(M) = a(N)$
- Denoting $T - M = N - T = \tau$, it can be shown that

$$1 > d\tau/dT > 0$$

Dynamic Effects of International Trade

Autarky Growth Rate and Rate of Technical Progress

- Rate of technical progress :

$$\frac{dT(t)}{dt} = \int_{T(t)}^{\infty} L(s,t)ds = \int_{T(t)}^{N(t)} a(s,t)X(s,t)ds = \frac{L(t)}{2}$$

- Growth rate of GDP per capita :

$$\begin{aligned} g(t) &= \frac{\int_B^{\infty} -\partial a(s,t)/\partial t X(s,t)ds}{L(t)} \\ &= \frac{L(t)}{2} \end{aligned}$$

Dynamic Effects of International Trade

Free trade : Preferences and Technology

- Two economies - LDC with population L and DC with population L^*
- Identical preferences
- $T^* > T$
- No spillovers of knowledge between the two economies and there's no international borrowing or lending
- Let $X = T^* - T$
- Let W be the numeraire and

$$\omega = W^*/W, \qquad p(s) = P(s)/W$$

- Under perfect competition, if the good is produced in the LDC, then $p(s) = a(s)$, whereas if it is produced in the DC, then$p(s) = \omega a^*(s)$
- Each good s is produced by the least cost producer

Dynamic Effects of International Trade

Free trade : General Equilibrium

- At each time t, consumers in both the LDC and the DC only consume goods priced at less than $p(M)$ and $p(M^*)$ respectively

$$p(s)C(s) = p(M) - p(s) \qquad p(s)C^*(s) = p(M^*) - p(s)$$

- Denote the set of goods produced in the DC and the LDC by GDC and GLDC
- M is determined by the B.C. of individuals in the LDC :

$$1 = \int_{s \in GDC} p(s)C(s)ds + \int_{s \in GLDC} p(s)C(s)ds$$

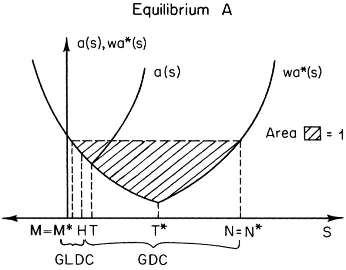
- M^* is determined by the B.C. of individuals in the DC :

$$1 = \int_{s \in GDC} p(s)C^*(s)ds + \int_{s \in GLDC} p(s)C^*(s)ds$$

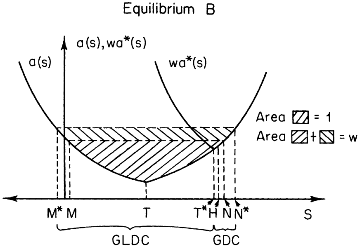
- The relative wage is determined by the trade balance condition :

$$\int_{s \in GDC} Lp(s)C(s)ds = \int_{s \in GLDC} L^* p(s)C^*(s)ds$$

- $\omega = 1$

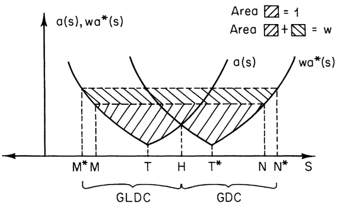


- $\omega = e^{2X}$

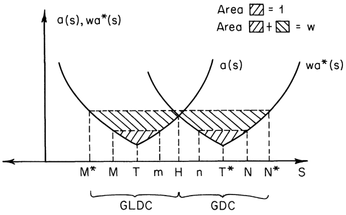


- $1 < \omega < e^{2X}$

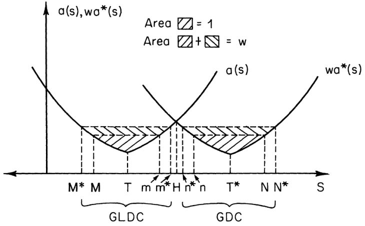
Equilibrium C



Equilibrium D



Equilibrium E



Technical Progress and Equilibrium Dynamics

- Equilibrium A :

$$\frac{dT}{dt}(A) = 0 \qquad \frac{dT^*}{dt}(A) = \frac{L^* + L}{2}$$

- Equilibrium B :

$$\frac{dT}{dt}(B) = \frac{L - \omega L^*}{2} \qquad \frac{dT^*}{dt}(B) = L^*$$

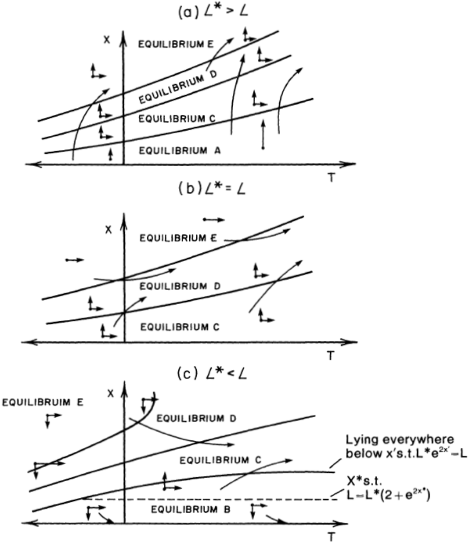
- Equilibrium C,D :

$$\frac{L}{2} > \frac{dT}{dt}(C, D) > 0 \qquad \frac{dT^*}{dt}(C, D) > \frac{L^*}{2}$$

- Equilibrium E :

$$\frac{dT}{dt}(E) = \frac{L}{2} \qquad \frac{dT^*}{dt}(E) = \frac{L^*}{2}$$

- To summarize, we have $\frac{L}{2} \geq \frac{dT}{dt} \geq 0$ and $\frac{dT^*}{dt} \geq \frac{L^*}{2}$



(a) $L^* > L$
X
T
EQUILIBRIUM E
EQUILIBRIUM D
EQUILIBRIUM C
EQUILIBRIUM A
(b) $L^* = L$
X
T
EQUILIBRIUM E
EQUILIBRIUM D
EQUILIBRIUM C
(c) $L^* < L$
X
T
EQUILIBRUIM E
EQUILIBRIUM D
EQUILIBRIUM C
EQUILIBRIUM B
Lying everywhere below x′s.t.$L^*e^{2x'}=L$
X*s.t. $L=L^*(2+e^{2x^*})$

Growth of GDP per capita and Intertemporal Welfare

- The growth rates are given by

$$g(t) = \frac{2(dT/dt)^2}{L} \leq \frac{L}{2} \qquad g^*(t) = \frac{2(dT^*/dt)^2}{L^*} \geq \frac{L^*}{2}$$

- Effect of free trade on welfare
  - Unambiguously improve the intertemporal utility (i.u.) of DC consumers if the DC is never overtaken by the LDC
  - Improve the i.u. of DC consumers even if the DC is overtaken by the LDC, provided that the time spent along the catch-up path is sufficiently large
  - Unambiguously improve the i.u. of LDC consumers if the LDC's population is sufficiently small relative to that of the DC
  - Unambiguously reduce the i.u. of LDC consumers if the LDC's population is sufficiently large relative to that of the DC and it is unable to overtake the DC